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A. D. MELVIN, CHIEF OF BUREAU.

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

I. VARIATION IN ANNUAL EGG PRODUCTION.

BY

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LETTER OF TRANSMITTAL.

U. S. Department of Agriculture,
Bureau of Animal Industry,
Washington, D. C., September 29, 1908.

Sir: I have the honor to transmit herewith and to recommend for publication in the bulletin series of this Bureau a manuscript entitled "A Biometrical Study of Egg Production in the Domestic Fowl. I. Variation in Annual Egg Production."

This paper is the first in a proposed series dealing with a biometrical study by Drs. Raymond Pearl and Frank M. Surface of the results obtained at the Maine Agricultural Experiment Station in breeding for egg production during the past nine years, the work during the past three years having been done in cooperation with this Bureau. The subjects to be discussed in the several parts are briefly outlined in the introduction.

While the investigation is the joint work of Doctors Pearl and Surface, Doctor Pearl is responsible for the actual writing of the whole paper.

Very respectfully,

A. D. Melvin,
Chief of Bureau.

Hon. James Wilson,
Secretary of Agriculture.

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CONTENTS.

Page.

Introduction 9

Material 11

Statistical methods used 20

Variation in egg production in Barred Plymouth Rocks 24

  Analytical discussion of variation in egg production 30

  Changes in egg production between 1899 and 1907 38

    Changes in mean egg production 39

    Changes in the variability of egg production 43

    Changes in other constants 46

Variation in egg production in White Wyandottes 49

Egg production in other breeds of fowls 50

The influence of certain housing conditions on annual egg production 58

Egg production in the second laying year 63

Selection and egg production 67

Summary of results and conclusions 73

Appendix I 76

ILLUSTRATIONS.

Page.

FIG. 1. Variation in annual egg production, 1903–4 33
2. Variation in annual egg production, 1904–5 33
3. Variation in annual egg production, 1905–6, 50-bird pens 34
4. Variation in annual egg production, 1905–6, 100-bird pens 34
5. Variation in annual egg production, 1905–6, 150-bird pens 35
6. Variation in annual egg production, 1906–7, 50-bird pens 35
7. Variation in annual egg production, 1906–7, 100-bird pens 36
8. Variation in annual egg production, 1906–7, 150-bird pens 36
9. Change in mean annual egg production, 1899–1907 42
10. Changes in annual egg production. Standard deviations, 1899–1907 45
11. Changes in annual egg production. Coefficients of variation, 1899–1907 46
12. Changes in annual egg production. Skewness, 1899–1907 47
13. Changes in percentages of extreme variates in egg production ,1899–1907 48
14. Mean annual egg production with different amounts of floor space per bird 60
15. Variability in annual egg production with different amounts of floor space per bird 62
16. Frequency polygons for egg production during the first and second years of laying 64
17. Change in intensity of selection, 1899–1907 70

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

I. VARIATION IN ANNUAL EGG PRODUCTION.[a]

INTRODUCTION.

In 1898 the Maine Agricultural Experiment Station began the use of trap nests[b] in connection with its poultry investigations. Since that time such nests have been used to obtain records of the egg production of all hens kept by the station. In consequence there has now accumulated a remarkable—in some respects, indeed, a unique—collection of exact statistical data regarding the phenomenon of egg production in certain breeds of the domestic fowl. So far as we are aware no other set of records of egg production exists which compares with this either in regard to the number of birds included in the statistics or in length of time covered by continuous daily records. On this account it has seemed desirable that these statistics be submitted to such thorough biometrical analysis as their intrinsic value obviously warrants. Hitherto practically no analytical study of egg production statistics (either the present collection or others) has ever been made. The need of such study, however, is apparent if it be considered that only by analysis of comprehensive statistics of egg production can we hope to reach a definite formulation of the fundamental biological laws underlying the process itself. The existing knowledge of egg production in fowls is on much the same plane as that regarding milk production in dairy cattle, for example. In both cases empirical methods have led to a knowledge of how to feed, care for, and, to some extent, breed the animals in order to get a high production of the thing desired. But of the biological factors concerned in improved egg production or milk secretion, and the laws according to which these factors operate, relatively little is definitely known.

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[a] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 2.

[b] The particular type of trap nest used was designed at this station and has been described and figured in the Annual Report of the Maine Agricultural Experiment Station for 1898, pp. 141–143, in various bulletins of the station issued subsequently, and in Bulletin 90 of the Bureau of Animal Industry. Recently a new and improved trap nest has been devised and has replaced the earlier form of nest in the experimental work of the station. For a description of this new nest see Bulletin 159 of the Maine station.

In the case of the fowl the frequency of ovulation in a given period of time has been enormously increased in the domesticated hen as compared with the wild *Gallus bankiva*. It is commonly said that the great egg production of present-day "improved" varieties is due to the long-continued process of selection for this quality or characteristic which has been going on since domestication of fowls began. Possibly this is true in a broad sense, but it does not tell us anything about the underlying physiological factors which are the things that have been improved by this selection, or even what these factors are. Yet it is about them that we must know if we are to make advance in the direction of increasing egg production. Selection must be directed with ever-increasing precision as it goes on if improvement is to continue, and to make it more precise the breeder must have more detailed knowledge regarding the thing for which he is selecting, and how this is inherited.

A point of particular biological interest in connection with statistics of egg production lies in the relation which they bear to statistics of fecundity and fertility in other organisms. In nearly all animals other than birds the only possible measure of fecundity is based on the number of young produced. But the number of young produced depends not alone on the mother's inherent reproductive capacity, but also on her opportunities for getting the ova she produces fertilized. Statistics such as those with which we are here dealing are statistics of *ovulation*, and they make it possible to study this single phase of the problem of fecundity free from disturbing factors.[a]

The general purpose of this study is to make as thorough an analysis of the phenomenon of egg production in the domestic fowl as may be possible. The detailed statistics which we have available are so extensive and the specific problems to be attacked are so numerous that it has seemed advisable to publish the results of the investigation in separate parts. The general topics which it is tentatively proposed shall be discussed in the several parts are briefly indicated in the following outline:

I. Variation in egg production based on the study of the records of annual production. (The present paper.)

II. The egg production in the several months of the year and the relation of monthly to total annual egg production. This portion of the investigation has as its aim the "dissection" of the gross annual curves of variation in egg production.

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[a] For a more detailed discussion of this point see Maine Agricultural Station Bulletin 166.

III. The effect of environmental influences on egg production.

IV. The relation of certain internal factors to egg production.

V. General summary and discussion of the results of the investigation.

The present paper (Part I) is in the first instance a necessary preliminary to those which are to follow. Its main purpose is, in a sense, to define the problems with which the subsequent parts of the work have to do. In it are set forth the facts regarding variation in first-year egg production.[a] These facts we shall attempt to analyze in later portions of the work. The subject of variation in first-year egg production has, however, a number of features of interest on its own account, and presents certain problems which can be answered without treating the more detailed statistics. First in importance is to learn definitely what the characteristics of variation in egg production in the individual are. To what kind of variation curves do statistics of ovulation lead? What are the biometrical constants of such curves? How does variation in annual egg production resemble or differ from variation in other characters, and in particular variation in fertility and fecundity? How are the variation constants for first-year production affected by environmental influences? In what way and to what extent has annual egg production been influenced by selective breeding? What relation exists between first-year and second-year production? These are some of the specific problems which will be considered in the paper.

MATERIAL.

Before undertaking a detailed account of the group statistics it will be well to make some statement regarding the homogeneity—from the biological standpoint—of the material from which these records have been obtained. As has been stated in publications of the Maine Agricultural Experiment Station, there has been conducted at this station during the last nine years an investigation under the direction of the late Prof. Gilbert M. Gowell on the subject of breeding hens for increased egg production. The statistical material with which the present paper has to do comprises the egg records obtained in the course of this breeding investigation. It will not be necessary to state here in detail the plan of this breeding work, since that has already been thoroughly done in Bulletin 90 of the Bureau of Animal Industry. In that bulletin (pp. 7–15) is given a detailed account of the methods used in this breeding work up to the year 1906. A general statement regarding this work will suffice for our present purpose.

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[a] The term "first-year egg production" will be used throughout as a convenient verbal contraction in the place of "the number of eggs produced by pullets in the year beginning the first day of the November following their hatching and ending a year from that date."

At the beginning of the experiment in 1898 three breeds of hens were used: (1) A strain of Barred Plymouth Rocks which had been bred by Professor Gowell for a period of some twenty-five years; (2) a strain of White Wyandottes which had been bred at the station for some years; and (3) a strain of Light Brahmas which similarly had been bred for some time at the station.

All the birds in hand at the beginning of the breeding experiment in 1898 were purebred in the ordinary sense and had for some years been bred by the same person—Professor Gowell. This insures that in regard to racial purity they form material which is, from a biological standpoint, very homogeneous. This is particularly true of the Barred Plymouth Rocks, which furnish the bulk of the statistical material with which the present investigation has to do. After October, 1902, no variety other than the Barred Plymouth Rock was used in the egg-production work. The White Wyandottes were used from 1898 to 1902, and egg records for them between these dates are available for study. On account of the small number of Light Brahmas available, together with the fact that they were dropped early in the course of the breeding experiment, they have not been included in the present investigation.

In breeding the birds from 1898 down to the present time the practice as stated in the published reports of the work has been to use each year, as mothers of the stock produced in that year, only hens which have, between November 1 of the year in which they were hatched and November 1 of the next following year, laid 160 or more eggs. The fathers of all stock raised since the breeding season of 1900 have been the sons of mothers whose production in their first laying year was 200 eggs or over. It will be seen again that this breeding practice has been such as to lead to great homogeneity of the statistical material from a biological standpoint. It is stated, however, that except in the first years of the breeding there has been no "close" inbreeding. No birds as closely related as first cousins are said to have been bred together since the first year of the breeding experiment.

The nature of the statistical records with which the present paper deals is briefly as follows: The first laying year of a hen has been arbitrarily taken to be from November 1 of the year in which she is hatched to November 1 of the following year. The "first year's egg record" of a hen, then, consists of the trap-nest record of the total number of eggs produced by that hen within such a year. Such "first-year" records constitute the basic material for Part I of this study.

Some important questions suggest themselves here. Has the "first-year" egg production any particular biological significance as compared with the egg production in any other (shorter or longer) period in the life of the bird? To what extent is there justification for taking first-year production as a measure of what might be called the total ability of a bird to produce eggs? In other words, why is annual egg production taken as the unit of the study of variation in this character? The answer to the last question is simplest: We are using annual production because it is the longest time unit for variation in egg production for which we have any extensive data available. A larger unit might, on some grounds, be desirable, but in such case as this the biometrician can only take the data as he finds them. It is obvious that the egg production in the first year can not be taken as a precise measure of the total number of eggs which hens would produce if allowed to live until natural death occurred. Whether it is a good measure or not depends on the degree to which first-year production is correlated with this unknown total possible production. No data exist from which this correlation can be precisely determined, but there are reasons to believe that if it could be evaluated it would be found to have a fairly high value. The chief of these reasons lies in the fact that in the domesticated fowl the average egg production of the first year is never equaled in any subsequent year, though with individual birds this relation may be changed. Thus, it is shown on page 65 that the mean production of 66 birds in their second laying year was but 61.7 per cent of that in their first year. That the average production falls off further in the third and subsequent years is known to be a fact. In other words, the first-year production, on the average, makes up a very considerably larger part of the unknown total production than does that of any other equal period of time. Hence we should expect it to be most highly correlated with that total.

The general conclusion which we have reached after careful consideration of the matter is that first-year production is to be regarded from the theoretical standpoint as a good measure of total possible production. It is a sufficiently large unit to include the influence of seasonal climatic changes, as well as those influences dependent on cyclical organic changes, such as, for example, original mating season, time of molting, and the like. From the practical standpoint first-year production is, for obvious reasons, in some respects the best unit which could be taken. It is highly desirable, however, that definite trap-nest records be obtained in considerable numbers for the egg production of the same birds during their entire lifetime or at least until they cease entirely to produce eggs. It is hoped that in time it will be possible to accumulate such records at the Maine station.

Another point to be noted is that the use of the term "first-year egg production" for the November-to-November laying is, in a sense, entirely arbitrary. Pullets very often begin to lay before

November 1 of the year in which they are hatched. Again, many pullets do not begin to lay until in December or later. They will not have laid a full year at the end of the following October. It is manifestly impossible to deal on any extensive scale with the actual first twelve months' egg production of each individual hen. One feasible procedure is to do as was done in this breeding investigation, namely, to take a calendar year as the unit of record, and choose the beginning of the year at a point which about averages the varying times of beginning egg production in the different individuals of the flock.

In extracting the egg records from the original record books certain complications arise which, for the sake of uniformity in the statistical treatment, we have had to decide in an arbitrary fashion. In the first place, it will always happen, of course, that death will prevent the completion of the "first laying year" by some of the birds which begin it. What shall be done with such records made incomplete by death? After careful study of the point it has been decided to omit entirely from our statistical treatment all records known to have been made incomplete by death. This is clearly the most logical method of dealing with such incomplete records which can consistently be applied. The same treatment has been given to the incomplete records of birds which were stolen during their first year. In the earlier years of the work it several times happened that birds were stolen after having completed only a part of the first year's record. All the records for stolen birds have been excluded from our statistics.

Another difficulty, and one in a sense more serious than either of those just mentioned, arises from the fact that hens do not invariably lay in the trap nests. It occasionally happens that birds will lay on the floor of the pen. Of course it then becomes impossible to connect the egg with the bird which laid it. When the study of these records was first undertaken it was for a time thought that this difficulty was a very serious one, and one which could not properly be allowed for in treating the data. Further study of the matter, however, led to the conclusion that this was not the case. In order that any conclusion regarding individual variation in egg production, or regarding the comparative production of flocks in different years or under different housing conditions, should be affected by the existence of unrecorded eggs laid on the floor, two conditions are necessary: (1) That the tendency to lay on the floor be not distributed at random (in the technical sense) among all the birds, and (2) that the total absolute amount of "floor" egg production should vary widely from year to year and in different pens. Are these conditions realized, as a matter of fact? We may discuss the two separately.

If it were a regular, constant, and undetected habit of some particular birds to lay outside the trap nests, and such birds formed a considerable portion of the flock, we could obviously draw no conclusions from trap-nest records. Such, however, appears not to be the case. Those who have had the birds in charge inform us that it has always been possible without any great difficulty to learn when a particular hen was beginning to form the habit of laying on the floor. Such hens when detected were trained, usually with comparative ease, to lay in the nests. A hen was never allowed to continue the floor-laying habit. All the available evidence and our own observations have convinced us that, could we know the birds that lay eggs outside the nests we should find them to be a random sample of the flock. That is to say, any bird is as likely as any other to lay an egg on the floor instead of in a trap nest. This being the case, the fact that there are eggs produced outside the nests will obviously not affect any conclusions respecting individual variation in egg production, if, as is the case, we deal with relatively large numbers of individuals.

Has the total number of eggs laid outside nests varied widely at different times during the period covered by the investigation, or has it been relatively constant? All the evidence that we have indicates that it has been relatively constant. Unfortunately, there exist no precise data on this point except for a single year (1904–5) out of the whole period covered by the statistics. We are informed by Mr. Walter Anderson, who has had immediate charge of the trap-nest records, that (*a*) the absolute amount of "floor" egg production has always been small, and (*b*) that in relation to the total number of eggs produced at any time the number laid on the floor has always been proportionately about the same. This means that figures for egg production based on the trap-nest records are absolutely a little too small in every case, but relatively are unaffected. It is relative figures with which we are chiefly concerned. It will obviously make no difference in our conclusions as to the comparative egg production of two years, say, if before making the comparison we add 2 per cent to the egg record for each year to cover the eggs laid on the floor.

The general conclusion, then, is that so far as the objects of the present investigation are concerned the unrecorded "floor" egg production is of no practical consequence. In the future poultry work of the Maine station an exact record will be kept of the eggs laid outside the nests.

It has further been decided to exclude from the statistical analysis the very few birds which, in the course of the trap-nesting experience of the station, have apparently laid no eggs at all in their first year. The number of such birds is so small (probably not 10 in the whole experience of the station) that it makes no significant difference in the statistical constants whether they are included or not. The specific reason for excluding such birds lies in an element of uncertainty in respect to the early records regarding them. That is, it is not possible now to be certain, in each of the cases where a bird has no record, whether the absence of record is due to the fact that she laid no eggs or to some other cause.

With this account of the general features of the statistics in hand, we may pass to the consideration of the records for particular years. We have decided to begin our discussion of the records with the data for the laying year 1899–1900; that is, with the records for birds hatched in April and May of the year 1899. The raw statistical data for this year have already been published.[a] They include records for the complete year (November 1, 1899, to November 1, 1900) for 70 Barred Plymouth Rocks and for the same number of White Wyandottes. An account of the housing and treatment of the birds for this year is given in the bulletin just referred to (pp. 31 and 32). So far as appears from the records, this laying year was in all respects a normal one. No serious accident or change of methods and management interfered in any way with the normal course of egg production.

The records for the laying year 1900–1901, that is, for birds hatched in April and May, 1900, have also been published.[b] Complete records for this laying year are available for 85 Barred Plymouth Rocks and 72 White Wyandottes. So far as may be judged from the records themselves and from the statements in Bulletin 79 (pp. 36 and 40) this may be considered to have been a normal laying year.

The egg records for the laying year 1901–2 have been published.[c] On account of lack of space relatively few pullets were carried over that winter. Complete records are available for but 48 Barred Plymouth Rocks and 33 White Wyandottes. The Barred Plymouth Rocks of that year were the first pullets sired by males bred from 200-egg-producing mothers to be tested in the trap nests. So far as we know this was a normal laying year.

The records for the years 1902–3 to 1906–7, inclusive, have never been published and were extracted by the authors of this paper from the original egg-record books in the archives of the station. In the year 1902–3, 147 of the Barred Plymouth Rock pullets tested in the trap nests completed first-year records. The egg production during this year can not be regarded as normal. The reason for this has been set forth by Professor Gowell[d] as follows:

> The pullets were hatched in April and May, and thinking to have them mostly in readiness for laying early in November, we fed them rather more beef scrap than usual

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[a] Maine Agricultural Experiment Station Bulletin 79, pp. 33–36. 1902.

[b] Loc. cit., pp. 37–40.

[c] Maine Agricultural Experiment Station Bulletin 93, pp. 72, 73. 1903.

[d] Maine Agricultural Experiment Station Bulletin 117, p. 96. 1905.

during the growing season, while they were out on the range, and before we were aware of their development they were laying—in August. They were nearly all laying heavily during September, October, and November. They were splendid birds, but almost every one of them molted completely in December, and we got very few eggs from them for more than two months. The most of the eggs secured from them were laid after the middle of January. Could they have commenced laying in October and continued for a year, molting would probably have been avoided and the showing would have been much better.

In the year 1903–4, 300 Barred Plymouth Rock pullets were tested in the trap nests. Of these 254 completed the first-year record and appear in the statistics of the present paper. The egg production of this year again can not be regarded as normal, for a reason which has been discussed by Professor Gowell (loc. cit.). Owing to a difficulty in the construction of a new house, the birds were not moved in from the range so as to begin their laying until December 6. Hence the records for 1903–4 as they appear in the statistics of the present paper are really eleven-month records instead of twelve-month records. They lack the potential contribution of the month of November, 1903, to the totals.

At the beginning of the laying year 1904–5 an experiment on the relation of the amount of floor space per bird to egg production was started. In carrying out this experiment the birds were divided into flocks of 50, 100, and 150 birds each. These were put into pens with floor space such that in the pens containing 50 and 100 birds there was 4.8 square feet of floor space per bird and in the 150-bird pens 3.2 square feet of floor space per bird. There were six 50-bird pens, and of the 300 birds which started the year in these pens 283 made complete records and appear in our statistics. There was one 100-bird pen, and 92 of these birds completed the year's record and appear in the present statistics. There was one 150-bird pen, and 140 of these birds completed the year's record and appear in the statistics. The records for this year are to be regarded as normal.

In the laying year of 1905–6 the birds tested in the trap nests were again divided into flocks of 50, 100, and 150 birds each. There were four of the 50-bird pens, and of these 200 birds 178 made complete records for the whole year and appear in the statistics of the present paper. There were two 100-bird pens, and 182 of these birds completed the year. There were also two 150-bird pens, and 275 out of the 300 birds completed the year. The laying year of 1905–6 was in nearly every respect a normal one. The only extraordinary factor which came in to interfere with the egg production during this year was a short period of sickness among the birds caused by the too free use of succulent feed. The details of this mishap have been stated by Professor Gowell [a] and need not be repeated here. The trouble occurred in April, 1906, and its effect on the egg yield is stated to have been as follows: "The egg yields were about 60 per cent of the number of birds just before the trouble began, but they were reduced to less than 10 per cent, and it was about twenty days before the birds regained their former production." This factor reduces the egg yields for the months of April and May of this year. The precise amount of this reduction would appear from the statistics not to be so great as is implied in the statement quoted. This will be more fully discussed in Part II of this study, which will deal particularly with monthly egg production.

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[a] Maine Agricultural Experiment Station Bulletin 144, p. 184. 1907.

In the laying year 1906–7 the experiments with flocks of different sizes were continued. We have records for this year of four 50-bird pens, two 100-bird pens, and two 150-bird pens. Of the 200 birds in the 50-bird pens 187 completed the year; of those in the 100-bird pens 185 completed the year; and, finally, of the 300 in the two 150-bird pens 281 have complete records for the year. In the early part of this laying year (December) a serious mishap occurred which considerably lowered the egg yields for this and the succeeding month. The matter is thus described by Professor Gowell:[a]

> Last December the roosts and walls of the roosting closets in all of the rooms but one were sprayed with two different brands of liquid lice killers which were warranted by their makers to destroy all the lice on fowl by the fumes penetrating among the feathers to the skins of the birds. The preparations were used according to directions. The roosts and woodwork near them were sprayed in the morning and left to dry until the birds went to roost. But the sprayed paint was not fully dry at that time, although it was thinned by heating before applying it. The curtains of the roosting closets were shut down at bedtime as usual and both ventilators, each of which are 3 feet long and 6 inches wide, were left wide open. Next morning it was very evident from the appearance of the birds that they had not enjoyed the night. They ate but little food during that and the succeeding three or four days and did not have their usual appetites for nine or ten days. The 700 birds were laying over 300 eggs per day before the trouble, but they laid less than 100 per day during the following week, and did not lay as many as before until twenty-four days had elapsed.
>
> Many of them molted partially, or quite fully, and these did not lay much for six or seven weeks. It is not thought that any birds died from the accident. Probably we were at fault in using the stuff in winter when it did not thoroughly dry out, as it might have done in a long, warm summer day. One pen at the end of the building was not sprayed, but the air in that room was loaded with the odor from the rest of the building, which easily found its way in around the loosely fitting door. The birds in that room fell off in their egg yields for several days, but none of them molted. Many of the birds in the sprayed rooms refused to go to roost in the closets again for several nights, until the odor had largely disappeared, while those in the unsprayed room went into their bedroom as usual.

The precise effect of this disturbance on the egg yields will be shown later in a discussion of the monthly production.

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[a] Loc. cit., p. 185.

Another point which should be noted regarding the records for 1906–7 is that they comprise the records for eleven months instead of a full year. The records were discontinued at the end of September, 1907, October of that year being entirely omitted.

From the account of the statistics of the several years included in this investigation it is evident that as they stand the data for any one year are probably not strictly comparable with those for any other year. Certainly this is true in the later years covered by the material. The statistics for each year must be treated separately, and whenever comparisons between years are instituted great caution must be exercised in drawing conclusions. It will obviously be necessary in some cases to apply corrective factors before any comparisons whatever can be made. The authors are aware that this fact constitutes the principal point at which these statistics are open to criticism, and that it is a serious defect. It should be remembered, however, when such a criticism of data like these is made, that the practical difficulties of keeping relatively constant the environmental conditions under which so many as a thousand laying hens are kept are very great. Especially is this true when the time unit of the experiments is twelve months. In nearly every case the factors which have come in to disturb the records of annual egg production, as noted above, are things which could not have been foreseen and avoided. They were accidents in the strict sense. It will be apparent in the later portions of the paper that in spite of these accidents in the several years the constants of the variation curves of egg production are very closely similar throughout the period covered by the statistics. In other words, the effect of these accidents has been decidedly more on the centering constants like the mean and the mode than upon the constants which have to do with the shape of the curve, such as, for example, the standard deviation and the skewness. On the whole, we believe that the data which we have, in spite of their shortcomings, give us a very fair picture of the true state of affairs relative to variation in annual egg production. When we come to the detailed monthly and daily statistics, which will be considered in the parts of the bulletin following the present one, these factors, which to a certain extent vitiate annual statistics, will be very much less serious, because in every case the accident has been confined to but a single month or at most two or three months.

In discussing the variation in egg production we shall consider the statistics for the Plymouth Rocks first and base the major portion of the discussion upon them. The records for the White Wyandottes will be taken up farther on in a separate section of the paper.

STATISTICAL METHODS USED.

In dealing with these egg-record statistics obviously the first thing to be done is to get the data in the form of frequency distributions. We have done this for each year's records of each set of pens (50, 100, and 150 birds) without grouping data in any way. These ungrouped frequency distributions, which form the basic raw material of the first part of this study, are given in detail in Appendix I, Tables I, II, and III. A glance at these distributions will indicate at once that the range of variation in egg production is very considerable. It is so great, in fact, as to make analytical treatment necessary if we are to form any adequate idea of variation in this character.

In this investigation we have made use of the ordinary methods of biometrical analysis. Summary accounts of these methods have been given by Elderton,[a] C. B. Davenport,[b] and E. Davenport.[c] For more detailed accounts the reader is referred to the memoirs of Pearson on the subject which have appeared in (*a*) the Philosophical Transactions of the Royal Society during the period from 1895 to 1905; (*b*) Biometrika; and (*c*) the Draper's Company Research Memoirs of the Department of Applied Mathematics of University College, London. In view of the existence of the treatises referred to it will not be necessary here to go into a general account of biometrical methods. It is, however, desirable to call attention to certain specific modes of treating these egg-production statistics which we have adopted.

The problem of grouping the material may be first considered. From the extent of the range it is obviously necessary that some grouping be resorted to in order to get a smooth frequency distribution covering the whole range of variation in annual egg production (more than 200 eggs). If the data were not grouped it would be necessary to have records of very many individuals, probably hundreds of thousands. With the smaller numbers which practical considerations make the only possible ones, it is clearly necessary that the data be grouped. When grouping such data the usual rule followed in biometrical work is to make as many classes as will give a fairly smooth distribution without too many classes having zero frequency. Obviously if the class units are sufficiently large all frequency distributions will be reduced to very much the same form. Only experience can decide what is the most appropriate value to take for the class unit in any specific case. After studying all the data on variation in egg production we have come to the conclusion that a class unit of 15 eggs is probably as satisfactory as any which could be taken. In adopting this unit we have set the class limits at the same points in the case of each distribution; that is to say, the first class includes birds laying from 0 to 14 eggs inclusive; the next class, birds laying from 15 to 29 eggs inclusive, and so on. The procedure sometimes followed of taking different limits for the class unit in the case of each frequency distribution, and subsequently treating these distributions as if they were in every way strictly comparable, is obviously not a logical one.

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[a] Elderton, W. P. Frequency Curves and Correlation. London, 1907. Pp. xiii and 172.

[b] Davenport, C. B. Statistical Methods with Special Reference to Biological Variation. Second edition. New York, 1904. Pp. viii and 223.

[c] Davenport, E. The Principles of Breeding. Boston, 1907. Pp. xiii and 727.

In deducing the moments from the grouped material we have in each case used Sheppard's corrections.[a] It will appear further on in the paper that the problem of getting satisfactory theoretical curves to graduate these statistics of egg production is an extremely difficult one. It occurred to us that possibly better success in the graduation of the curves might be had by using raw instead of corrected moments. Actual trial, however, showed that somewhat worse instead of better results were obtained when the raw moments were used. On the whole, this is what is to be expected on theoretical grounds, as will be apparent when the grouped frequency distributions are examined. Our material is grouped in rather large units, and at the same time we have apparently fairly close approximation to high contact at both ends of the range. These conditions, of course, indicate that Sheppard's corrections are demanded.

In deducing constants from the ungrouped material we have calculated the mean and the median by ordinary arithmetical methods. In calculating standard deviations in these cases we have used the method of moments rather than the method of calculating from the formula $\sigma^2 = \frac{S(x^2)}{n}$, where x denotes deviation from the mean. In calculating the mode in the case of the ungrouped frequency distributions, use has been made of the approximate relation:

$$\text{Mean to mode} = 3 \times \text{(mean to median)},$$

the median of the curve lying between the mean and the mode. This approximation to the position of the mode was first worked out empirically by Pearson[b] and found to give reasonably close values in most cases. Recently Charlier[c] has derived this relation directly in his treatment of a certain type of frequency curve. He shows definitely why this approximation to the mode must give a fairly close value in all cases where the frequency curve is not too widely divergent from the normal curve of errors. In order to determine for the class of curves with which we are here dealing how good this approximation to the mode is, we have calculated for six curves, (*a*) the true value of the mode from the moments, by the usual general formula,[a]

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[a] Sheppard, W. F. Biometrika, Vol. V, pp. 450–459. 1907.

[b] Cf. Pearson, K. Biometrika, Vol. I, pp. 260, 261. 1902.

[c] Charlier, C. V. L. Researches into the Theory of Probability. Meddelanden från Lunds Astronomiska Observatorium, Serie II, No. 4, Kongl. Fysiografiska Sällskapets Handlingar, Bd. 16, pp. 1–51. 19 fig. Lund, 1906.

$$d=\frac{1}{2}\,\frac{\sqrt{\mu_2}\,\sqrt{\beta_1}\,\left(\beta_2+3\right)}{5\beta_2-6\beta_1-9}$$

and (*b*) the approximate value of the mode by the method under discussion. The results are shown in the following table:

Year and group.	True mode.	Mode by approximate method.	Difference.
1903	139.20	143.25	−4.05
1904, 50-bird pens	149.14	148.30	−0.84
1905, 50-bird pens	149.48	149.88	−0.40
1905, 100-bird pens	135.00	133.00	+2.00
1906, 50-bird pens	123.21	120.44	+2.77
1906, 150-bird pens	103.73	105.89	−2.16

When it is remembered that the probable errors of the centering constants (mean and median) are in these curves of about the magnitude ±2, it is apparent that the approximation gives values as close to the true mode as could reasonably be expected.

Having determined the mode in the manner indicated, in the case of the ungrouped material, the distance from mean to mode was used to find the skewness by the following relation:

$$\text{Skewness}=\frac{\text{Distance from mean to mode}}{\text{Standard deviation}}$$

Respecting the probable errors of the various constants discussed, the following should be said. The probable error of the mean was calculated from the usual formula,

$$P.\ E._{mean}=0.67449\frac{\sigma}{\sqrt{n}}$$

using Gibson's tables.[b] Theoretically, in the case of the grouped material, where Sheppard's corrections are applied to the moments, the uncorrected value of σ should be used in this formula.[c] But practically, with the very large values of σ with which we have here to do, and the consequent large absolute values of the probable errors, it makes no difference, to the number of decimal places tabulated in the probable errors, whether the corrected or uncorrected value is used. We have consequently used the corrected values to save labor. The probable error of the median is given by Sheppard[a] as follows:

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[a] Pearson, K. Philosophical Transactions of the Royal Society, vol. 198 A, p. 277. 1902.

[b] Gibson, W. Biometrika, Vol. IV, pp. 385-393. 1906.

[c] Cf. Sheppard, W. F. Biometrika, Vol. V, p. 455. 1907.

$$P.\ E._{median} = 0.84535\frac{\sigma}{\sqrt{n}}$$

For the sake of economy in calculation we have made use of the obviously simpler relation:

$$P.\ E._{median} = 1.25332 \times P.\ E._{mean}$$

In the case of the probable error of the standard deviation it has recently been pointed out by the writer[b] that caution must be exercised in using the formula usually given, viz:

$$P.\ E.\ \sigma = 0.67449\frac{\sigma}{\sqrt{2n}} \text{ ------------------ (i)}$$

When curves deviate much from the normal curve of errors $P.\ E.\ \sigma$ by this formula may be seriously in error. Resort should be had in all cases of importance to the general formula

$$P.\ E.\ \sigma = 0.67449\sqrt{\frac{\mu_4 - \mu_2^2}{4\mu_2}\Big/ n} \text{ --------------- (ii)}$$

which for convenience in computation may be written

$$P.\ E.\ \sigma = \chi_1\sqrt{\frac{\mu_4 - \mu_2^2}{4\mu_2}}$$

where χ_1 is the value $0.6744898/\sqrt{n}$ tabled by Miss Gibson (loc. cit.). In the present investigation we have made it a rule to calculate this probable error by equation (ii) in every case where it was necessary for some other reason to determine the value of μ_4. In all other cases equation (i) was used. In the tables it will be specifically indicated which formula was used in each particular case.

The usual formula for the probable error of the coefficient of variation V,

$$P.\ E._V = \frac{.067449}{\sqrt{2n}}\left[1 + 2\left(\frac{V}{100}\right)^2\right]^{\frac{1}{2}}$$

was used throughout.

In the case of the skewness the question of the proper value of the probable error is a difficult one. It has been shown by Pearson and Filon[c] that the probable error of this constant has a different value for each different type of frequency curve. A maximum limit, however, is always given by the value in the case of the normal curve, i. e.,

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[a] Sheppard, W. F. Philosophical Transactions of the Royal Society, vol. 192 A, pp. 101–167. 1898.

[b] Pearl, R., Biometrika, Vol. VI, pp. 112–117. 1908.

[c] Pearson, K., and Filon, L. N. G. Philosophical Transactions of the Royal Society; vol. 191 A, pp. 229–311. 1898.

$$P.\ E._{Sk.} = 0.67449\sqrt{\frac{3}{2n}}$$

The authors cited point out (loc. cit., p. 271, footnote) that a very close approximation of the probable error of the skewness generally is given by the relation

$$P.\ E._{Sk.} = 0.67449\sqrt{\frac{3}{2n}}\ \frac{1}{\sqrt{\{1+3(Sk.)^2\}}}$$

This formula we have used throughout, first putting it in the somewhat simplified form

$$P.\ E._{Sk.} = \frac{1.7320508\chi_2}{\sqrt{\{1+3(Sk.)^2\}}}$$

where χ_2 is the function tabled by Miss Gibson (loc. cit.).

All the computations in this investigation have been carried out with the aid of a large Brunsviga arithmometer and an adding machine. All possible precautions have been taken to exclude arithmetical errors in the work. It is the intention to publish all the raw material which was available to the authors (in the present or later parts) of this paper, so that it will be possible for any interested persons to check up the computations.

VARIATION IN EGG PRODUCTION IN BARRED PLYMOUTH ROCKS.

Turning now to the discussion of the results of the work, we have first the grouped frequency distributions for variation in first-year egg production in Barred Plymouth Rock pullets during a period of eight years set forth in Table 1. In this table are given both the absolute frequencies and the frequencies per cent for each year.

TABLE 1.—*Frequency distributions for variation in first-year egg production, Barred Plymouth Rocks.*

Year and group.	Annual egg production.																	
	0–14.	15–29.	30–44.	45–59.	60–74.	75–89.	90–104.	105–119.	120–134.	135–149.	150–164.	165–179.	180–194.	195–209.	210–224.	225–239.	240–254.	Total.
1899–1900:																		
Absolute frequency	....	....	3	2	4	4	4	2	10	11	9	12	6	1	1	1	....	70
Percentage frequency	....	....	4.29	2.86	5.71	5.71	5.71	2.86	14.29	15.71	12.86	17.14	8.57	1.43	1.43	1.43	....	100.00
1900–1901:																		
Absolute frequency	....	1	....	4	4	2	6	7	6	15	5	14	12	7	1	1	....	85
Percentage frequency	....	1.18	....	4.71	4.71	2.35	7.06	8.24	7.06	17.65	5.88	16.47	14.12	8.24	1.18	1.18	....	100.03
1901–1902:																		
Absolute frequency	....	....	....	....	1	1	2	8	4	1	11	8	3	5	3	....	1	48
Percentage frequency	....	....	....	....	2.08	2.08	4.16	16.67	8.32	2.08	22.92	16.67	6.25	10.42	6.25	....	2.08	99.98
1902–1903:																		
Absolute frequency	....	2	....	1	5	8	17	18	17	26	17	18	9	2	6	1	....	147
Percentage frequency	....	1.36	....	.68	3.40	5.44	11.56	12.24	11.56	17.69	11.56	12.24	6.12	1.36	4.08	.68	....	99.97
1903–1904:																		
Absolute frequency	7	5	5	10	10	20	24	29	52	37	29	16	8	2	....	....	....	254
Percentage frequency	2.76	1.97	1.97	3.94	3.94	7.87	9.45	11.42	20.47	14.57	11.42	6.29	3.15	.79	....	....	....	100.01
1904–1905:																		
50-bird pens—																		
Absolute frequency	3	3	14	7	9	15	16	39	24	40	33	28	16	25	7	3	1	283
Percentage frequency	1.06	1.06	4.95	2.47	3.18	5.30	5.65	13.78	8.48	14.13	11.66	9.89	5.65	8.83	2.47	1.06	.35	99.97
100-bird pens—																		
Absolute frequency	....	2	4	1	3	8	10	11	5	7	11	11	8	9	2	....	....	92
Percentage frequency	....	2.17	4.35	1.09	3.26	8.70	10.87	11.96	5.43	7.61	11.96	11.96	8.70	9.78	2.16	....	....	100.01
150-bird pens—																		
Absolute frequency	7	6	3	9	6	15	10	18	12	10	19	11	6	6	2	....	....	140
Percentage frequency	5.00	4.29	2.14	6.43	4.29	10.71	7.14	12.86	8.57	7.14	13.57	7.86	4.29	4.29	1.43	....	....	100.01
1905–1906:																		
50-bird pens—																		
Absolute frequency	....	....	1	2	4	9	13	25	24	22	32	17	20	9	....	....	....	178
Percentage frequency	....	....	.56	1.12	2.25	5.06	7.30	14.04	13.48	12.36	17.98	9.55	11.24	5.06	....	....	....	100.00
100-bird pens—																		
Absolute frequency	1	3	3	3	8	14	17	26	25	23	27	16	12	3	....	....	1	182
Percentage frequency	.55	1.65	1.65	1.65	4.40	7.69	9.34	14.29	13.74	12.64	14.84	8.79	6.59	1.65	....	....	.55	100.02
150-bird pens—																		
Absolute frequency	1	1	4	10	21	23	35	46	40	35	25	19	8	6	1	....	....	275
Percentage frequency	.36	.36	1.45	3.64	7.64	8.36	12.73	16.73	14.55	12.73	9.09	6.91	2.91	2.18	.36	....	....	100.00
1906–1907:																		
50-bird pens—																		
Absolute frequency	2	2	5	5	9	16	30	39	26	21	19	12	1	....	....	....	....	187
Percentage frequency	1.07	1.07	2.67	2.67	4.81	8.56	16.04	20.86	13.90	11.23	10.16	6.42	.53	....	....	....	....	99.99
100-bird pens—																		
Absolute frequency	4	....	8	11	7	25	29	25	30	20	11	10	5	....	....	....	....	185
Percentage frequency	2.16	....	4.32	5.95	3.78	13.51	15.68	13.51	16.22	10.81	5.95	5.41	2.70	....	....	....	....	100.00
150-bird pens—																		
Absolute frequency	10	8	8	15	29	32	48	39	36	25	18	6	5	....	2	....	....	281
Percentage frequency	3.56	2.85	2.85	5.34	10.32	11.39	17.08	13.88	12.81	8.90	6.40	2.14	1.78	....	.71	....	....	100.01

The percentage distributions for 1903, for the 50-bird pens of 1904, the 50, 100, and 150 bird pens of 1905, and the 50, 100, and 150 bird pens of 1906 are shown graphically in figures 1 to 8, inclusive. Of the other distributions the majority involve so few individuals relatively that they give rather irregular polygons. On this account, and since the general form in all is very similar to that of the polygons which are shown graphically, it will not be necessary to figure them separately.

From the data given in Table 1 and the diagrams we note at once the following general characteristics of variation in egg production:

1. The distribution of variation is not usually symmetrical about the center or type of egg production. The distributions nearly all show some degree of asymmetry or skewness. Further, this skewness is obviously negative in all cases—that is, the distributions tail off more gradually toward the side of low egg production.

2. The range of variation in egg production is very considerable, even in a stock so closely selected to type as this has been. The observed range extends from an egg production of zero to one of nearly 250. The manner in which this great range is covered is perhaps even more strikingly shown in the tables of Appendix I than in Table 1.

3. The distributions generally have more than one observational mode or peak. This fact, however, is probably not to be interpreted as meaning that the normal egg-production variation curve is a multimodal one. It is in all probability merely a result of random sampling with respect to a character having a great range of variation, when, furthermore, each sample contains only a relatively small number of individuals. The general form of the frequency distributions would indicate that the variation in the character is of the usual unimodal skew type. This point, however, will be more particularly discussed later on in the paper, when the evidence from the analytical constants is at hand.

4. Not only is the range of variation in egg production great, but also the absolute amount of variation in the character, as indicated by the "scatter" of the distributions, is relatively large. This will be made clearer by the constants of variation, to the examination of which we may next turn.

In Table 2 are given the values of the chief physical constants of the frequency distributions of Table 1, together with their probable errors. The methods by which the calculations were made have been described in the previous section. Six constants are included in the table. A brief examination of their meaning may be useful. The first of the constants tabled—the mean or average annual egg production—needs no explanation. The median (the second constant tabled) is the value at which the flocks divide into two equal parts in regard to egg production. Thus, in 1902–3 one-half of the birds laid more than 137.5 eggs and one-half laid fewer than that number. The mode is that amount of annual egg production which is exhibited by the largest number of birds. It is the "fashionable" egg production. For example, in 1902–3 more hens laid 139.53 eggs in the year than laid any other single number, either greater or less. The standard deviation is a measure of absolute variation; in the present instance it expresses the degree of variation in annual egg production in terms of eggs. It is a measure of the "spread" or "scatter" of the frequency distribution. The coefficient of variation is a relative measure of variation. It is the percentage of the standard deviation in the mean. The skewness is the measure of the degree to which the frequency curve is asymmetrical. A positive skewness means that the curve tails off more gradually toward large values than toward small values of the character under discussion, while a negative skewness means that, as in the present cases, the curve tails off more gradually toward the small values of the character. With positive skewness the mean is greater than the mode, while with negative skewness the reverse relation holds.

In the last three lines of Table 2 are given the weighted mean values of each constant for the years 1904–5, 1905–6, and 1906–7, respectively. In each of these years we have data for flocks of 50, 100, and 150 birds each. In order to get a single value which shall be in a measure representative of each constant for each of these years the only feasible procedure is to take a weighted mean of the values of the particular constants for each of the three flocks. This we have done. We have weighted the several flock constants in each case with the number of birds they involve, as shown in Table 1. A single example worked out in extenso will make plain just how these weighted values are obtained:

1905–6, 50-bird pens: Mean 140.31 × 178 = 24,975.18
1905–6, 100-bird pens: Mean 127.50 × 182 = 23,205.00
1905–6, 150-bird pens: Mean 119.43 × 275 = 32,843.25

Sums = 635 and 81,023.43

81,023.43 ÷ 635 = 127.60 = "weighted mean" egg production in year 1905–6.

TABLE 2.—*Constants of variation in annual egg production, Barred Plymouth Rocks.*

Laying year and group.	Mean.[a]	Median.	Mode.	Standard deviation.	Coefficient of variation.	Skewness.
1899-1900	136.36±3.55	142.00±4.45	153.29	44.03±2.51	32.29±2.02	−0.384±0.082
1900-1901	143.44±3.29	146.25±4.12	151.88	45.02±2.33	31.38±1.78	−.188±.081
1901-2	155.58±3.79	158.50±4.75	164.33	38.94±2.68	25.03±1.83	−.225±.104
1902-3	136.48±2.19	137.50±2.74	139.54	39.34±1.55	28.82±1.22	−.078±.067
1903-4	117.87±1.75	126.33±2.20	139.20	[c] 41.43±1.31	35.14±1.17	−.515±.039
1904-5:						
50-bird pens	134.60±1.98	139.17±2.49	149.14	[c] 49.49±1.29	36.77±1.17	−.294±.044
100-bird pens	133.61±3.47	141.00±4.35	155.78	[c] 49.41±2.46	36.98±2.07	−.449±.068
150-bird pens	114.54±3.02	117.00±3.79	121.93	[c] 53.04±2.38	46.31±2.23	−.139±.068
1905-6:						
50-bird pens	140.31±1.81	143.50±2.26	149.48	[c] 35.75±1.11	25.48±0.97	−.256±.052
100-bird pens	127.50±2.03	129.33±2.54	135.00	[c] 40.59±1.49	31.84±1.23	−.185±.056
150-bird pens	119.43±1.54	[b] 118.86±1.93	120.07	[c] 37.88±1.01	31.72±1.00	−.017±.050
1906-7:						
50-bird pens	114.16±1.74	116.25±2.18	123.21	[c] 35.22±1.29	30.91±1.18	−.263±.055
100-bird pens	108.53±1.92	[b] 110.75±2.41	109.55	[c] 38.71±1.31	35.54±1.39	−.016±.061
150-bird pens	101.08±1.64	102.50±2.06	103.73	[c] 40.87±1.16	40.43±1.32	−.065±.050
1904-5, weighted mean	128.97	133.47	142.93	50.44	39.40	−.280
1905-6, weighted mean	127.60	128.77	132.59	38.06	30.01	−.132
1906-7, weighted mean	106.94	108.77	110.96	38.64	36.32	−.108

[a] Beginning with the year 1902-3, the means (and also the other constants except the medians) are calculated from the grouped frequency distributions of Table 1, instead of from the ungrouped distributions given in the appendix. This leads to slightly different values than are obtained if one calculates directly from the ungrouped material. The differences, however, are always very small, and are not sensible in comparison with the probable errors of the constants. In passing it should be said that on theoretical grounds there is every reason to suppose that the means from the grouped figures represent more nearly the unknowable true means than do those from the ungrouped material.

[b] The failure of the median in these cases to fall between the mean and the mode arises from the fact that the skewness of these curves is very small. Consequently mean, median, and mode very nearly coincide. Each is subject to a considerable probable error, great enough indeed so that twice the range included within the probable error of any one of these constants includes the other two within its limits.

[c] These probable errors were calculated by the formula involving μ_4. (Cf. p. 23, supra.)

Attention may first be called to certain general points brought out by the constants in this table, and then turned to the discussion of specific problems, for which the table furnishes data. We note first that the mean or average egg production ranges in amount between about 100 eggs (150-bird pen, 1906-7) and 156 eggs (1901-2). The median and the mode are in every case larger than the mean, indicating, as does also the last column of the table, that all these egg-production variation curves are negatively skew. The amount of variation in annual egg production is large both absolutely and relatively. The lowest coefficient of variation in the table is 25.03 per cent (1901-2). Even this is a distinctly large coefficient, when compared with what has been found for various characters in other organisms than the domesticated fowl. The values of the constants measuring variation obtained in this case indicate that egg production is to be classed, so far as variation is concerned, with purely physiological characters. It has been pointed out by the writer[a] that in the case of man characters which in general may be regarded as strictly physiological (even though they may be, as in the present case, measured in morphological units) vary much more than do such morphological characteristics as bone measurements and the like. This point has recently been confirmed by Kellicott[b] in his study of variation in the toad. We hope later to have quantitative data regarding variation in other characters of the domestic fowl, so that direct comparison of various characters and organs in respect to variation may be made.

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[a] Pearl, R. Biometrika, Vol. IV, pp. 31-36. 1905.

[b] Kellicott, W. E. Journal of Experimental Zoology, Vol. IV, pp. 576-614. 1907.

It is of interest to compare these results on variation in egg production with the results of other workers regarding variation in fertility and fecundity in other organisms. In Table 3 we have collected the existing data, from which such comparisons may be made. We have included data on variation in number of seeds produced in a plant, this being obviously a measure of gross fertility in such a case.

TABLE 3.—*Constants of variation in fertility and fecundity in various organisms.*

Subject.	Character.	Coefficient of variation.	Skewness.	Authority.
Nelumbium luteum........................	Number of seeds......	17.445	+0.0164	Pearl.[a]
Poland-China swine........................	Size of litter...........	27.411	+ .0701	Surface.[b]
Duroc-Jersey swine........................	....do.................	25.997	+ .0539	Surface.[b]
Horse..	Fecundity [c]	24.771	− .1286	Pearson.[d]
Man...	Number of children...	48.41	+ .0802	Powys.[e]
Domestic fowl..............................	Annual egg production	[f] 34.21	[f] − .205	This paper.

[a] American Naturalist, Vol. XL, pp. 757-768. 1906.

[b] Reductions of data given by Rommel (Bureau of Animal Industry Circular No. 95). A paper giving a detailed account of these reductions is now in press.

[c] Fecundity in this case means the fraction which the actual number of offspring arising from a given number of coverings is of the possible number of offspring under the circumstances.

[d] Biometrika, Vol. I, pp. 289-292. 1902. Actually only the moments of this fecundity curve are given at the place cited. From the moments we have calculated the coefficients of variation and the skewness.

[e] Biometrika, Vol. V., p. 251. 1905.

[f] Weighted mean value of the constant for the whole period 1899-1900 to 1906-7, inclusive. The weighting is in proportion to the number of birds involved in each year.

Further data for man might be included in this table, but that is hardly necessary, as the example given of a human fertility curve is based on very extensive data and may be regarded as typical. It is clear from the table that these egg-production curves are in no way exceptional in regard to the degree of variation which they exhibit. It appears to be generally the case that fecundity, fertility, and closely allied characters are highly variable. There would seem to be a tendency for the coefficients measuring variation in these characters to be above a value of 20 per cent. In general it may be said that, so far as indicated by the present data, the domestic fowl exhibits a relative degree of variation in annual egg production distinctly less than that shown in normal human stocks in respect to size of family, and rather greater than that in Thoroughbred brood mares in respect to fecundity.

In Table 3 have also been included comparative data respecting the skewness of variation in the several characters enumerated. It is evident at once that there is even less uniformity in this particular than in the relative amount of variation. Even the sign of the skewness is not the same in all cases. In general the degree of skewness is small. The only cases in which the direction of the asymmetry of the distribution is negative are those of fecundity in brood mares and egg production in fowls. Human fertility curves all agree in showing a distinct positive skewness. It was at first thought that the negative skewness of the egg-production curves might owe its origin to the practice which has been followed of selection for high production. It seems improbable, though, that this is the case, because, as indicated in the last column of Table 2, environmental conditions appear to have a great deal to do with the shape of the curves of variation in egg production.

ANALYTICAL DISCUSSION OF VARIATION IN EGG PRODUCTION.

Having now considered the general facts respecting variation in annual egg production, we may next turn our attention to the more complete mathematical analysis of the variation curves, using the methods of Pearson.[a] We have carried out the analysis in detail and fitted curves to the following distributions given in Table 1: 1903–4; 1904–5, 50-bird pens; 1905–6, 50, 100, and 150 bird pens; 1906–7, 50, 100, and 150 bird pens. The other distributions involve so few individuals as to make their detailed treatment unprofitable. The analytical constants are exhibited in Table 4. The moments are given in terms of the unit of grouping, which in each case is equal to 15 eggs.

TABLE 4.—*Analytical constants of variation in annual egg production.*

Constant.	1903–4.	1904–5, 50-bird pens.	1905–6.			1906–7.		
			50-bird pens.	100-bird pens.	150-bird pens.	50-bird pens.	100-bird pens.	150-bird pens.
N	254	283	178	182	275	187	185	281
μ_2	7.6269	10.8868	5.6818	7.3233	6.3761	5.5117	6.6601	7.4221
μ_3	− 14.6883	− 13.3924	− 3.8863	− 7.0879	− .4478	− 6.2369	− 4.5912	− 3.0085
μ_4	187.6675	318.7131	81.2274	168.5017	111.2316	97.7818	126.9814	165.2723
β_1	.4860	.1390	.0823	.1279	.0008	.2323	.0714	.0221
$\sqrt{\beta_1}$	.6971	.3728	.2869	.3577	.0278	.4820	.2671	.1488
β_2	3.2262	2.6890	2.5161	3.1419	2.7360	3.2188	2.8627	3.0002
β_2-3	+ .2262	− .3100	− .4839	+ .1419	− .2640	+ .2188	− .1373	+ .0002
κ_1	− 1.0055	− 1.0389	− 1.2149	− .0999	− .5303	− .2594	− .4886	− .0660
Skewness	− .5148	− .2937	− .2564	− .1848	− .0171	− .2631	− .0160	− .0650
d . . . . . Eggs	21.33	14.53	9.17	7.50	.65	9.26	.62	2.66
Standard deviation, eggs	41.43	49.49	35.75	40.59	37.89	35.22	38.71	40.87
Mean . . . . . Eggs	117.87	134.60	140.31	127.50	119.43	113.94	108.93	101.08
Mode . . . . . do	139.20	149.14	149.48	135.00	120.07	123.21	109.55	103.73
Range . . . . . do	331.82	328.21	221.29	1265.08	344.88	631.22	599.98	. . . .
+end of range . . do	194.41	248.18	208.55	313.77	291.87	226.11	358.93	. . . .
−end of range . . do	−137.41	− 80.02	12.74	−951.31	− 53.01	−405.11	−141.06	. . . .
y_0 per cent	13.47	11.46	15.32	14.86	14.49	17.19	15.18	14.64
P. E. $\sqrt{\beta_1}$ [b]	± .1037	± .0982	± .1238	± .1224	± .0996	± .1208	± .1214	± .0986
P. E. β_2 [b]	± .2074	± .1964	± .2476	± .2448	± .1992	± .2416	± .2428	± .1972
P. E. skewness [b]	± .0518	± .0491	± .0619	± .0612	± .0498	± .0604	± .0607	± .0493

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[a] Mathematical Contributions to the Theory of Evolution. II.—Skew Variation in Homogeneous Material. Philosophical Transactions of the Royal Society, vol. 186 A, pp. 343–414. 1895. Math. Contr., etc. X.—Supplement to a Memoir on Skew Variation. *Ibid.*, vol. 197 A, pp. 443–459. 1901.

[b] These are the probable errors in the case of the normal curve, and are introduced to show the degree to which the present curves deviate from normality.

Examining first the criterion $\kappa_1 = 2\beta_2 - 3\beta_1 - 6$, we note that it is in every case negative. This indicates that unless $\sqrt{\beta_1} = 0$ within the limits imposed by its probable error the curves demanded are of Pearson's Type I, with range limited in both directions, and skewness. From the value of $\sqrt{\beta_1}$, given in the table, we conclude that the first four of the distributions, and also that for the 50-bird pens in 1906–7, fulfill the conditions for a Type I curve. The three remaining distributions have a very small (practically insignificant) skewness. Consequently $\sqrt{\beta_1}$ approaches zero closely in its value in these cases, and is in each instance not significant in comparison with its probable error. Hence, the conclusion is that a symmetrical curve must be used to graduate these three distributions. This limits the choice to Pearson's Type II, a symmetrical curve with the range limited at both ends, on the one hand, or to the normal or Gaussian curve of errors, symmetrical but of unlimited range, on the other hand. The criteria which indicate the proper choice in such a case are, first, that if κ_1 be negative (as is the case with all of our three curves) it indicates a Type II curve, and second, that if β_2 be significantly different from 3, again a Type II curve is indicated. In our three distributions $\beta_2 - 3$ can not in any single case be asserted to be significant in comparison with the probable error of β_2. In the case of the 1905–6 150-bird pen distribution the probability is that $\beta_2 - 3$ is significant. There is little doubt that in this case we shall get a better graduation with a Type II curve than with a normal curve. Accordingly, such a curve has been fitted in this distribution. In the case of the other two distributions we should undoubtedly get sensibly as good graduations with either Type II or normal curves. We have used Type II in the case of the 100-bird pen curve (1906–7) and the normal curve in the case of the 150-bird pen distribution (1906–7).

The equations to the eight curves follow.

1903–4:

$$y = 34.22\left(1 + \frac{x}{18.4410}\right)^{6.9897}\left(1 - \frac{x}{3.6804}\right)^{1.3950}$$

1904–5, 50-bird pens:

$$y = 32.42\left(1 + \frac{x}{15.2774}\right)^{4.8540}\left(1 - \frac{x}{6.6032}\right)^{2.0980}$$

1905–6, 50-bird pens:

$$y = 27.42\left(1 + \frac{x}{9.1157}\right)^{3.3313}\left(1 - \frac{x}{4.7876}\right)^{1.7496}$$

1905–6, 100-bird pens:

$$y = 27.05\left(1 + \frac{x}{72.4209}\right)^{102.1468}\left(1 - \frac{x}{11.9179}\right)^{16.8097}$$

1905–6, 150-bird pens:

$$y=39.86\left(1-\frac{x^2}{132.1586}\right)^{8.8636}$$

1906–7, 50-bird pens:

$$y=32.14\left(1+\frac{x}{35.2214}\right)^{36.7812}\left(1-\frac{x}{6.8600}\right)^{7.1638}$$

1906–7, 100-bird pens:

$$y=28.08\left(1-\frac{x^2}{277.7664}\right)^{19.3530}$$

1906–7, 150-bird pens:

$$y=41.15\,e^{-0.0674x^2}$$

The frequency distributions and their fitted curves are shown graphically in figures 1 to 8, inclusive. In drawing these variation curves the frequencies have all been reduced to percentages. Hence, since the abscissal units are taken equal, all the curves have equal areas, and any given ordinates are directly comparable in the different diagrams.

It is clear from these figures that the curves give graduations of the data which are on the whole very good. They are quite as satisfactory as could reasonably be expected with such relatively small numbers in the observational series and the consequent irregularity of the polygons.

From Table 4 and the diagrams we note—

1. That the skewness or asymmetry of the variation curves, while in all but three cases distinct and significant, is in none of these curves extreme. Variation in egg production tends to approach the Gaussian type of distribution rather closely. The uniform tendency for the skewness in these curves to be in the negative direction when it exists at all is evident from the diagrams.

2. There is no uniform tendency for these ovulation curves to be either leptokurtic or platykurtic (that is, more or less peaked than the normal curve of equal standard deviation). In four out of the eight curves the sign of β_2-3 is positive, in the remainder negative. The deviation from the condition of mesokurtosis is in no case great, however. In view of the magnitude of the probable errors of β_2, it is impossible to assert that any of the deviations from mesokurtosis are certainly significant. In the case of the 1905–6 50-bird pen curve the value of the kurtosis β_2-3 is probably significant. In all the other cases the probability is in the other direction.

3. The total range is clearly overestimated in all but one case (1905–6, 50-bird pen), and in several of the curves this exaggeration of the range is very great. A total range of 328 or 332 or 345, while rather large, is not absolutely impossible theoretically. But when we come to a range of 1,265 eggs for the individual variation in egg

production within a year, it is clear that the exaggeration has gone far beyond the limits of what we must regard as possible. In spite of the great overestimation of the total range it will be observed that the upper (+) end of the range in every case has a value which is theoretically possible. While the production of over 300 eggs in a year by an individual hen has never been recorded so far as we are aware, it still would be very difficult to prove that such an event is theoretically impossible. The really serious difficulty with the range,

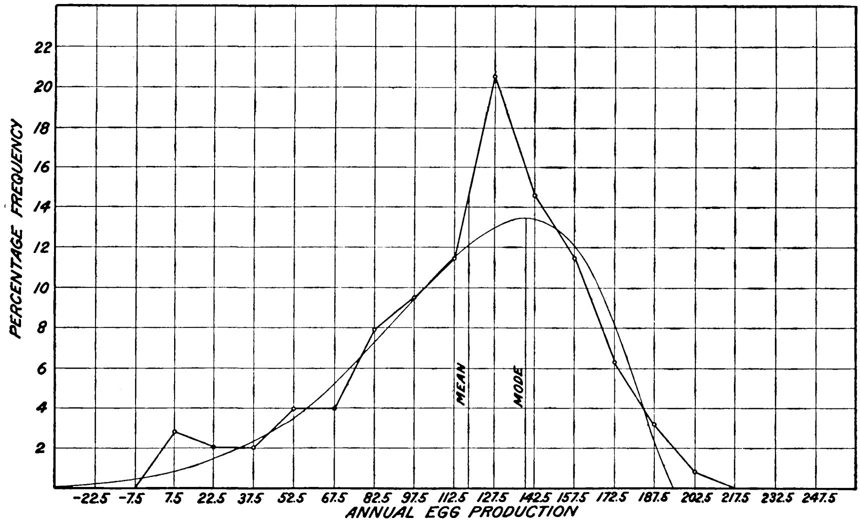

FIG. 1.—Variation in annual egg production, 1903-4.

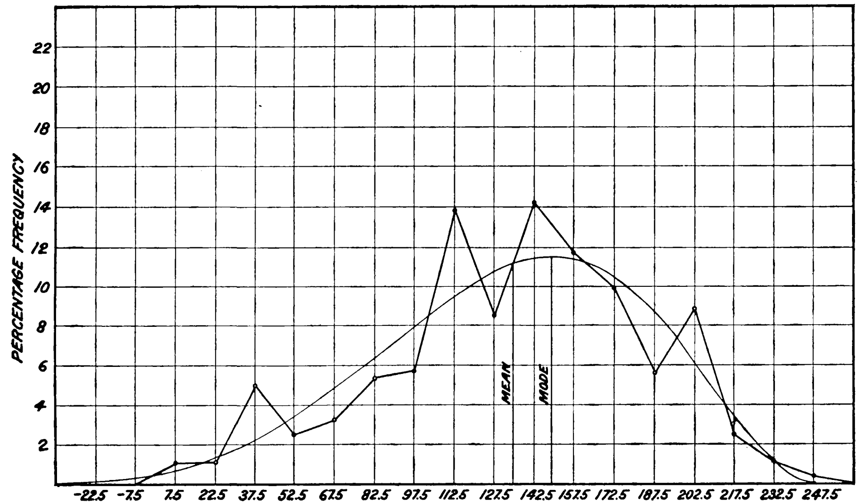

FIG. 2.—Variation in annual egg production, 1904-5.

as given by the curves, lies in the location of the lower (−) range end. In only one of the eight curves (1905–6, 50-bird pen) does the lower range end have a value which is physically possible. It is obvious

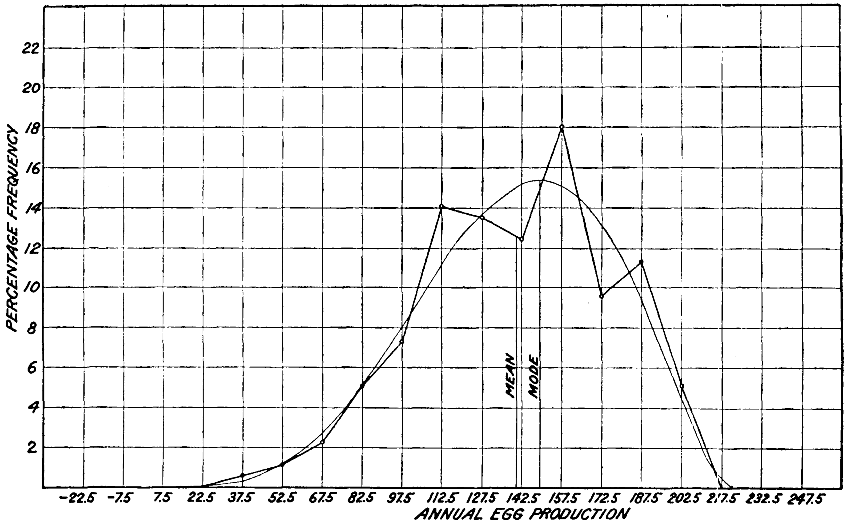

FIG. 3.—Variation in annual egg production, 1905–6, 50-bird pens.

that no hen is capable of laying fewer than no eggs at all in a year, yet, with the single exception noted, the lower range end has a negative value in every case.

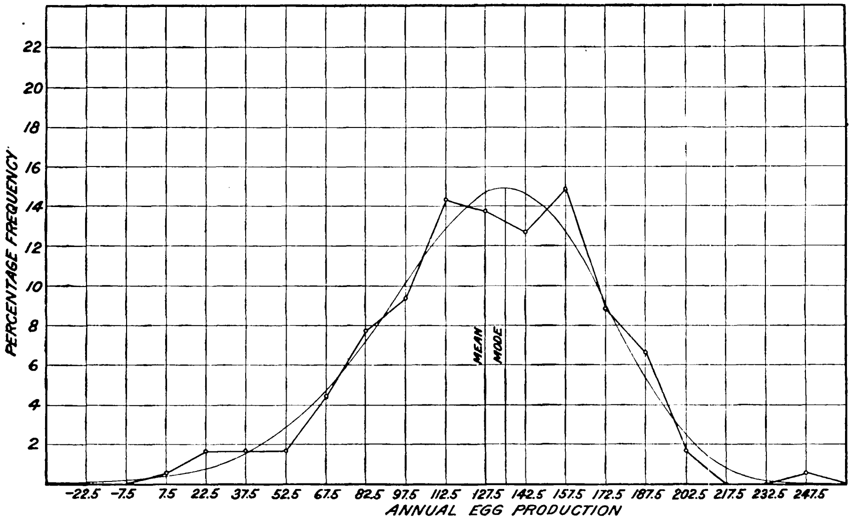

FIG. 4.—Variation in annual egg production, 1905–6, 100-bird pens.

The nearly uniform failure of the fitted curves to give reasonable estimations of the range limits of variation in egg production is remarkable. Anyone who has had any large experience with Pearson's limited range curves knows that they almost never lead to impossible estimations of the range ends. We have given the matter very careful study in the present case, and while we are able easily to understand and explain the results we get with these curves, we have not been able to modify them in any way so as to get a better representation of the actual range by the theoretical. It occurred to us that possibly by using the unmodified moments in the place of those corrected by Sheppard's formulæ we might get a better estimation of the range. Trial, however, proved this not to be the case. In these

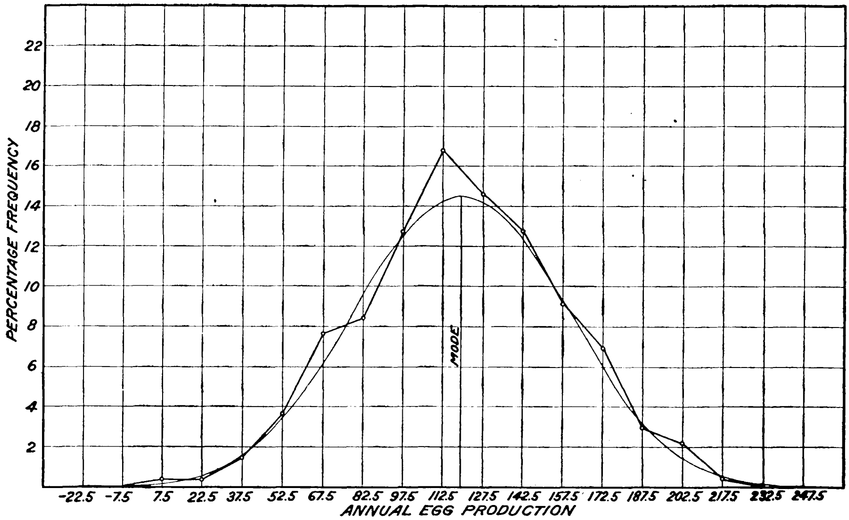

FIG. 5.—Variation in annual egg production, 1905–6, 150-bird pens.

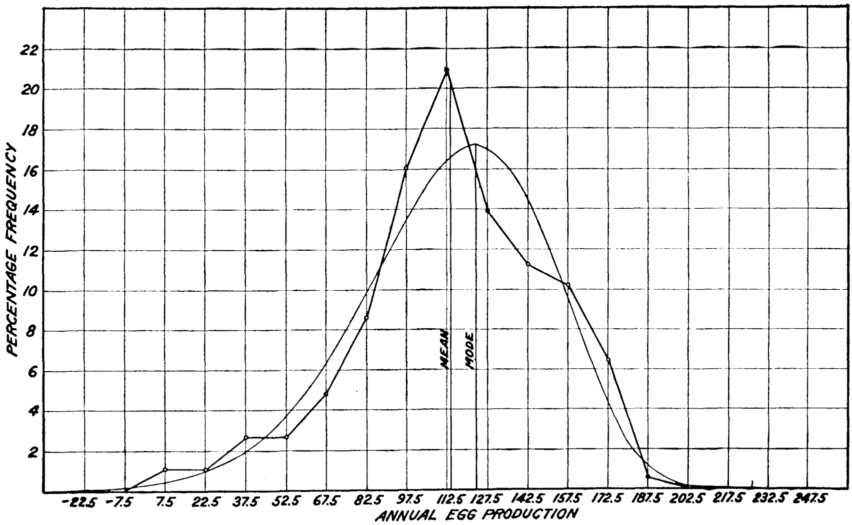

FIG. 6.—Variation in annual egg production, 1906–7, 50-bird pens.

particular curves it made no substantial difference in the range if Sheppard's corrections were or were not used. It is obvious from the diagrams that the general "fit" of all these curves is excellent.

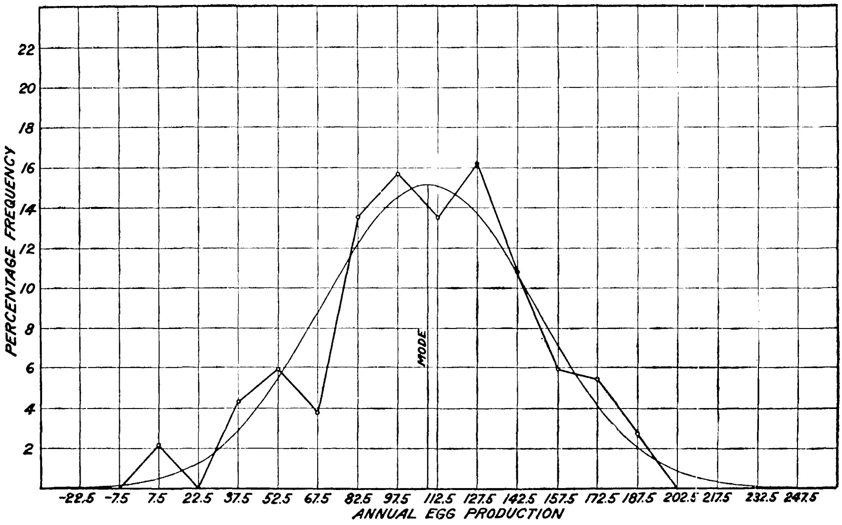

FIG. 7.—Variation in annual egg production, 1906–7, 100-bird pens.

In fact, it would be difficult to get any better graduations than those given by the present curves. Curves of Type I and Type II are the only limited range curves which can possibly be used on these frequency distributions, and it is obvious from Table 4 that one of these types is not superior to the other in the matter of the estimation of the range. Hence we can not get over our difficulty by using some

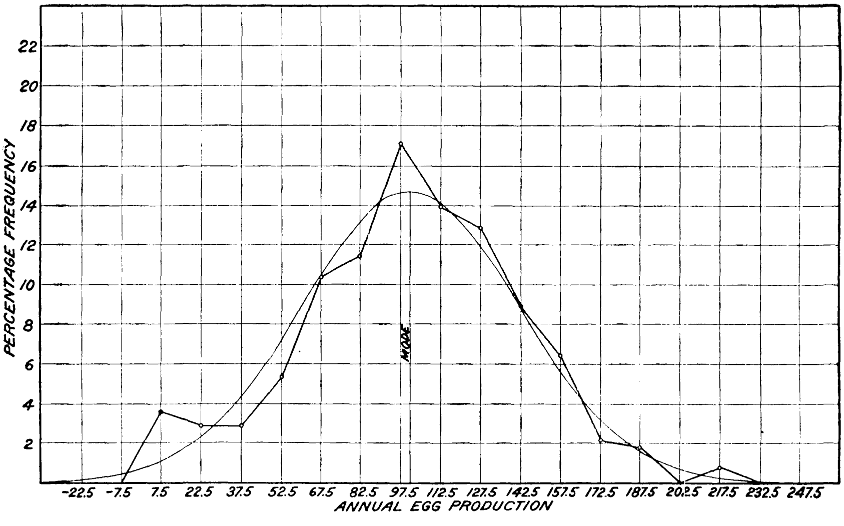

FIG. 8.—Variation in annual egg production, 1906–7, 150-bird pens.

other types of curve than those which actually have been used. A remarkable thing is that in every one of the egg-production distributions the value of the criterion κ_1 is such as to indicate that the distribution deviates from normality in the direction of a limited range curve (either Type I or II). In so far mathematical theory and biological fact agree excellently, because annual egg production is a thing which can only vary within certain limits. The remarkable thing is that while the data demand a limited range curve they lead to impossible values of the range.

Consideration of the matter leads to the conclusion that the chief reasons for the great overestimation of the range by these curves are: (*a*) That in the majority of cases the curves approach very close to the normal type. The range of the normal curve is from $-\infty$ to $+\infty$. As the limited range curve approaches the normal its range tends very rapidly to increase. (*b*) That the distributions are in every case rather irregular, owing to the relatively small number of individuals included in each. It will be observed that in each of the worst cases of extension of the range into the negative there is a group of hens laying very few eggs in the year and producing a pronounced hump in the curve near its lower end. Now the theoretical curve attempts to make allowance for this irregularity, but only does so at the expense of sending a long tail off into the negative. With a larger number of observations there is no doubt that much of the irregularity would disappear from the observational polygons, and with it would go that which in considerable measure causes the great overestimation of the range by the present curves.

The points just mentioned should be given due weight before criticism is made of the theoretical curves used because of their failure to give good estimation of the range. In addition the following facts should not be forgotten in the same connection. First, that the total frequency given by the theoretical curves beyond the observed range is in every case so small as to be practically negligible; and second, that the range is subject to a probable error, both in respect to its extent and its position. In the case of the present distributions both these probable errors must be very large for two reasons—one, that the curves approach closely the normal type; the other, that the number of individuals is statistically small.

There is some reason to believe that characters like fertility and fecundity as a rule lead to theoretical curves with considerable overestimation of the range on the negative side. This is certainly the case with our present data. It is true to an even more marked degree in Powys's [a] curves of fertility in human stock. Examination of the data and diagrams which this author gives shows that in spite of the large numbers of individuals included in his statistics and the consequent smoothness of the polygons the theoretical curves cross the zero line (no children in the family) with a frequency which is sensible in every case, and which in some cases is relatively much greater than anything we have found in the egg curves.

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[a] Loc. cit.

The general conclusion we reach is that the failure of the theoretical curves to give good estimations of the observed range is to be regarded as the result of peculiarities of the observational data rather than an inherent fault of the curves.

(4) It is of interest to note that in these curves the height of the modal ordinate y_0 when expressed as a percentage of the total frequency, is very nearly constant in all cases. Davenport [a] has recently called attention to the significance of the modal frequency as a variation constant.

CHANGES IN EGG PRODUCTION BETWEEN 1899 AND 1907.

In attempting to reach a definite conclusion regarding the obviously important matter of what has been the character and amount of change in the annual egg production during the period covered by our statistics, we are met at once by the difficulty noted earlier in this paper (p. 19). The data are not strictly comparable from year to year. Improvements were made in feeding, housing, and handling the birds during the period covered by the investigation. Proper allowance can not be made for the influence of these factors. Further, various accidents lowered the egg production in particular years. The only feasible way of getting some definite idea of how the variation constants change appears to be the following: First, to determine the changes in the actual values of the constants as they stand. These may be regarded as lower limiting values. The general trend of these having been determined, we may next proceed to modify the constants, for the influence of accidents and the like, from the actual values they show. In every case we can be sure of the general direction in which such modification should be applied. As to what its exact amount in each case should be there are, of course, no criteria. But, if we make the modification relatively large, we shall establish what may be regarded as a series of upper limiting values for the constants. We may then fairly conclude that had all conditions remained normal throughout the period covered by the data, the values exhibited by the constants would have fallen somewhere between the limits set by the actual and the modified values. While such a method can not give us precise values, it is capable of showing what the general trend of the data is. We may first examine the changes which took place in the mean or average annual egg production in the period from 1899 to 1907.

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[a] The Principles of Breeding, p. 422.

CHANGES IN MEAN EGG PRODUCTION.

In considering this problem the question immediately arises as to how the data for 1904 and subsequent years shall be treated. In these years shall we use for comparison with earlier years an average based on all the birds together (i. e., those kept in 50, 100, and 150 bird pens), or only those birds kept in flocks of one size (either 50, 100, or 150)? A little consideration leads to the conclusion that there can be but one answer to this question. We must use the records of flocks of one size only, otherwise the heterogeneity of the material will vitiate conclusions. This being the case, it is further advisable to use the data from the 50-bird flocks, since in the earlier years of the experiment the birds were kept in small flocks, with which the 50-bird flocks of later years are most nearly comparable. Consequently for the years 1904 to 1907, inclusive, we shall use the 50-bird pen records only.

The data for the changes in mean egg production are given in Table 5. The averages given here in the column headed "Actual average production" are those obtained by dividing the figures in the third column of the table by those in the second column. In other words, they are the averages from the ungrouped data given in Table I of Appendix I. On this account they differ slightly from the grouped means of Table 2.

TABLE 5.—*Changes in average egg production between 1899 and 1907.*

Year and pen.	Birds completing the year.	Eggs laid.	Actual average production.	Added to actual average.	Modified average.
1899-1900	70	9,545	136.36	0	136.36
1900-1901	85	12,192	143.44	0	143.44
1901-2	48	7,468	155.58	0	155.58
1902-3	147	19,906	135.42	[a] 23.73	159.15
1903-4	254	29,947	117.90	[b] 11.24	129.14
1904-5, 50-bird pens	283	37,943	134.07	0	134.07
1905-6, 50-bird pens	178	24,827	140.14	[b] 13.95	154.09
1906-7, 50-bird pens	187	21,175	113.24	[b] 29.53	142.77

[a] Cf. page 40.

[b] Cf. page 41.

The first point which will be noted from this table is that the averages given in the fourth column do not agree with those which have been published in the several bulletins of the Maine station, in which reports have been given of the progress of the experiment in breeding for increased egg production. Regarding this discrepancy it need only be said that the previously published averages have several times been in error from one or the other of two causes, viz, (*a*) faulty methods of handling the statistical material, and (*b*) arithmetical mistakes. Any interested person may verify for himself the averages given in Table 5, from the figures given in Table I of Appendix I, which comprises all the data which exist at the station respecting annual egg production in Barred Plymouth Rocks.

As has been pointed out earlier in the paper (p. 19), more or less serious accidents happened to lower the egg production in four out of the eight years covered in the investigation. The "actual averages" of Table 5 give the lower limit to the egg production of each year. The hens could not have produced fewer than the recorded number of eggs. In the years 1902–3, 1903–4, 1905–6, 1906–7 they probably would have produced more than the recorded number of eggs, had not the various accidents occurred. To obtain an upper limiting value for the egg production of each of these years we must add an allowance for the accident so great that it can only be regarded as unreasonably liberal. Then we can say that under entirely normal circumstances the average egg production could not have been greater than that given by the modified averages. We have then to consider separately the allowances to be added to each year's average.

In the year 1902–3 (cf. p. 16, supra) the birds are stated to have molted in December and to have laid very few eggs in that and the succeeding month. This molting was attributed to too heavy feeding during the summer and early fall months. As a matter of fact the average egg production of the 147 birds was 4.46 eggs in December, 1902, and 5.92 eggs in January, 1903; 76 birds laid in December and 77 in January—a little more than half the flock. Now, a reference to Table 2 will show that the egg production of the 50-bird pens in the year 1905–6 was the highest obtained in any year after 1901–2. Taking into account the number of birds involved it ranks highest, since in the early years there were relatively few birds. In the year 1905–6 the only point at which the laying can be regarded as abnormal is in April and May (cf. p. 18). The egg production in the other months of that year must be regarded as normal. If, then, we add to the 1902–3 average the mean egg production for the whole of the two months of December, 1905, and January, 1906, we shall have made a most liberal allowance for the accident. The mean December, 1905, (50-bird pens) production was 9.90 eggs; that for January, 1906, was 13.83 eggs. We may then add 23.73 eggs to the 1902–3 actual average, giving a modified average of 159.15 eggs. There can be no question that this figure may be taken as a safe upper limit to this year's production.

In the year 1903–4 the only disturbing factor in the records is that the birds were not moved into the houses until December 6. Hence they lack any November record, and also that for the first five days in December. The average production for the month of December was 5.67 eggs. Nevertheless, in order to establish a sufficiently high upper limit to the year's production, we may add to the actual 1903–4 average not only the normal average production for the whole month of November (1905, 50-bird pens), but also one-half of the normal average production of December (1905, 50-bird pens). This gives altogether 11.24 eggs, making the modified average for 1903–4 129.14 eggs. There is no valid reason for adding, as a modifying factor, more than the egg production of (*a*) the month during which the birds made no records and, of (*b*) one-half of the month in which they missed five days.

In the year 1905–6 all the conditions were normal, and the egg production was exceptionally high, save for a single mishap, which is said to have affected the April and May totals (cf. p. 18, supra). There is some reason for thinking that only a small allowance should be made for this mishap. The mean production for April is 16.48 eggs and that for May 11.42. In order to be sure of a sufficiently high upper limiting value, however, we may add 50 per cent of the recorded production of each of these months, making the modified April mean $16.48 + 8.24 = 24.72$, and the modified May mean $11.42 + 5.71 = 17.13$. The total modifying factor to be added to the actual mean of this year is then 13.95 eggs, making the modified mean 154.09.

In 1906–7 the application of lice killer to the roosts is stated to have caused a very serious lowering of the egg production in the months of December, 1906, and January, 1907. The actual average egg production (50-bird pens) was for the month of December 4.97 eggs and for January 5.70 eggs. Inasmuch as there is no record of the exact date when the lice killer was applied, we are compelled to add to the actual egg production of 1906–7 as a modifying factor the normal average production for the whole of the months of December and January (based, as before, on 1905 50-bird pen records). A further addition must be made to allow for the fact that this year's records are only for eleven months. There is no record for October, 1907. We may, as in the other cases, add the mean production for this month of the 1905–6 50-bird pens. This gives us the modifying term 29.53 eggs and as the modified average 142.77 eggs.

With the actual and modified averages in hand we may inquire: What has been the general trend of the mean annual egg production during the period covered by the investigation? The clearest answer to this question may be obtained by plotting the figures in the fourth and sixth columns of Table 5, and then striking through each of the two zigzag lines so obtained the best fitting straight line, as determined by the method of least squares. Such a diagram is given in figure 9.

The equations of the two straight lines are as follows:

Actual averages, $y = 148.48 - 3.10\,x$.

Modified averages, $y = 144.13 + 0.043\,x$.

In these equations y denotes the mean annual egg production and x the year. The origin of x is taken at 1898–99—that is, one abscissal unit to the left of the first observation. The reason for so taking the origin is obvious.

Taking the line of the actual averages, it is seen that during the first three years there is a steady rise in mean egg production. The highest mean production during the whole period of eight years is in 1901–2. From this year on the line drops, reaching a very low point in 1903–4. Rising again, the line reaches a second maximum (not so high, however, as the first) in 1905–6. The next year, 1906–7, shows the lowest average production of any year in the experiment. The general trend of the actual averages is obviously downward. This is clearly shown by the fitted line *ab*.

Turning to the line of the modified averages, we note that, while still showing the same individual year fluctuations, its general trend is almost exactly horizontal. As shown by the equation to the straight line *ac*, however, there is an extremely slight upward tendency in the modified averages. It is so small as to be entirely negligible.

Now, the truth must lie between these two lines *ab* and *ac*. One is the graduation of the lower limiting values for each year's average production, the other the graduation of the upper limiting values of what the egg production might have been had not particular accidents occurred. There appears to be but one possible conclusion. In the period from 1899 to 1907 the general trend of average annual egg production has been downward. During the period there have been, as would of course be expected, fluctuations up and down in individual years. These have been no greater, however, than might be expected as purely chance phenomena.

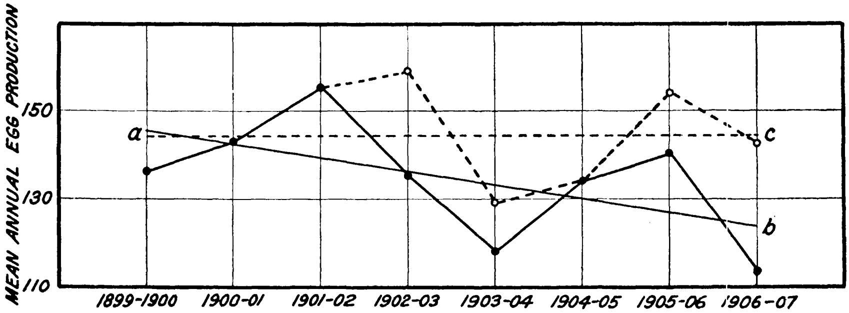

FIG. 9.—Change in mean annual egg production, 1899–1907. The solid lines show the actual averages and best-fitting straight line; the broken lines the modified averages and best-fitting straight line.

It might possibly be contended that we should reach a different conclusion if we took into consideration all the birds of the years 1904 to 1907, instead of those in 50-bird flocks. A little inspection of Table 2 shows, however, that if the mean productions of all the birds of these last three years were to be used, the general trend of the line representing the change of the mean during the whole period would be more sharply downward than it is in figure 9. This must obviously be the case, because, as the last three lines of Table 2 show, the weighted mean of the means is in each case very considerably lower than the 50-bird pen mean of the same year.

The discussion of the relation of the changes in mean annual egg production to the method of breeding practiced is deferred to a later section of the paper.

CHANGES IN THE VARIABILITY OF EGG PRODUCTION.

It has been shown that in spite of fluctuations up or down in particular years there is evidence of a gradual but steady change in mean egg production during the period covered by the breeding investigation. Is there any evidence of a similar change in the constants measuring variability in egg production? On a priori grounds we should expect to find a progressive change in variability because of the selection practiced. During the eight generations included in the investigation the immediate ancestry of the birds was strictly selected. On this account we should expect a reduction in variability until a minimum had been reached.[a] Has there been such a reduction?

In attempting to get light upon this matter one is met, as before, by the difficulty that modification ought to be made in the constants for four of the years to allow for the accidents. The matter is not quite so simple in the case of the variation-measuring constants as it is in the case of the means. The problem may be stated as follows: Let A be a character (say egg production during eleven months) having in a given population of individuals a mean value of m_1 and a standard deviation of σ_1, and let B be another character (say one month's egg production) of the same population having a mean given by m_2 and a standard deviation given by σ_2. What will be the mean and standard deviation of the sum of these two characters $A+B$ in the same population?

We have, if M be the mean and Σ the standard deviation of the combined characters,

$$M=m_1+m_2 \text{ . . . . (i)}$$

and

$$\Sigma=\sqrt{(\sigma_1^2+\sigma_2^2+2r_{12}\,\sigma_1\,\sigma_2} \text{ . . . . (ii)}$$

where r_{12} is the coefficient of correlation between A and B.

Equation (ii) presents a number of points of interest and significance. In the first place it is seen that so long at least as r_{12} is positive Σ will necessarily be greater than either σ_1 or σ_2. Now, as a matter of fact, the egg production of each single month is positively correlated with the total production for the year, as we should obviously expect it to be. That is, r_{12} is in this case always positive. Hence, if we raise the mean egg production of the year by adding as a modifying term the production of one or more months we shall at the same time increase the absolute variability in annual production as measured by the standard deviation.

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[a] For a full discussion of this matter see Pearson, K., The Law of Ancestral Inheritance. Proceedings of the Royal Society, vol. 62, pp. 386–412.

This leads to another consideration. When the total production of one group of hens is greater in a given time than that of another group in the same time it merely means that the aggregate laying within the period has been increased in one lot over what it is in the other. But, according to (ii) above, this means that at the same time the standard deviation will be greater in one lot than in the other. In other words, when the mean egg production changes we should expect the standard deviation to change in the same direction, unless some real alteration in the innate variational tendency in respect to egg production has occurred. We hence must conclude that if, as in the present case, we find the general trend of mean egg production, and at the same time that of the standard deviation in the same character to be downward, we are not justified in assuming from the standard deviation alone that there has been any progressive change in the innate variational tendency. We may expect the standard deviations to become smaller for the same reasons that the means do. The practical conclusion is that judgment of changes in variation in egg production must be based on a relative measure, such as is given by the coefficient of variation.

Further, if an increase in mean egg production means an increase in the standard deviation except there be a real change in the variational tendency of the population, then clearly it is futile to modify the standard deviations to allow for accidents, as has been done in the case of the means, because such a modification is purely artificial and represents no real change in variational tendency. Hence the coefficient of variation will be unchanged after the modification of mean and standard deviation from what it was before. To show that facts accord with theory on this point an actual case may be cited.

Table 6 gives in parallel columns the values of the chief constants of the distributions for 1904–5, when (*a*) the records of twelve months' laying (November 1, 1904, to October 31, 1905) are used, and when (*b*) the totals of the first eleven months of the same laying year (November 1, 1904, to September 30, 1905) are used.

TABLE 6.—*Egg production in 1904–5.*

Constant.*	50-bird pens.		100-bird pens.		150-bird pens.	
	(*a*) October totals.	(*b*) September totals.	(*a*) October totals.	(*b*) September totals.	(*a*) October totals.	(*b*) September totals.
Mean	134.60	129.41	133.61	127.18	114.54	107.96
Median	139.17	132.90	141.00	140.25	117.00	110.67
Mode	149.14	139.71	155.78	166.38	121.93	116.08
Standard deviation	49.49	47.25	49.41	45.90	53.04	49.56
Coefficient of variation	36.77	36.51	36.98	36.09	46.31	45.91
Skewness	−.294	−.218	−.449	−.854	−.139	−.164

* Probable errors are not given in this table, since they are already given in Table 2 for the constants in the (*a*) columns, and those in the (*b*) columns will obviously be of substantially the same magnitude, since the number of individuals involved remains always the same.

From the table it appears that the change in standard deviations made by using twelve instead of eleven month distributions is almost exactly proportional to the change in the means. The coefficients of variation are not sensibly different in the two cases. The results then are, first, that it will not be necessary to modify the standard deviations to allow for accidents, and, second, that conclusions respecting variation in egg production must be drawn from the coefficient of variation rather than the standard deviation.

Taking the data given in Table 2, figure 10 has been prepared. It shows the change in the standard deviations during the eight years covered by the investigation. In the last three years the constants for the 50-bird pens are used as in the case of the means. The straight line is the graph of

$$y = 45.12 - 0.88\,x$$

wherein y denotes standard deviation and x number of years from 1898–99. This is the line which best fits the observations, as determined by the method of least squares.

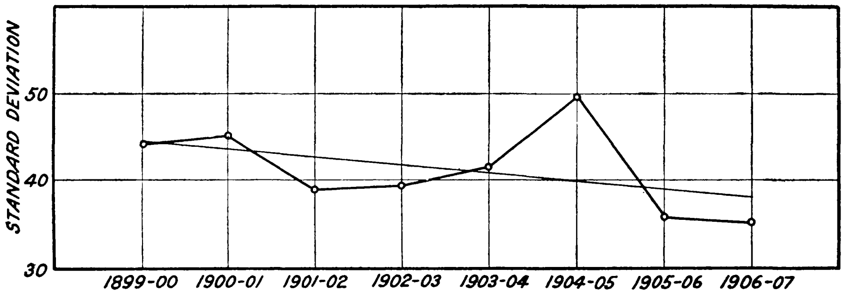

FIG. 10.—Changes in annual egg production. Standard deviations, 1899–1907.

The general trend of the observations as brought out in this diagram is obviously downward. However, since the tendency of the means was downward during the same period, it can not be concluded that there has been a real reduction in germinal variability in respect to annual egg production.

Turning to the coefficients of variation and using the data in Table 2 as before, we have the results shown in figure 11. The constants of 50-bird pens are used in the last three years.

As in the other cases, a straight line was fitted to the observations by the method of least squares. The equation to the line is in this case

$$y = 30.56 + 0.039\,x$$

where y denotes coefficient of variation and x the number of years from 1898–99.

From this diagram it is seen that the lowest relative variability was shown in the year 1901–2 and the highest in the year 1904–5.

A second minimum, nearly as low as the earlier one, occurs in 1905–6. Finally, the variability in the last year of the period (1906–7) is very nearly the same as that at the outstart eight years before. The fitted straight line is almost exactly horizontal; in fact, the change in the eight years is a little less than 0.3 of a unit. The slight slope which the line does show is upward, but no stress can be laid on so small a change on account of the values of the probable errors concerned. The general conclusion to be drawn is that during the period covered by the statistics the degree of variability in egg production in proportion to the mean has not substantially changed. There have been rather wide fluctuations during the period, but these have been as frequent and great in one direction as in the other.

CHANGES IN OTHER CONSTANTS.

Nothing in particular is to be gained by a special examination of the changes which have occurred in the median or modal egg production during the period under investigation. Both these constants run nearly parallel to the means, since the skewness is negative throughout.

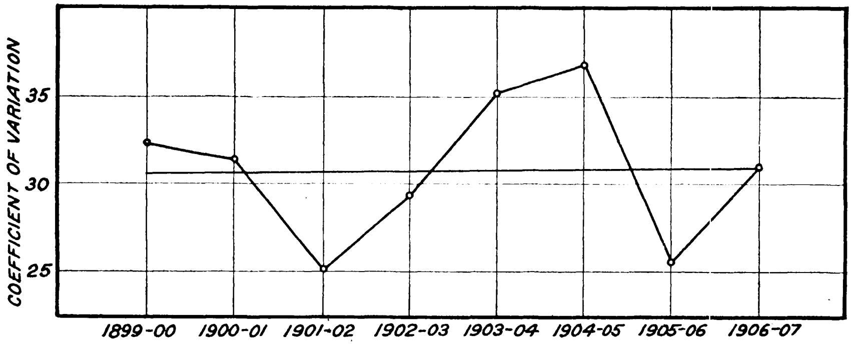

FIG. 11.—Changes in annual egg production. Coefficients of variation, 1899-1907.

It is of some interest to see the trend of the skewness. To this end figure 12 has been prepared. The data are taken from Table 2, and the 50-bird pen constants are used in the last three years. The equation to the straight line is

$$y = -0.265 - 0.0022\,x$$

where y denotes the value of the skewness and x has the same significance as in the other similar equations.

The individual year fluctuations here are rather marked, but there is no evidence of a steady progressive change in the degree or character of the asymmetry of the curves. The fitted straight line is very nearly horizontal.

Another matter of some importance is to determine the proportion of extreme variates in both directions to the total number of birds kept during the several years of the investigation. In this connection the following questions present themselves: In the period from 1899 to 1907 did the number of very exceptionally high-laying hens in proportion to the total number in the flocks increase or diminish, and how great was the change? Similarly, did the porportionate number of exceptionally poor-laying hens increase or diminish, and by what amount? One would expect, of course, in view of the selectionpracticed in the breeding, that in successive years there would be more "good" layers in the flock and fewer "drones." The data to answer these questions are given in Table 7. We have taken as an exceptionally high-producing hen one that laid 195 or more eggs in her first laying year, and as an exceptionally poor-laying hen one that laid less than 45 eggs in her first laying year. The fairness of these designations can hardly be questioned.

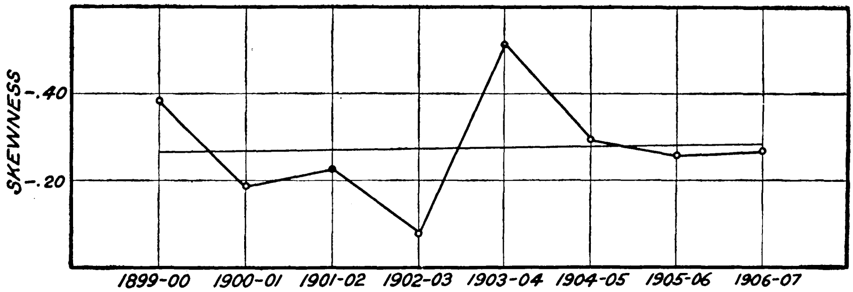

FIG. 12.—Changes in annual egg production. Skewness, 1899-1907.

Table 7 gives for each year the percentage (*a*) of birds laying less than 45 eggs and (*b*) of birds laying 195 or more eggs. In the last three years data from 50-bird pens are used.

TABLE 7.—*Percentage of the flocks laying (A) less than 45 eggs, and (B) 195 or more eggs in a year.*

Annual egg production.	1899–1900.	1900–1901.	1901–2.	1902–3.	1903–4.	50-bird pens.		
						1904–5.	1905–6.	1906–7.
	Per cent.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
Less than 45	4.29	1.18	0	1.36	6.70	7.07	0.56	4.81
195 or more	4.29	10.60	18.75	6.12	.79	12.71	5.06	0

The facts brought out in Table 7 are shown graphically in figure 13, in which the continuous lines show the change in the percentage of exceptionally high layers in the successive years and the broken lines the change in the percentage of very poor layers. The straight lines are the graphs of the following equations, deduced by the method of least squares:

$$\text{Poor layers: } y = 1.795 + 0.3225x$$
$$\text{Good layers: } y = 11.639 - 0.966x$$

In these equations y denotes percentage and x years from 1898–99.

It is noted at once from figure 13 that the changes in the proportionate numbers of exceptionally good layers in successive years very closely parallel the changes in the mean annual egg production as shown in figure 9. In 1899–1900 a little over 4 per cent of the flock laid over 195 eggs in the year. The number increased in 1900–1901 and 1901–2, reaching in this latter year the maximum point of the whole period. This maximum was a high one. Nearly 19 per cent of the flock laid 195 eggs or over. The next two years witnessed a marked decrease, the percentage of high layers reaching in 1903–4 less than 1. In 1904–5 there was another rise in the percentage, with a falling off again in 1905–6 and 1906–7. In the last year there were no birds in the 50-bird pen flocks which laid 195 eggs in the year. As is shown by the continuous straight line, instead of there having been any steady increase in the percentage of high-laying birds in these eight years there has, on the contrary, been a distinct decrease in the percentage.

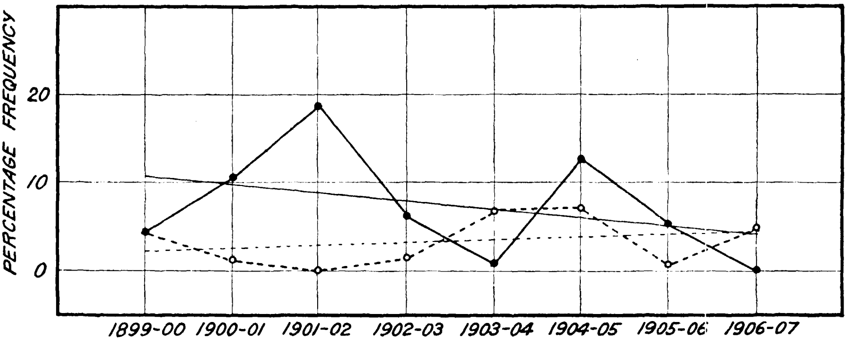

FIG. 13.—Changes in percentages of extreme variates in egg production, 1899–1907. The solid line shows the percentage of birds laying over 195 eggs; the broken line the percentage of birds laying under 45 eggs.

Turning to the poor layers, it is seen that in 1899–1900 there were exactly the same number laying less than 45 eggs as of those laying more than 195. The percentage fell to zero in 1901–2, but during the next three years steadily rose until in 1904–5, 7 per cent of the flock laid fewer than 45 eggs in the year. In 1905–6 there was a drop, with a return in the last year of nearly 5 per cent. Taking all the figures together, it is obvious that the only conclusion which can be drawn is that at the beginning of the selection experiment there were relatively few extreme "drones" in the flock; all the available evidence indicates that after eight years of selection there are as many or more such than there were at the beginning.

Further discussion of the significance of these changes in the number of extreme variates is deferred until a later section of the paper is reached.

VARIATION IN EGG PRODUCTION IN WHITE WYANDOTTES.

Up to this point we have discussed the variation in annual egg production of the Barred Plymouth Rocks only. As was pointed out in the introduction, the Maine station egg records also furnish some data on the same matter for White Wyandottes. To the discussion of these data we may next turn.

The raw material for the White Wyandottes is given in Appendix I, Table II. The grouped frequency distributions (absolute and percentage) are given in Table 8.

TABLE 8.—*Frequency distributions for variation in annual egg production of White Wyandottes.*

Eggs laid.	1899-1900.		1900-1901.		1901-2.	
	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.
0 to 14	1	1.43			1	3.03
15 to 29	1	1.43				
30 to 44	1	1.43	1	1.39		
45 to 59			2	2.78	1	3.03
60 to 74	5	7.14	5	6.94		
75 to 89	5	7.14	6	8.33	2	6.06
90 to 104	6	8.57	4	5.56		
105 to 119	9	12.86	7	9.72	5	15.15
120 to 134	11	15.71	8	11.11	7	21.21
135 to 149	7	10.00	9	12.50	8	24.24
150 to 164	6	8.57	12	16.67	6	18.18
165 to 179	10	14.29	7	9.72		
180 to 194	2	2.86	4	5.56	3	9.09
195 to 209	4	5.71	4	5.56		
210 to 224	2	2.86	1	1.39		
225 to 239			2	2.78		
Total	70	100.00	72	100.01	33	99.99

It will be evident at once from Table 8 and the detailed statistics given in the Appendix that the general features of variation in egg production are very similar in White Wyandottes to what we have already found for Barred Plymouth Rocks. The range of variation is substantially the same in the two cases. As before, the number of observations is in each case too few to fill this range with anything approaching a smooth frequency distribution. It is further apparent that there is a general tendency toward negative skewness in these as in the Barred Plymouth Rock distributions.

The chief physical constants for the distributions of Table 8 are given in Table 9.

TABLE 9.—*Constants of variation in egg production, White Wyandottes.*

Laying year.	Mean.	Median.	Mode.	Standard deviation.	Coefficient of variation.	Skewness.
1899-1900	130.13±3.64	128.50±4.56	125.24	45.15±2.57	34.69±2.20	+0.108±0.097
1900-1901	135.43±3.54	139.50±4.44	147.64	44.53±2.50	32.88±2.04	− .274± .091
1901-2	130.36±4.37	137.50±5.48	151.77	37.22±3.09	28.55±2.56	− .575± .102

From Table 9 we note the following points:

1. The mean annual production of the White Wyandottes is lower in each of the three years than is that of the Barred Plymouth Rocks (Table 2) in the same year. It is extremely doubtful, however, whether this can be taken to represent a true varietal difference from which to conclude that White Wyandottes are poorer layers than Barred Plymouth Rocks. There are several reasons which make such a conclusion too doubtful to carry any weight. In the first place, the number of birds involved is too small. Further, there is no assurance that the same method of feeding is as well suited to egg production in the case of White Wyandottes as in the case of Barred Plymouth Rocks. Further, there is reason to believe that in subsequent years the descendants of the station strain of White Wyandottes came to have a mean annual egg production as high or higher than that of the station Barred Plymouth Rocks. This statement is based on a verbal communication to the writer by Prof. Gilman A. Drew, of the University of Maine, who bred these White Wyandottes after the station dropped them.
2. The White Wyandottes show substantially the same degree of individual variability in annual egg production as do the Barred Plymouth Rocks.
3. In two of the three years the distributions show negative skewness, just as in all cases with the Barred Plymouth Rocks. In the first year the skewness is positive, but its amount is so small and the number of individuals involved is so small that no particular significance is to be attached to this exception to the usual rule for curves of variation in egg production.

The conclusion, so far as any may be drawn from the limited data at hand, is that the general features of variation in annual egg production appear to be substantially the same in White Wyandottes and Barred Plymouth Rocks.

EGG PRODUCTION IN OTHER BREEDS OF FOWLS.

Reliable and extensive statistical data regarding egg production are exceedingly rare in the literature. Furthermore, whenever any such records have been published it has almost invariably been the custom not to give detailed statistics for individual birds, but rather flock averages based on larger or smaller numbers. From such material it is obviously impossible to learn anything regarding variation in egg production. While this fact is greatly to be regretted, there are certain points of interest to be gained by a comparative study of egg-production averages.

A question which will have occurred to the reader is as to how the average annual productions shown in the different years by the present statistics compare with the experience of others regarding egg production with other breeds of fowls and under different environmental conditions. Fortunately there is an abundance of data from which light may be gained on this question. For some years past there have been conducted at various places in Australia egg-laying competitions in which exact records of the number of eggs laid by pens of various breeds have been kept. The most extensive and thorough of these tests have been carried out by the Hawkesbury Agricultural College and Experimental Farm at Richmond, New South Wales. The data which that institution has accumulated on egg production are of great value. It is the fashion among poultrymen in this country to discredit these Australian egg-laying competitions and to aver that the published records of their results are wholly untrustworthy. The authors of the present paper have very carefully studied the published reports of these contests and can find no evidence that those conducted at or under the direction of agricultural colleges or government experimental farms (e. g., Hawkesbury and Rockdale competitions) at least, have not been carried out in a strictly scientific spirit and in a manner well calculated both to accumulate accurate scientific data on egg production, and, as judged by the results, to give data which are valuable from the purely practical point of view. We can see no more reason on a priori grounds for repudiating the data from these contests than one would have in repudiating any other quantitative data published by an Australian college or university.

It may be well to state briefly the manner in which these contests were carried out. The Hawkesbury competitions may be taken as types. The unit of the competition is a pen of six birds of the same breed. Intending competitors send the birds they wish to enter at a specified time to the college authorities. Each pen of six has a separate house and yard. All yards and houses are of the same size. The birds are cared for throughout the year, and the egg records are kept by an official of the college. The attempt is made to locate each house and yard in such way that the environmental conditions shall be as nearly as possible identical for all birds in the competition. One would judge from the photographs and descriptions which have been published that this ideal of environmental uniformity has been unusually well attained. At the inauguration of the first of these contests the following regulations for its conduct were drawn up:[a]

1. The competition to commence on the 1st of April and extend to the 30th of September,[b] a period of six months calendar.
2. The competitors will be bound to pen their birds during March, each pen to consist of six pullets or hens of any age, no male birds to be included.
3. All birds to be bred by the competitor.

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[a] Agricultural Gazette of New South Wales, Vol. XVIII, p. 522. 1907.

[b] The reader will not overlook the fact of the reversal of the seasons in the southern hemisphere. This first contest was a winter egg-laying competition.

4. All birds to be examined by the poultry expert on arrival at the college, and any found to be suffering from infectious or contagious disease to be rejected; in the event of a bird dying, the competitor to be allowed to replace it.

5. All eggs to become the property of the department of agriculture.

6. All competition to be decided by the greatest total number of eggs laid by each pen, eggs under $1\frac{1}{2}$ ounces not to count.

7. The market value of the eggs laid to be recorded and the weight of eggs from each pen, and prizes given for the greatest aggregate weight.

8. Records to be kept of the total quantities of the various feeds consumed and the average cost per head.

After the first contest, rule 1 was changed to include a period of one year in the test. Later rule 2 was also changed in the direction of restricting the age of competing birds, so that the year of the test was the bird's pullet year in each case.

From the standpoint of the scientific study of egg production there is one point open to serious criticism in these Australian statistics. It lies in the fact that no account is kept of the performance of individual birds. The pen instead of the individual is made the unit. This obviously detracts considerably from the scientific availability of the statistics. Another point which is somewhat open to criticism is that involved in the last clause of rule 4, which states that in event of a bird dying the owner may replace it with another. This procedure, however, probably does not seriously affect the averages for comparative purposes. The mortality is stated to have varied in the five years of competition at the Hawkesbury College between 3 and 9 per cent. Further, the plan of replacing each hen that dies with another will give averages for yearly production which we have ascertained to be very closely comparable with those of this paper which exclude the dead, i. e., which count only birds which laid during the whole year. The reason for this is apparent. Suppose a hen dies after having laid during three months. If her record is included with that of birds completing twelve months' laying, its effect will obviously be to lower the flock average. Suppose, however, that when she dies, her place is taken and the twelve months completed by another bird of the same age, and the two are counted as one bird laying twelve months. The effect on the flock average will obviously be to make it very close to what it would have been if the first bird had not died. The only difference which can be produced is that which will arise if the second bird is a distinctively better or poorer layer than the one she replaces. With a small mortality, and any care whatever in the selection of birds, the effect on flock averages must be so small as to be practically negligible. For purposes of comparison with our own statistics, in which only the birds laying the full twelve months are included, it is much better that dead birds should be replaced than not.

The following example will show how small is the effect produced on the mean production by substituting birds for those that die.

From the records for 1905–6 we first chose six birds quite at random. This was done by taking every twentieth bird, beginning with No. 23, i. e., Nos. 23, 43, 63, 83, 103, and 123. Now, let us suppose that two of these birds, say Nos. 43 and 83, died during the year of the test, and assume that No. 43 died January 31 and No. 83 July 31. Now assume that the laying year of each of these two birds was completed by another bird taken at random from the flock. How will the average of the six birds for the year, provided the substitutions are made, differ from what it would have been supposing the two birds, 43 and 83, to have completed their year for themselves? For the substituted birds two numbers were written down at random. They were Nos. 494 and 251. We have the following figures:

Bird Nos.	Actual laying of birds during whole year.	Until time of assumed death, laying of two birds assumed to have died was—	Substituted birds laid, from assumed time of death to end of year—	Assuming that substituted birds finished year for those dying the laying was—
23	152			152
43	159	Laying to Jan. 31, 33	113 (No. 494)	146
63	111			111
83	124	Laying to July 1, 116	27 (No. 251)	143
103	181			181
123	147			147
Total	874			880

Average production with no death occurring, 145.67 eggs.

Average production assuming that two birds died and that other birds completed the year for them, 146.67 eggs.

The difference between the two averages is obviously insignificant.

For purposes of comparison and illustration some of the data obtained in these Australian contests may be introduced here.

The results of the fifth and most recent Hawkesbury competition[a] which are of significance for our present inquiry are reproduced in Table 10.

TABLE 10.—*Average annual egg production of various breeds.*

[Data obtained in fifth annual egg-laying competition conducted by the Hawkesbury Agricultural College, New South Wales.]

Breed.	No. of birds.	Average egg production per hen in first laying year.	Breed.	No. of birds.	Average egg production per hen in first laying year.
Cuckoo Leghorns	6	190.16	Partridge Wyandottes	6	164.16
Langshans	18	188.88	Buff Wyandottes	12	163.75
Black Orpingtons	120	178.41	Buff Leghorns	18	160.55
S. C. Brown Leghorns	30	177.00	Buff Orpingtons	18	150.11
S. C. White Leghorns	138	174.93	White Wyandottes	24	146.70
R. C. Brown Leghorns	12	173.50	Black Leghorns	6	138.33
R. C. White Leghorns	12	172.66	Houdans	6	137.33
Golden Wyandottes	12	171.33	Faverolles	3	126.66
Silver Wyandottes	126	170.51			
Minorcas	24	168.91	Total	597	171
Rhode Island Reds	6	166.66			

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[a] Loc. cit., p. 532.

Similar data from the second Rockdale competition[a] are presented in Table 11, in which the averages given have been calculated from the raw data given in the report of the contest.

TABLE 11.—*Average annual egg production of various breeds.*

[Data obtained in second annual Rockdale egg-laying competition.]

Breed.	No. of birds.	Total eggs laid.	Average production per hen in first laying year.
Brown Leghorns	24	5,100	212.50
White Wyandottes	18	3,765	209.17
Andalusians	6	1,255	209.17
White Leghorns	96	19,763	205.86
Houdans	6	1,196	199.33
Minorcas	6	1,174	195.67
Black Orpingtons	96	18,459	192.28
Silver Wyandottes	78	14,748	189.08
R. C. Minorcas	6	1,116	186.00
Buff Orpingtons	18	3,097	172.06
Langshans	6	774	129.00
Total	360	[a] 70,447	195.69

[a] This total is incorrectly given as 70,437 in the original table (loc. cit., p. 595).

From these tables it will be noted, first, that an annual egg production very considerably higher than anything observed in the experience of the Maine station is possible even with large numbers of birds. Averages of 205.86 eggs for 96 White Leghorns and 192.28 eggs for the same number of Black Orpingtons (Table 11) are far in excess of any American experience of which the authors are aware. It must, of course, always be kept in mind that these Australian averages are from selected birds, but also (in the case of figures here given) that the selection is made in advance of any direct test of the laying capacity of the selected birds. To be able to select from a flock of pullets which have not yet begun laying a pen of individuals capable of making an annual average such as those we find in these Australian statistics would certainly seem indicative of high laying capacity in the general flock. As a further illustration of this point we may cite a specific instance. In the second Rockdale competition (Table 11) the highest single pen consisted of 6 White Leghorns. In the year they laid 1,473 eggs, making an average of 245.5 eggs per bird. Now, as an examination of Table I of the appendix will show, if we select, a posteriori, the 6 best laying birds which have been found in the whole eight years covered by our statistics, we get the figures on the opposite page.

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[a] Loc. cit., pp. 591–691.

	Eggs laid.
1899–1900	237
1901–2	240
1904–5	234
1904–5	234
1904–5	248
1905–6	246
Total	1,439

These figures give an average per bird of 239.83. Or, in other words, taking the 6 best laying birds that have ever appeared in the Maine station's Barred Plymouth Rock flocks, their average annual laying is nearly six eggs lower than that of the best pen in the particular Australian competition cited. In general we must conclude that the strain of Barred Plymouth Rocks with which we are dealing can not be regarded as an exceptionally high egg-producing one in comparison with other strains and breeds.

A question which constantly recurs to the mind of one studying the problem of egg production is: How does the egg production of the "improved" strains and breeds of the present day compare with that of the domesticated fowl of earlier times?

It is the common opinion that, with the greater attention which has been paid in recent years to poultry raising as a definite and independent branch of agriculture, there has been a great improvement in the so-called "utility" points of the domestic fowl. There have unquestionably been great improvements made in the last twenty-five years in the management of poultry. Equally there can be no doubt that with this improvement in methods of management there has been a decided betterment in the general average condition, in respect to "utility" points, of the domestic fowl. It is not so clear, however, whether within modern times there has been any marked amelioration in the innate qualities on which high egg production depends. This may at first sight appear to be an unwarranted statement. A study of the available evidence, however, we believe, can only lead to the position of doubt on the question just expressed

It should be understood clearly that the question is not as to whether, for example, the average egg production, within a given period, of hens in general is at the present time greater than it was fifty or a hundred years ago. It undoubtedly is. Rather, the significant point is whether, if a given lot of hens of, say a century ago, had been fed, housed, and handled in the same way that our so-called best laying strains are to-day, their egg production would not have been substantially the same as that of the present-day flock. The answer to this question, so far as egg production is concerned, is by no means certain.[a]

The difficulty, of course, lies in getting adequate and trustworthy evidence as to the egg-producing ability of fowls in any but the most recent times. In the older literature one finds plenty of statements that such and such a breed is "noted for its excellence in laying," and the like, but any precise statement as to the criterion of "excellence in laying" then in vogue is almost invariably wanting. Further, when numerical data are given they demand the most careful scrutiny before they can be accepted, because too often it appears from the context that the recorder has some personal interest in making the egg-production record a high one. In such cases the data must obviously be taken with a grain of salt. We have been able, however, to find a few definite records of egg production in earlier times, which appear to be in every respect trustworthy. There is no more reason to doubt their accuracy than there is to doubt the accuracy of any quantitative data of the same period. A few such records may be cited.

In the Journal of Agriculture (English), Volume XI, pages 339 and 340, the following detailed records of egg production are given, on the authority of the editor, in an article on artificial hatching:

Produce of three Poland pullets hatched in the preceding June, from December 1, 1835, to December 1, 1836.

	Eggs laid.		Eggs laid.
December	12	July	55
January	50	August	55
February	48	September	55
March	50	October	25
April	54	November	9
May	55		
June	56	Total	524

These figures give as an average production for the year 174.67 eggs per bird, certainly a very creditable performance. We can discover no reason for doubting the accuracy of these figures. They are not presented as in any way exceptional, but rather so far as can be judged from the context, as a definite numerical statement, by way of illustration, of what any poultry raiser might reasonably expect the performance of his hens to be under ordinarily favorable conditions.

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[a] It is of course certain if the inquiry concerns itself with the so-called "show" points of poultry, such as "shape," "feather," color, and the like. In respect to these characters fanciers have wonderfully improved all the standard breeds. A glance at the plates in such works as Tegetmeier's Poultry Book, for example, affords sufficient evidence of the improvement here.

Another statement of the same general tenor is to be found in an "Essay on the rearing and management of poultry" by William Trotter, published in the Journal of the Royal Agricultural Society in 1851 (vol. 12, pp. 161–202). The essay was awarded a prize by the Royal Agricultural Society in the year mentioned, and hence, we may feel reasonably sure, was subjected to careful and critical scrutiny before publication. It seems, under the circumstances, altogether unlikely that any serious misstatement of fact would have occurred in the essay in the first instance, or if it had occurred, would have been allowed to pass into print. On page 169 (loc. cit.) the following statement is to be found: "Hens of the best laying varieties will lay in a season from 160 to 270 eggs each, averaging 215." If any credence whatever is to be put in this statement, it would appear to indicate that as high egg production as any that we know now was regarded as at least possible fifty years ago.

In another essay which was awarded a prize by the Royal Agricultural Society, and was published in the journal of that society in the year 1867 (series 2, vol. 3, pp. 520–532) a still more definite and precise statement respecting egg production is to be found. The essay is by Mrs. F. Somerville, and its title is "On the rearing and management of poultry on an ordinary farm." The same conclusions as were reached concerning Mr. Trotter's essay appear to apply regarding the trustworthiness of statements in this latter one. On page 532 (loc. cit.) it is stated that in one year "104 hens produced 13,739 eggs *exclusive of those set.*[a] This leads to an average production per hen of 132.11 eggs in the year. The author goes on to state that the 104 hens "reared 372 chickens, besides hatching the ducks and guinea fowls." Now, assuming that it took but 400 eggs to produce 372 chickens, we have the total egg production 14,139 in the year. This gives an average of just under 136 eggs. This can only be regarded as comparing very favorably indeed with the best of our modern records of egg production in equally large flocks, especially when it is considered that these hens had to do all the incubating of at least 372 hen eggs, 79 duck eggs, and 42 guinea-fowl eggs, these being the numbers of each hatched.

We have a number of other references to statements similar to those quoted, but what have been given will suffice for our present purpose. They show that there is trustworthy evidence that at about the middle and the first half of the last century there existed individual birds and flocks of poultry whose ability as egg producers was at least the equal of that to be found in poultry at the present time.

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[a] Our italics.

THE INFLUENCE OF CERTAIN HOUSING CONDITIONS ON ANNUAL EGG PRODUCTION.

As has been noted earlier in this paper (p. 17), there has been conducted during the last three years covered by our statistics an experiment to determine the influence of size of flock and amount of floor space per bird on egg production. The general plan of this experiment has already been described and need be only briefly reviewed here. The birds in the experiment were handled in flocks of 50, 100, and 150. The pens were so constructed that in the flocks of 50 and 100 birds there was an allotment of 4.8 square feet of floor space, while in the flocks of 150 birds there was an allotment of 3.2 square feet of floor space per bird. The general plan of construction of the houses in which these birds were kept throughout the course of the experiment has been fully described in earlier publications[a] and need not be taken up again here. The data accumulated in this experiment furnish evidence on two different questions. In the first place, we have data on the influence of the number of birds associated together in one pen on the egg production when the floor space per bird is the same. Evidence of this sort one can, of course, get by comparing the egg records for 50-bird pens and 100-bird pens. In addition there are data on the further question of the influence of amount of floor space per se on the egg production when one brings into the comparison the 150-bird pens.

In conducting this experiment on the influence of floor space on egg production great care has been exercised to make constant all conditions other than housing. The birds used in the experiment have all been of substantially similar breeding. They have been in every case birds bred from mothers laying between 160 and 199 eggs in a year and from fathers whose mothers and earlier ancestors in the female line were birds laying 200 or more eggs in their first laying year. Furthermore, in sorting the birds into the various pens in the fall of each year great care has been taken to make a pro rata distribution of the pullets of different qualities in the different pens. Thus, for example, the pullets on the range would be examined and the very best of them selected, then this selected group would be apportioned in the proper proportion to the different pens in the experiment. The 100-bird pens would receive twice as many of this class of birds as did the 50-bird pens and the 150-bird pens three times as many. Then the second choice would be made of the pullets on the range, and these again divided among the houses in the same proportional manner. This procedure assured as even distribution of birds in the several components of the experiment as probably could in any way be obtained.

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[a] Maine Agricultural Experiment Station Bulletin 100 and Bureau of Animal Industry Bulletin 90.

It is the purpose at this point to discuss the results of this floor-space experiment only in so far as they have to do with the influence of this factor on annual egg production. In later parts of the work we shall discuss the results of the experiment in respect to the laying in different months of the year, and in other ways shall attempt to make a closer analysis of the data.

We may examine first the influence of the factors size of flock and amount of floor space per bird on the mean or average annual egg production. The data on this matter are presented in Table 12. In addition to the actual means of the different pens for each year there is also given in the last column of this table the excess in the number of eggs produced per bird in the 50-bird pens over that in the 150-bird pens. It will be noted that in this table the actual means are given. It is, for obvious reasons, not necessary to make any modification of any of these averages to allow for the effect of accidents, as was done earlier in the paper in discussing the change in average annual production from year to year. Here the figures which we desire to compare are all based on records of the same year; hence there is every reason to suppose that the influence of any accident which may have occurred will be equally distributed in all parts of the data. That is to say, for example, there is no evidence for supposing that either of the accidents which occurred in the years 1905–6 and 1906–7 affected the 50-bird pens in a different way from what they did either the 100 or the 150 bird pens. Consequently we are justified in using the actual means as they stand.

TABLE 12. —*Relation of size of flock and floor space to mean annual egg production.*

Year.	Mean annual egg production.			Excess of 50 over 150 bird pens.
	50-bird pens.	100-bird pens.	150-bird pens.	
1904–5	134. 60±1. 98	133. 61±3. 47	114. 54±3. 02	20. 06±3. 61
1905–6	140. 31±1. 81	127. 50±2. 03	119. 43±1. 54	20. 88±2. 38
1906–7	114. 16±1. 74	108. 53±1. 92	101. 08±1. 64	13. 08±2. 39
Mean	129. 69	123. 21	111. 68	18. 01

The data in Table 12 are shown graphically in figure 14. From the table and the diagram we note at once the following points:

1. It is obvious that the mean annual egg production is very markedly affected by differences in the environmental factors with which we are dealing. In each year there is a decided difference in the average production of the different pens. Further, it is clear that the general trend of the annual averages is downward as the number of birds in the pen increases. In every instance the mean for the 100-bird pens is smaller than that for the 50-bird pens of the same year, and the mean for the 150-bird pens is again smaller in every case than is that for the 100-bird pens.

2. The decrease in the mean annual production as we pass from the 50-bird pens to the 100-bird pens is, in round numbers, taking the average figures for the whole three years, about 6 eggs. The difference between these two types of pens is not altogether regular from year to year. In the year 1904–5 it is so small as to be insignificant in comparison with its probable error. In 1905–6 the difference is relatively large, amounting to approximately a dozen eggs. In 1906–7 it is about the average, namely, 6 eggs. These results, it will be observed, are perfectly consistent in their general trend throughout the whole course of the experiment, and there appears to be but one conclusion possible. That conclusion is that with the general plan of feeding and housing followed at the Maine station, even though the amount of floor space per individual bird remains the same, the average egg production per bird in a year's laying is distinctly and significantly smaller when the birds are in flocks of 100 than it is when they are in flocks of 50 birds each.

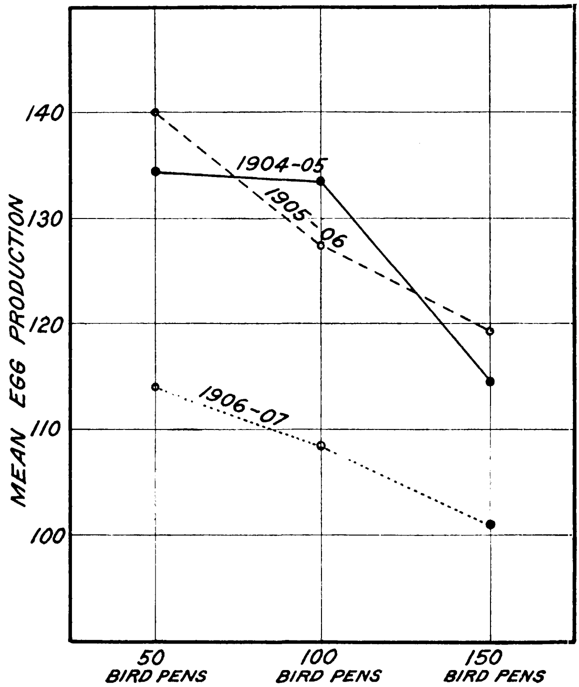

FIG. 14.—Mean annual egg production with different amounts of floor space per bird.

3. The differences between the averages for 100-bird pens and 150-bird pens are, on the whole, distinctly larger than those between the 50-bird and the 100-bird pens. Taking average figures, the mean annual production is approximately a dozen eggs smaller in the 150-bird pens than it is in the 100-bird pens. As before, the absolute amount of the decrease varies in different years, although the general trend is entirely consistent throughout the course of the experiment.

From these results it must be concluded that under the conditions of the experiment, when we have the combined influence of an increase of number of birds in the flock together with a decrease in the amount of floor space per bird, there is a notable decrease in the average annual egg production.

4. If we consider the maximum difference due to the influence of different amounts of floor space and flock size, as shown between 50 and 150 bird pens, it is seen to be large in amount and relatively constant in different years. In round numbers, the birds in the 50-bird pens averaged to lay a dozen and a half eggs more per bird than did those in the 150-bird pens. Such a large difference is of special interest, as showing what a considerable effect on egg production the environmental factors with which we are dealing may have.

We may next turn to a consideration of the influence of size of flock and floor space per bird on the variability in annual egg production. An examination of Table 2 shows that as the number of birds per pen increases, the standard deviation in egg production also tends to increase. This indicates at once that there is a real increase in the variability of egg production of the large as compared with the small flocks because, as has just been pointed out, the mean egg production decreases as the size of the flock increases. In order to obtain a fair notion of the change in relative variability, we may, as before, resort to the coefficient of variability as the measure. In Table 13 are given the coefficients of variation for each of the different sized flocks and for each of the three years over which the experiment extended. In Table 13, as in Table 12, there is also given in the last column a statement of the maximum differences shown in the pens of different sizes.

TABLE 13.—*Relation of size of flock and floor space to variation in annual egg production.*

Year.	Coefficient of variation in egg production.			Excess of 150 over 50-bird pens.
	50-bird pens.	100-bird pens.	150-bird pens.	
	Per cent.	*Per cent.*	*Per cent.*	*Per cent.*
1904-5	36.77±1.17	36.98±2.07	46.31±2.23	9.54±2.52
1905-6	25.48±0.97	31.84±1.23	31.72±1.00	6.24±1.39
1906-7	30.91±1.18	35.54±1.39	40.43±1.32	9.52±1.77
Mean	31.05	34.45	39.49	8.43

The data of Table 13 are shown graphically in figure 15. From the table and the diagram we note the following points:

1. The relative variability in annual egg production is evidently influenced very decidedly by the environmental conditions. The general trend of the variability, as has been pointed out, is to increase as the size of the flock increases, although there is not so great a degree of regularity in this change as there is in the case of mean egg production.

2. As to the amount of change in variability with the increase in the size of the flock, if we take average figures, it would appear that it is fairly uniform from 50 to 100 and from 100 to 150 bird pens. In each case there is an increase on the average of about 4 per cent in the coefficient of variation. In individual years the increase may not be uniform. For example, in 1904–5 there is practically no difference in the variability in the 50 and 100 bird pens. Again, in 1905–6 the 100-bird pens and the 150-bird pens show substantially the same degree of variation. This apparent equality in variability in egg production in different pens is probably to be explained, however, merely as a result of random sampling, the real tendency being that indicated in the average figures already discussed.

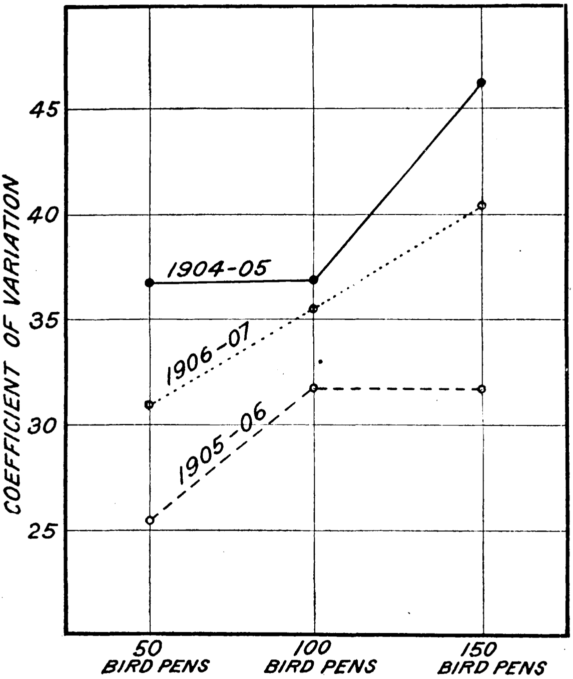

FIG. 15.—Variability in annual egg production with different amounts of floor space per bird.

3. Taking the maximum differences in variability produced by the environmental differences, we see that they are in the neighborhood of 8 to 10 per cent. Since the probable errors of these differences are about 2 per cent, it is obvious that the effect on variation in annual egg production of the environmental factors, with which we are dealing, is an entirely real and definite one. The general result, from present data, would appear to be that an environmental influence which tends to lower the mean annual egg production also tends to increase the variability of the egg production, so that poor-producing flocks are, at the same time, very variable in egg production.

Putting all the results of the floor-space experiment together, they lead to the conclusion that as the size of the flock is increased there tends to be a decrease in the average annual egg production per bird, and this relation holds true whether the floor space per bird is or is not diminished. Further, those conditions relative to size of flock and floor space which tend to cause lower mean annual egg production also tend to increase variation in the production of the flock as a whole. It must be borne in mind that these conclusions apply only to annual egg production, and that from them one must not draw the practical conclusion that it is desirable under all conditions to keep laying hens in small flocks. It will be shown in Part II of this paper, when we come to examine the influence of size of flock and floor space on the egg production during different seasons of the year, that the practical conclusion, relative to the proper floor space and size of flock for laying birds, will be somewhat modified from what one would conclude from the data here presented. For this reason we shall defer drawing any practical conclusions from this experiment or making any suggestions relative to flock management in regard to floor space until the data to be brought out in Part II are at hand.

It will no doubt have occurred to the reader that the results set forth in this section regarding the effect of a strictly environmental factor on egg production have a very significant bearing on the results of the breeding experiment which has been conducted with these birds. It has been shown that a comparatively slight environmental change can in one year produce relatively great changes in mean annual egg production, and, still more important, in the variability of egg production. From this fact alone it would seem doubtful if the method of breeding which has been followed has made any real change whatever in the innate characteristics of the strain relative to egg production. A full discussion of this point will, however, be deferred until a later section of the paper, where the whole question of the influence of selection on egg production will be taken up.

EGG PRODUCTION IN THE SECOND LAYING YEAR.

Up to this point we have dealt entirely with the egg production in the first laying year, all the egg records ending with the second October following the hatching of the birds. In an earlier section of the paper it has been shown that this first-year egg production may be taken as a fair measure of the innate egg-producing capacity of a bird. It is, however, a matter of some interest to study egg production during the later portion of a hen's life. Unfortunately the data available for such a study are exceedingly meager. In the records of the Maine station there are returns of more than one year's laying for less than a hundred birds. Furthermore, these records of the laying during more than one year are not all that could be desired in another respect. They are records of selected birds and not of a random sample of any particular flock. The birds chosen to be carried through more than one year and subjected to trap-nest records are in most instances birds which laid unusually well during their first laying year. As a matter of fact, as will appear later, the mean egg production of these birds in their first laying year was higher than any other average production which has ever been observed at this station. This, of course, is merely the result of selecting especially high-laying birds. The available data on egg production in the first two laying years have already been published.[a] From these published data have been taken only the birds which are recorded as having laid some eggs, at least, in both of the first two laying years. This has given for the present discussion the records of the egg laying for two years of 66 birds. The egg records for the two years of these birds are given in Table III of Appendix I.

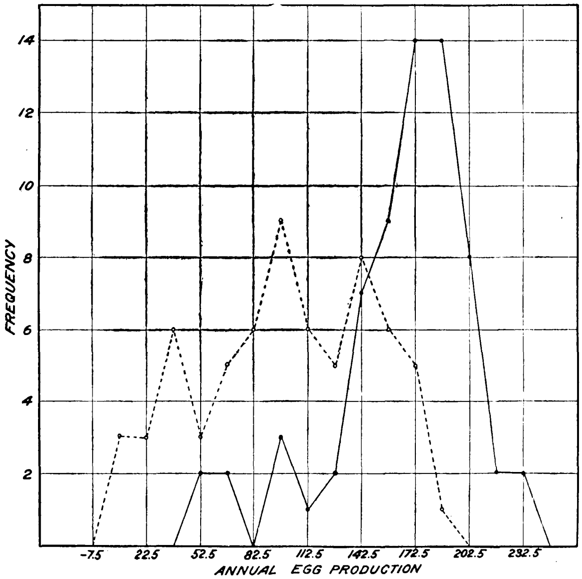

FIG. 16.—Frequency polygons for egg production during the first and second years of laying. The solid line gives the record of the first year, the broken line those of the second year.

By grouping the egg production as in the other cases into class units of 15 eggs each and forming separate frequency distributions for the first and second years' laying, we have the results shown graphically in figure 16.

The number of individuals involved is so small that the variation polygons are rather irregular. However, they suffice to show the general trend of the variation in these two periods. It is noted at once that the variation polygon for the first year's laying possesses the same general characteristics as the others which have been discussed earlier in the paper. Further, as is of course to be expected, the diagram shows that the second year's production is on the average very decidedly smaller than that of the first year. The material shows in the first year an observational range of variation of 190 eggs—the lowest-yielding bird of the lot having laid 47 eggs during the year and the highest 237 eggs. In the second laying year of the same individuals the lowest-yielding bird laid only 8 eggs during the year and the highest-yielding bird 185 eggs, giving a total observed range of 177 eggs.

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[a] Maine Agricultural Experiment Station Bulletin 93, p. 75. 1903.

While the polygons are much too irregular, on account of the relative fewness of the individuals involved, to enable any final conclusion to be drawn regarding such matters as skewness, yet it seems to be indicated that the polygon of variation for the second year's laying is much more symmetrical about the mean than is that for the first year's laying. The first year polygon is clearly negatively skew, like the great majority of the other egg-production frequency distributions which we have examined.

Turning to the constants of variation in egg production for the two years, we have the results set forth in Table 14.

TABLE 14.—*Constants of variation in first and second year egg production.*

Year.	Mean.	Standard deviation.	Coefficient of variation.
First laying year	162.95±3.30	39.71±2.33	24.37±1.50
Second laying year	100.47±3.99	48.06±2.82	47.84±3.38

The first point to be noted from Table 14 is the relatively high mean production for the first year—in round numbers, 163 eggs. As has been pointed out, this is due merely to the fact that it is based on a group of birds selected for their egg production and not on a random sample of any year's flock. In the second year, in this particular case, the mean production decreased by rather more than a third of what it was in the first year. Whether the difference between the laying of the first and second years would be the same in a random sample of birds it is impossible now to say. We hope later to get some definite evidence on this point.

It is obvious from the table that, however it is measured, the second year's egg production is very much more variable than is that of the first year. The standard deviation for the second year's production is nearly 10 eggs greater than is that for the first year's. Since at the same time the mean egg production has decreased, it is obvious that the relative variability, as measured by the coefficient of variation, must be very much greater in the second year. This is in fact the case. The relative variability in the second year is nearly twice as great as it is in the first. It will be noted that the variability of this group of 66 birds, in spite of the fact that they were a selected lot, is not widely different from what we have found for samples not especially selected.

A point of much interest is to determine what degree of correlation exists between the laying of the first and the second years. If a bird is an especially good layer in its pullet year will it, on the average, be an especially good layer in its second laying year; or, conversely, if it is a very poor layer in the pullet year will it, in the average case, be an exceptionally poor layer in its second year of life? The only way in which a definite answer can be obtained to this question is to measure the degree of correlation which actually exists between the first and the second year's laying, and then see whether the coefficient of correlation so obtained is significant in comparison with its probable error. Accordingly, we have calculated this coefficient for our present material. On account of the relatively small number of birds represented in the statistics we have not grouped the material in any way in making the calculation. For this material the correlation between the first and the second year's egg production is given by a coefficient,

$$r = 0.032 \pm 0.083$$

while this correlation is positive, yet it is extremely small and obviously insignificant in comparison with its probable error. We can only conclude that so far as any evidence from our present material is concerned there is no sensible correlation between the egg production of the first laying year and that of the second.

We may next consider the characteristics of variation in the combined egg production of the first and second laying years. This is a point of practical as well as theoretical interest, because it has been contended that the proper measure of a bird's producing capacity is the laying of the first two years rather than that of the first year. At the present time there are being conducted in Australia egg-laying competitions in which the egg production of the first two years is taken as the unit. If we add together for each of our 66 birds the production in the first and the second year and then form a frequency distribution, using a class unit of 40 eggs, we get the distribution given in Table 15. In addition to the frequency distribution itself, there are also included in Table 15 the values of the chief physical constants deduced from the distribution.

TABLE 15.—*Distribution and constants of variation in combined first two years' egg production.*

Eggs laid.	Frequency.	Eggs laid.	Frequency.
80–119	3	360–399	4
120–159	3		
160–199	5	Total	66
200–239	12		
240–279	15	Mean	263.42±5.84
280–319	12	Standard deviation	70.29±4.13
320–359	12	Coefficient of variation	26.68±1.67

It is obvious that the general characteristics of variation in the egg production of the first two years combined are very similar, so far as may be judged from the present material, to what has been found for the production of the first year alone. The frequency distribution is apparently unimodal and negatively skew. The comparatively low mean egg production given by the two-year distribution is obviously to be explained by the fact just brought out that there is no sensible correlation between the egg production of the two years. Obviously, if the bird which was a relatively high producer in its first laying year was also always or usually a relatively high producer in the second laying year we should get a higher mean egg production in the same lot of birds than that which we have here. With the increase in the mean egg production over that of a single year is found, as would be expected, an increase in the standard deviation. This increase, however, is almost precisely proportional to that which occurs in the mean; hence we are compelled to conclude that the relative variability in respect to the first two years' egg production is not substantially different (as is indicated by the coefficient of variation) from that of the first year alone.

So far as may be judged from such meager data as we now have, there seems no particular reason for regularly taking as a unit in egg-production investigations or contests two years' laying instead of one year's. The important characteristics of variation appear not to be essentially different for the longer unit from what they are for the shorter. Such a conclusion can for the present, however, be only provisional.

It is much to be regretted that more extensive material on this matter of egg production in years subsequent to the pullet year is not at hand. What we have attempted to do in this section is to discuss the very meager material which now exists. It is a part of the plans for future poultry work at the Maine station to make further study of this point, and we hope that later we may be able to present a fuller and more adequate discussion of the matter.

SELECTION AND EGG PRODUCTION.

We may now turn to the consideration of the results of the breeding experiment which gave origin to the statistics on which this paper is based, so far as these results concern the annual egg production. An account of the experiment has been given earlier in this paper (pp. 11, 12). Its underlying idea was to improve the egg-laying quality of a particular strain of hens by breeding only from the best producers and their sons. It should be said that until the last year of the experiment (1906–7) practically no records were ever kept by which it could be told what birds were the parents of any particular individual offspring. It was simply known that only eggs of high-producing hens were incubated and that the males that fertilized these eggs were sons of high-producing mothers. In a very few cases records were kept of the mothers of individual birds, and in still fewer cases of the fathers. These facts will make it plain why it is necessary to discuss the results of this breeding experiment in the manner which will be followed in this section. The lack of the most fundamental data of all in any breeding investigation—namely, the pedigree data—makes it impossible to attack the problem of the influence of selection on egg production in the usual straightforward fashion. Obviously, the proper way to study the matter would be to form correlation tables between parent and offspring in regard to egg production and then evaluate the degree of intensity of inheritance of this character, compare the variability in successive generations following selection, etc. It is impossible to proceed in this way, since we have only mass data instead of individual data. The only conclusions which can be reached must be such as can be deduced from mass data.

We may first consider the question of the degree of intensity of the selection which has been practiced. In the several accounts of the breeding experiment [a] it is stated that only birds which laid 160 or more eggs in their pullet year were ever used as breeders. Now, while at first thought this may seem like an entirely uniform procedure, a little consideration shows that it is far from being such, so far as intensity of selection is concerned. All selection must be of or about a type. This is equally true whether the one making the selection has a definite and fixed type in mind and picks out individuals as nearly like it as possible, or, as in the present case, the selection is made by truncating the general population distribution and taking as the selected individuals all those falling in the portion cut off. One thinks of selection as most intense when the selected individuals conform closest to type. The most intense selection possible would be to take out of a population only those individuals which exactly conform to the type within the limit of the unit of measurement. Thus, in the present material, suppose it to be agreed that the type to which selection was to be made was an egg production of 180 eggs in the year. Then we should get the most intense selection possible by taking as breeders only birds laying exactly 180 eggs. As birds having productions differing from 180 eggs were taken, we should be selecting with less and less stringency.

These considerations indicate at once a method whereby one may measure with precision the intensity or stringency of the selection applied in any particular case. From what has been said it is apparent that the smaller the variation exhibited by the selected stock in comparison with that exhibited by the general population from which the selection is made, by so much is the selection more intense or stringent. Hence, if one takes the ratio of the standard deviation of the selected stock to the standard deviation of the general population, the result will be a coefficient measuring intensity of selection. If we use for this coefficient the letter S we have

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[a] See particularly Bureau of Animal Industry Bulletin 90.

$$S=\frac{\sigma_M}{\sigma_P}$$

wherein σ_M denotes the standard deviation of the selected stock and σ_P that of the general population. Now obviously the limits of this coefficient S are 0 and $+1$. When the selection is as stringent as possible and each selected individual is exactly like the type (within the limits of the unit of measurement), there will be no variation in the selected stock, and hence σ_M will equal zero, and consequently S will also. If on the other hand the selected individuals are chosen at random, they will approach no more closely to the type than do the individuals of the general population. In that case we shall have $\sigma_M=\sigma_P$ and $S=1$. Between these two limits, 0 and 1, S will vary in magnitude inversely as the intensity of the selection. We may for convenience call S the coefficient of selection.

In the breeding experiment on which we are here reporting, only hens laying more than 160 eggs in their pullet year were used for breeding. Of such hens, so far as we are able to learn, practically all were used. There were a few rejections because of small or badly colored eggs, but the number of such birds was very small. As has been pointed out above (p. 67), it is now impossible to tell exactly what birds were used as breeders in earlier years. Consequently, we can not give absolute figures for the intensity of selection. What can be done, however, is to determine the intensity of selection in each year of the experiment, assuming that all birds laying 160 or more eggs in their pullet year were used as breeders. The results so obtained, while not absolutely correct, will come very close to the true facts. For all practical purposes one may consider the figures to give the actual intensity of the selection practiced. In Table 16 it is assumed that all birds laying over 160 eggs in the pullet year were mothers of the flock of the second year following their hatching.

TABLE 16.—*Intensity of selection practiced in the breeding, 1899–1907.*

Year.	General population mean.	Mother's mean.	Difference in means.	General population standard deviation.	Mother's standard deviation.	Difference in standard deviations.	Intensity of selection.	Percentage of mothers to general population.
1899–1900	136.36	179.83	43.47	44.03	20.59	23.44	0.468	34.28
1900–1901	143.44	184.83	41.39	45.02	16.06	28.96	.357	42.36
1901–2	155.58	188.20	32.62	38.94	20.56	18.38	.528	47.92
1902–3	136.48	182.62	46.14	39.34	17.86	21.48	.454	27.88
1903–4	117.87	172.70	54.83	41.43	10.86	30.57	.262	15.76
1904–5 [a]	129.07	186.87	57.80	51.30	18.78	32.52	.366	30.10
1905–6 [a]	128.44	174.09	45.65	39.07	13.86	25.21	.355	20.94
1906–7 [a]	106.99	172.33	65.34	39.09	10.84	28.25	.277	8.88

[a] For these years, just as in the earlier ones, all the birds in the flock are taken together in determining the constants in this table. In selecting breeding stock no account was made of the size of the pens from which they came. Hence we have no reason to treat 50, 100, and 150 bird pens separately.

From Table 16 we note the following points:

1. The difference between the mean of the selected stock (mothers' mean) and that of the general population varies rather widely in different years. It is, for obvious reasons, least when the general population mean is highest, and greatest when the general population mean is lowest. This difference ranges in value from about 33 eggs in 1901–2 to 65 eggs in 1906–7, and averages 48.41.

2. Similarly there is considerable fluctuation from year to year in the differences between the standard deviation of selected stock and the general population. These differences range between 18 and 32 eggs, with an average of 26.10.

3. The intensity of selection, as measured by the coefficient, is rather great throughout the whole experiment. In the years when the selection is most stringent the coefficient only drops to a value of about one-fourth, while in the year of least intense selection the value is more than one-half. These figures mean that never in the course of the experiment was the selection more than three-fourths as stringent as it was theoretically possible to make it, and it was usually decidedly less close than that.

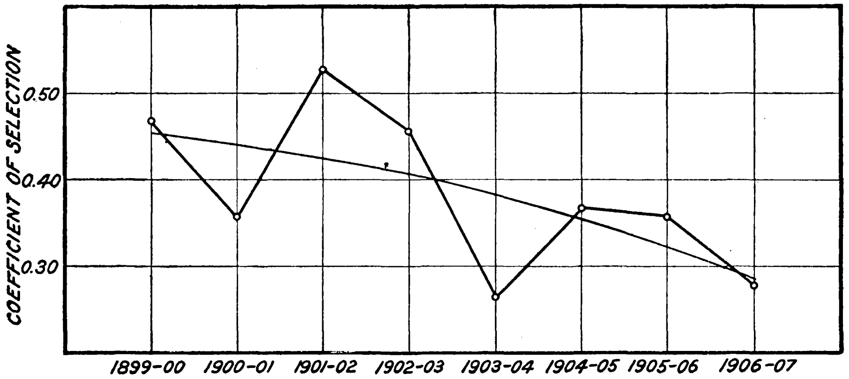

FIG. 17.—Change in intensity of selection, 1899–1907.

4. The general trend of selection during the period was to become relatively more stringent; that is, the value of the coefficient of selection diminished during the experiment. This is clearly shown in figure 17. In this figure the zigzag line shows the fluctuation in the value of the coefficient in the different years. The smooth line is the parabola

$$y = 0.4620 - 0.0062x - 0.0020x^2$$

the constants of which were determined by the method of least squares.

The reason for the decrease in the value of the coefficient of selection is clearly to be found in the fact that there was a steady decrease in the egg production of the flocks during the experiment, as has been shown above (p. 42). As the laying becomes poorer there will be fewer and fewer of the flock producing over 160 eggs. Consequently the variation exhibited in the selected stock will steadily become smaller both absolutely and in proportion to that of the general population (coefficient of selection).

5. The last column of the table shows for each year the proportion of the flock which produced over 160 eggs. As we should expect from what has gone before, this proportion diminishes in the later years of the experiment.

Having examined the method and degree of selection practiced, we may next consider the results. According to current theories regarding the inheritance of fluctuating variations, the following are the primary and fundamental results which one would expect to get in such an experiment as this:

1. A gradual and steady tendency for the mean annual egg production to increase with each successive generation. Since all the ancestry for at least eight generations was selected, it would be expected that at the end of this period there would be a marked gain in egg production over what existed at the beginning.

2. A small but not the less distinct and definite decrease in the variability of egg production during the course of the experiment.

3. A considerable degree of fixity of the improvement gained, such that ordinary environmental fluctuations would not in a single generation cause all that had been presumably gained by selection to be lost.

Now, it has been shown in this paper that, so far as annual egg production is concerned, the actual results obtained in this breeding experiment are as follows:

1. The only change which occurred in the mean egg production, aside from individual year fluctuations, was a small but entirely definite decrease during the eight years (pp. 39–42). Further the percentage of exceptionally high producers in the flocks has somewhat decreased during the course of the experiment, and at the same time the proportion of extremely poor producers has slightly increased (p. 48).

2. The variation in annual egg production has remained unchanged throughout the course of the experiment except for minor fluctuations in individual years (p. 46).

3. There is no evidence that the quality of high egg productiveness is any more fixed in the strain at the end of the experiment than it was at the beginning. During the last three years of the experiment it has been definitely shown that a relatively small environmental change is able to produce a very large difference in the mean egg production in flocks of hens of exactly the same selected ancestry (pp. 58–63). Such a result could not occur if the character had been fixed by the selective breeding.

These results we believe to be of great interest and importance, alike from the standpoint of evolution theory and that of practical poultry husbandry. While in a sense they mark a failure to attain that for which the work was undertaken—namely, an improvement of egg laying by this particular method of breeding—yet they possess much educational value. We have here an extensive selection experiment carried out in the traditional time-honored fashion, but with records of a character which make it possible to appreciate the net results with certainty. These results are not in accord with current theoretical ideas, but they are in remarkable accord with the experience of Nilsson at the Svalöf experiment station in breeding cereals, as recently described by De Vries.[a] This is sufficiently shown by the following quotation. Speaking of the earlier experience of the Svalöf station in breeding cereals by the ordinary method of continued selection, De Vries says (loc. cit., pp. 62 and 63):

> The deficiencies of the method could not escape observation. All those which were of purely technical nature could be overcome, and on this side of the problem a high degree of perfection was attained. But there were other weak points which were related to the very foundation principles of the method. Among these, the principal one was that the continuous selection of the best specimens in an arbitrary direction did not lead to improvement in all cases. Quite on the contrary, success was rare; so rare, even, that it could almost be looked upon as an exception. The fact itself was not new, since in Germany, also, only in exceptional cases a real improvement had been obtained. But, of course, usually only successful instances are published, and concerning the remainder ordinarily no evidence is at hand.
>
> In Svalöf, however, where numerous experiments of the same kind, but with different varieties of cereals, were conducted side by side, the fact could not escape observation. Soon the idea suggested itself that if success is an exception, the principle involved in the method can hardly be a valuable one, at least, not one on which the breeder may confidently rely.

The fact that in the present case of long-continued selection for a particular kind of animal production we get results so exactly paralleling those obtained in plant breeding when the same method of selection is practiced is certainly an interesting one. A strong argument of those who have endeavored to controvert the views of De Vries regarding the significance of fluctuating and mutational variation in the creation or amelioration of varieties has been the history of domesticated animals. Is the idea that all improvement of breeds on the animal side has been brought about by the continued selection of small "fluctuating" variations as well grounded as it is commonly supposed to be?

It appears to us to be idle to attempt any explanation at present as to why this breeding experiment resulted as it did. Of the objective results we are certain, and we have endeavored to set them forth clearly and distinctly. But for the interpretation of them we need much additional data which we do not as yet possess. Such data we hope in course of time to accumulate. Beginning with the breeding season of 1908 we shall have an exact individual pedigree record of every individual bird in the station's poultry plant. These records must ultimately throw light on the meaning of our present selection results.

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[a] De Vries, H., Plant Breeding, pp. xiii and 360. Chicago, 1907.

The practical conclusion to be drawn from the results of this breeding experiment seems to us to be clear. It is that the improvement of a strain of hens in egg-producing ability by selective breeding is not so simple a matter as it has been supposed to be. Nothing could be simpler than breeding from high producers to get high producers. But if this method of breeding totally fails to get high producers—in other words, if the daughters prove not to be like the mothers in egg production—it can not fail to excite wonder as to whether the simplicity of the method is not its chief (possibly its only) recommendation. Anyone who makes a thorough first-hand study of an extensive selection experiment carried out, as was this one, by the so-called German method without testing the transmitting power of individual organisms, can not fail to be impressed, we believe, with the fact that the improvement of a race by selective breeding is a vastly more complicated matter than it is assumed to be by those who maintain that one need only to breed from the best to insure improvement. The supposed "facts" of heredity on which the practical stock breeder (working for utility points) operates are in very large part inferences rather than facts. What is needed more than anything else for the advancement of the stock-breeding industry in all its phases is an accumulation of definite knowledge of the fundamental principles of the hereditary process. All breeding operations must be based on the laws of inheritance in organisms. The practical stock breeder is able to work out the applications of these laws for himself. What he most needs is broader and deeper knowledge of the laws themselves. This knowledge must come from thoroughgoing purely scientific investigations.

SUMMARY OF RESULTS AND CONCLUSIONS.

This paper constitutes Part I of a general study of egg production in the domestic fowl. It deals with variation in annual egg production. For the purposes of this investigation "annual egg production" is taken to be the total laying in a year beginning with November 1. For the most part the present paper is concerned with the egg production of the pullet year. The data on which the work is based are the trap-nest records kept since 1898 at the Maine Agricultural Experiment Station in connection with an experiment in breeding by selection for high egg production.

The chief results of this part of the investigation are summarily stated in the following paragraphs. Unless something directly to the contrary is said, all the statements in this summary have to do with the egg production in a single variety, namely, the Barred Plymouth Rock.

1. Variation in annual egg production exhibits the following characteristics: (*a*) The observed range of variation is from zero to about 250 eggs; (*b*) the distributions are usually unimodal and unsymmetrical. The asymmetry or skewness, when it exists, is always in the negative direction; that is, the modal egg production is always larger than the mean egg production. (*c*) The amount of variation in egg production is both absolutely and relatively large. The mean value of the coefficient of variation from all our data is about 34 per cent. (*d*) Variation in egg production, so far as our statistics show, belongs to the type of continuous, so-called "fluctuating" variation.

2. Analytically considered, such of the variation polygons as are skew are found to conform to Pearson's Type I curve. The symmetrical distributions belong either to his Type II or the normal curve of errors. The range of variation is in nearly all cases greatly overestimated by the theoretical curves. It is pointed out that this appears to be a characteristic of fecundity curves frequently.

3. It is shown that during the period covered by the statistics (1899–1907), which covers practically the whole period of the breeding experiment, there has been, apart from fluctuations up and down in individual years, a small but steady decrease in the mean or average annual egg production.

4. During the same period (1899–1907) the variability in annual egg production has not sensibly changed. There have been chance fluctuations up and down in individual years, but there has been no steady trend toward lower or higher variability. The same statement applies to the skewness of the distributions in the period covered by the investigation.

5. The percentage of extremely high layers (producing more than 195 eggs in the pullet year) in the flock decreased during the period from 1899 to 1907. The percentage of exceptionally poor layers (producing less than 45 eggs in the pullet year) in the flock increased during the same period.

6. The general characteristics of variation in annual egg production in White Wyandottes are essentially similar to those described above for variation in Barred Plymouth Rocks.

7. Evidence from the literature is presented tending to show (*a*) that it is possible to get average annual egg yields higher than any of those which have been observed at the Maine Experiment Station, and (*b*) that records exist showing that in exceptional cases average annual egg yields were obtained during the middle and earlier half of last century, which were just as high as any we now know.

8. When the laying hens were kept in flocks of 100 birds each the average annual egg production per bird was distinctly and significantly lower than when they were kept in flocks of 50 birds each, though the number of square feet of floor space per bird was the same in the two cases and all other environmental conditions were made as nearly as possible identical.

9. Laying birds kept in flocks of 150 birds each, and with somewhat less floor space per bird than those kept in flocks of 50 and 100 birds each, have in every case an average annual egg production significantly smaller than that of the birds kept in smaller flocks. It is pointed out that great caution must be shown in drawing practical conclusions from these results relative to housing because we are here dealing only with annual egg production.

10. Those conditions of housing and flock size which tend to lower the mean annual egg production are found to tend to increase the variability of the production, so that poor producing flocks are at the same time flocks very variable in production, and vice versa.

11. Egg production in the second laying year is found, on the basis of rather meager data, to average about a third lower and to be distinctly more variable than that in the pullet year. There is no sensible correlation between the production of the first and second year.

12. During the period covered by the statistics all birds used for breeding have been the offspring of mothers laying 160 or more eggs in their pullet year and of fathers which were the sons of high-producing mothers. It is now possible to determine what have been the results of this extensive experiment in selection. It is shown that the intensity or stringency of selection became relatively greater during the progress of the experiment, though the absolute standard of selection remained the same. It is further shown that there is no evidence that the selective breeding practiced has improved the strain in respect to egg production. On the contrary, the data show that (*a*) the mean egg production has diminished during the experiment, (*b*) the variability in egg production has remained unchanged, and (*c*) in the last years of the experiment relatively slight environmental changes caused very marked changes in the flock productiveness. This is obviously inconsistent with the view that any particular type of egg production has in any way been fixed in the strain by the breeding.

APPENDIX I.

In the following three tables are given the detailed raw data on which the present paper is based:

TABLE I.—*Egg production of Barred Plymouth Rocks.*

Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.													
	1899–1900.	1900–1901.	1901–2	902–31	1903–4	1904–5 50-bird pens.	1904–5 100-bird pens.	1904–5 150-bird pens.	1905–6 50-bird pens.	1905–6 100-bird pens.	1905–6 150-bird pens.	1906–7 50-bird pens.	1906–7 100-bird pens.	1906–7 150-bird pens.
1					1	1		1						1
2					2	1		1						
3								2					1	4
4								2						2
5					2									
6												1	1	1
7					1			1				1		
9														2
10													1	
11					1						1		1	
14						1				1				
15								1		1				
17										1		1		
18								1						
19							1							1
20						1								1
21								1						1
22							1			1	1			
23		1			2			1						
24														1
25				1	1	1						1		
26								1						1
27								1						
28					1									2
29				1	1	1								1
30						2	1					1		
31										1	1		1	
32	1					1						1		
33													1	1
34						1								
35														1
36	1				1	2					1			
37						1								1
38						1					1			
39					1	2	2					2	1	1
40						2	1	1	1	1			1	1
41					1									2
42	1				1	1							1	
43					1					1		1	1	
44						1		2			1		2	1
45							1	2					1	1
46						1		2				1	1	1
47		1			1						1		1	
48					1	1					1			2
49						2		2					1	
50					1							1	1	2
51		1			1	1			1		1	1	1	
52				1					1	2		1		
53						1					1	1		
54		1			1	1		1			1			
55	1				1			2					1	2
56		1			2						3			2
57					2									1
58											1		3	
59	1									1	1		1	4
60				1		2	1	1			1	1		
61	1	1									1			3
62					1					1	1	1		2
63					1				1	1	1			1

TABLE I.—*Egg production of Barred Plymouth Rocks*—Continued.

Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.													
	1899–1900.	1900–1901.	1901–2	1902–3	1903–4	1904–5 50-bird pens.	1904–5 100-bird pens.	1904–5 150-bird pens.	1905–6 50-bird pens.	1905–6 100-bird pens.	1905–6 150-bird pens.	1906–7 50-bird pens.	1906–7 100-bird pens.	1906–7 150-bird pens.
64					1	2		1		3	3	2		2
65						1							1	2
66										1	2	1		1
67											3			1
68	2				2	1					1		1	5
69				1	1		1		1	1	1			2
70	1	1	1	2				1	1		2		1	2
71					1		1	1				2	1	1
72		1				2		1	1	1	1		2	1
73				1	2	1					2	1	1	4
74		1			1			1			2	1		2
75								2	1	1	3	1	2	1
76	1	1			1	1	1	2	1			1	2	2
77					1	1		2			3	1	2	1
78						1		1	1	2		2	2	1
79									1	1		2	3	
80	1			1	1	2	2	1	1				3	3
81				1	1	3		2			4	1	1	3
82				2	2	1		2			3		2	3
83				1	6	1		1		2		3	1	5
84					1	1	1			3	1	1		2
85					1							1		1
86				2	1	1	1	2	1	1	2	1	4	2
87		1				1	1		1	4	2		1	4
88	1				1				1		2	1	1	3
89	1		1	1	4	2	2		1		3	1	1	1
90	1			1	1	2					3	2		1
91		1		2	1		1		1			1	4	3
92		1		2		2	2	2		2	2	6	3	2
93			1				1	2			5	1	2	3
94		1		2	3		1	1	1	1	3		1	4
95		1		1				3		1	2	1	1	4
96				1	1	1			1	1	2	3	1	3
97					1	2	1		1	1	3	3	3	4
98	1				5	2	1				2	2	3	4
99		1	1	1	1		2	1	1	2	3	2	1	2
100	1	1		2	2	1			1	1	1	2		4
101				1	2	1			2	2	1	1	2	3
102				2	2	1	1		1	2	4	2	1	3
103					1	1		1	2	2	2	2	2	3
104	1			2	4	3			2	2	2	2	5	5
105		1	2	2	1	2	1	1	1	4	4			1
106				1	1	2			2	1	1	4	2	4
107		1		6	5	2	2			3	4	4	3	1
108				2	3	3		1	2	2	1			1
109					3	4		1	1	3	2	4	2	3
110		2		1	4	1	1	2	3	1	2	2	2	
111			2	1	2	3	2	2	1	1	7	1	2	4
112					3	4	1	1	3		9	4	2	2
113			1	1		2			2		1	1	2	2
114				2		1		1	3	1	4	3	2	1
115	1	2		1	3	3		1		3	3	1	1	6
116			2			3		4	2	3	2	2	2	5
117	1	1			2	2	1	3	2		2	5	2	3
118			1		1	3	2	1	1	3	2	2	2	4
119				1	1	4	1		2	1	2	6	1	2
120	1			1	1	2		1	1		1	4	3	2
121				1	1	2		1	2	1	1	2	1	3
122			1		5	2	1			1	3	1		2
123		1		3	2			1	4	3	6		2	4
124			1		2			1	7	1	3	2	6	3
125	1			1	5	1			3	1	2	4	2	2
126			2	3	3	3				2	3	1		4
127				1	2	2	1	1		5	1	2		1
128	1				1		2	2		1	2	2	2	1
129	2	2		1	8	2	1	1	1	3	2	1	2	2
130				2	1	1			1		2	3	2	
131	1	1		1	6	4		1	1	2	1	2	4	1
132	1	2		1	5	3		1		1	6	1	2	
133	1			2	3	2			2	2	1	1	2	9
134	2				7			2	2	2	6		2	2
135	1			1	3			2	2	6	2	2	4	2
136		1		4	4	2	1		1	3	3	2	3	4
137	1	4		1	3	5		1	1	3	5	1		
138	1	1		3	3	4		1	1	2	6			1
139	1	2		1	2	3			1			2	2	3

TABLE I.—*Egg production of Barred Plymouth Rocks*—Continued.

Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.													
	1899-1900.	1900-1901.	1901-2	1902-3	1903-4	1904-5 50-bird pens.	1904-5 100-bird pens.	1904-5 150-bird pens.	1905-6 50-bird pens.	1905-6 100-bird pens.	1905-6 150-bird pens.	1906-7 50-bird pens.	1906-7 100-bird pens.	1906-7 150-bird pens.
140				4		4	1		2	1	2	1	1	1
141	2	1		1	4	3				2	2	3	2	1
142				1	3	2		2	1		2	2	1	4
143	1			1	2	1		2	1	1	1		2	2
144	1	1		2	3	1		1	2	2	1	1		2
145	1	2		2	1	1			2	3	2	3	1	1
146		2		1	3	2	2	1	2		3		1	
147	1			1	1	4	2		3		2	4	1	
148			1	2	3	4	1				2		1	1
149	1	1		1	2	4			3		2		1	3
150	1					5		3	2	2	1	1	2	3
151				1	3		1	1	1	1	1	2		
152	1			1	3	4			4	3	4	1	1	
153	2		2	2	4	1			3		4	3	1	1
154		1		1	1	2	2	2	2	1	1	1	1	1
155		2	3	2			1	1	3	5	1	1	1	
156				1	1	3	2	1	2	3	2	1		2
157		1	1	3	2	3		3	2	1	2			3
158	1		1	1		4	1		4	1	2	2	1	1
159	1		1		1	2		1	2	1	2	1		
160		1		1			1	2	1	4	1			4
161				1	1	2		2	2	2	1		1	2
162			1		5	5	1		1					
163	3		1	2	3	1	1	2	1	3	1	1	1	
164			1	1	5	1	1	1	2		2	5	2	1
165		1		3	1	2	1		1		2	1		2
166	1	1		2	3	6		2	2	2	2	1		1
167	1	2		2	1	1	1	1	3	1	3	2		1
168			2	1	1	3	1	1			1		4	1
169	1	1		1		1	1		3	1				
170	2	3	1	1		2	1	3	3	1		1	1	1
171	1			2	2			1	1	1		1		
172				1	1	3	3	1	1	1	3	1		
173			2			1				2	1	1		
174	1	1	1		1	2	1		1	1	2	2	2	
175	2	1	1	1	1						1	1	1	
176	1			1	3	5		1	1	3	1			
177		2		1	1			1	1	1	3		1	
178	1	1		1	1	1	1					1		
179	1	1	1	1		1	1			2			1	
180	1			2	1		1	1	2	1	1			2
181	2	2				1		1	2	2				2
182		1		1	2	1	1				2			
183	1	1			1	1		2			2		1	
184					1	2	1		1		1		1	
185	1	2		3		3	1		3	1				
186					1	1	1	1	2	1		1	1	
187					1		1		1					
188			1						3	2				
189		1	1	1	1	1			2		1			
190		3		1		3			3	1			1	1
191		1		1			1			1				
192		1				1	1		1		1		1	
193	1		1			2				1				
194								1		2				
195		1						1	2	1	1			
196		1	1		1				1		2			
197						1	1		2	1				
198			1								1			
199		1					1	1	1					
200		2		1		4								
201			1			5	2	1		1				
202						2	1				1			
203						1	3	1	2					
204						2		2						
205			1	1		4								
206			1			3								
207						1								
208	1					2			1		1			
209		2			1		1							
210				1		2								1
211	1		1			2								1
212						1	1	1						
213		1	1											
214											1			

TABLE I.—*Egg production of Barred Plymouth Rocks*—Continued.

Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.													
	1899-1900.	1900-1901.	1901-2	1902-3	1903-4	1904-5 50-bird pens.	1904-5 100-bird pens.	1904-5 150-bird pens.	1905-6 50-bird pens.	1905-6 100-bird pens.	1905-6 150-bird pens.	1906-7 50-bird pens.	1906-7 100-bird pens.	1906-7 150-bird pens.
215					1									
217				1		1								
220				2				1						
221				1		1								
222			1	1										
225				1										
226						1								
234		1				2								
237	1													
240			1											
246										1				
248						1								
Total	70	85	48	147	254	283	92	140	178	182	275	187	185	281

TABLE II.—*Egg production of White Wyandottes.*

Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.			Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.			Eggs produced in pullet year.	Number of birds producing in each year the specified number of eggs.		
	1899-1900.	1900-1	1901-2		1899-1900	1900-1	1901-2		1899-1900	1900-1	1901-2
1			1	115	1			160			2
10	1			117	1			161		3	1
22	1			118	1	1		162			1
36		1		119	1	3	1	163	1	2	
41	1			120	2		2	164	1		
52		1		121		1		165		1	
54		1		124		2		166	1	2	
55			1	125	1			167	1	1	
60		1		126	2			171	2		
61	1			127	1			173	1	2	
62	1	1		128	2	2	3	175	1		
64		1		129	1		1	176		1	
65		1		130	1		1	178	2		
66	1			131		1		179	2		
68	2			132	1	1		180	1		
72		1		134		1		181		1	
78	2	1	1	136	1	1		183		1	
80		1		137			1	184			1
81		1		138		1		185			1
82			1	139		1	1	186		2	
83	1			140		1	2	191			1
86		1		141	1	1		193	1		
87	2	2		142	2	1		195	1	1	
90		2		143			2	196	1		
93	1			144	1	2		203		1	
98	1			146		1		207		1	
99	1	1		147	1			208	1	1	
100	1			149	1		2	209	1		
103		1		150		1		214	1		
104	2			152			1	217	1	1	
107	2	1	1	153	1	1		226		1	
108	2	1	1	154		1		233		1	
109			1	155	1						
110			1	156	2	1	1	Total	70	72	33
113	1	1		159		3					

TABLE III.—*Egg production of the same individual hens in the first and second laying years.*

Eggs laid in—		Eggs laid in—		Eggs laid in—		Eggs laid in—	
First year.	Second year.	First year.	Second year.	First year.	Second year.	First year.	Second year.
161	61	135	161	195	166	190	88
175	8	137	168	200	176	170	103
155	118	165	165	185	147	189	13
165	112	151	82	209	138	95	82
175	94	154	94	234	150	177	58
181	159	120	115	191	73	175	98
142	89	143	46	99	67	157	86
191	94	129	125	183	124	167	51
165	151	191	147	107	22	200	185
169	40	151	150	61	20	209	102
140	103	138	148	213	137	170	107
160	145	208	127	185	106	51	86
201	30	237	102	92	35	181	70
136	130	163	71	190	143	47	44
184	99	211	145	169	30	192	133
152	162	196	16	74	12		
155	116	170	172	182	34		

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Issued February 28, 1911.

U. S. DEPARTMENT OF AGRICULTURE,

BUREAU OF ANIMAL INDUSTRY.—BULLETIN 110, PART II.

A. D. MELVIN, CHIEF OF BUREAU.

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

II. SEASONAL DISTRIBUTION OF EGG PRODUCTION.

BY

RAYMOND PEARL, PH. D.,
Expert in Poultry Breeding, Bureau of Animal Industry, and
Biologist, Maine Agricultural Experiment Station,
AND
FRANK M. SURFACE, PH. D.,
Associate Biologist, Maine Agricultural Experiment Station.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
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1911.

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LETTER OF TRANSMITTAL.

U. S. Department of Agriculture,
Bureau of Animal Industry,
Washington, D. C., March 10, 1910.

Sir: I have the honor to transmit herewith a manuscript entitled "A Biometrical Study of Egg Production in the Domestic Fowl. II. Seasonal Distribution of Egg Production," by Drs. Raymond Pearl and Frank M. Surface. Part I of this work, dealing with the variation in annual egg production, was published last year as Bulletin 110, Part I, of this Bureau. I respectfully recommend that the present paper be published as Part II of that bulletin.

Respectfully,

A. D. Melvin,
Chief of Bureau.

Hon. James Wilson,
Secretary of Agriculture.

CONTENTS.

Page.

Introduction........ 81

Material and methods........ 84

Mean egg production in each month........ 86

  The monthly distribution of egg production........ 89

Variation in egg production in each month of the year........ 93

The natural divisions of the laying year........ 99

Changes in monthly egg production due to selective breeding........ 101

  Effect on average monthly production........ 102

  Effect on absolute variation as measured by the standard deviation........ 106

  Effect on relative variability........ 109

Influence of certain housing conditions on monthly egg production........ 112

  Influence of housing conditions on average monthly production........ 113

  Influence of flock size on variability in egg production........ 118

The frequency distribution of variation in monthly egg production........ 123

  Position of modes and antimode in monthly distributions........ 130

  Relative height of modal ordinates........ 131

Analysis of the monthly egg-production distributions........ 133

  Statement of the hypothesis........ 136

  The fitting of the curves........ 140

    Components and fitted curves for November distribution........ 142

    Components and fitted curves for April distribution........ 148

    Components and fitted curves for July distribution........ 151

    Conformity of remaining months to general type........ 154

Summary of results and conclusions........ 155

Appendix II........ 159

ILLUSTRATIONS.

Page.

FIG. 1. Diagram showing the weighted mean monthly egg production for each month of the pullet year.. 90

FIG. 2. Integral curve of weighted mean monthly egg production, showing the absolute and relative number of eggs laid from the beginning of laying year up to the end of each month........................... 91

FIG. 3. Diagram showing the fluctuations of mean monthly egg production in the two months of maximum production, March and April......... 92

FIG. 4. Diagram showing the fluctuation of mean monthly egg production in the two months of minimum production, October and November... 93

FIG. 5. Diagram showing weighted mean value of the standard deviation for each month in the laying year.................................... 97

FIG. 6. Diagram showing weighted mean value of the coefficient of variation for each month in the laying year................................. 97

FIG. 7. Diagram showing the course of the observed mean egg production in each of the months of November to April, inclusive, 1899–1907..... 103

FIG. 8. Diagram showing the course of the observed mean egg production in each of the months of May to October, inclusive, 1899–1907......... 104

FIG. 9. Diagram showing the trend of the observed absolute variability in monthly egg production as measured by the standard deviation in each of the months of November to April, inclusive, 1899–1907...... 107

FIG. 10. Diagram showing the trend of the observed absolute variability in monthly egg production as measured by the standard deviation in each of the months of May to October, inclusive, 1899–1907..... 108

FIG. 11. Diagram showing the course of the observed relative variability in egg production as measured by the coefficient of variation in each of the months of November to April, inclusive, 1899–1907......... 110

FIG. 12. Diagram showing the course of the observed relative variability in egg production as measured by the coefficient of variation in each of the months of May to October, inclusive, 1899–1907............. 111

FIG. 13. Diagram showing the average excess in mean egg production of flocks kept in 50-bird pens over flocks kept in 100-bird pens for the first eleven months of the laying year, November to September........ 116

FIG. 14. Diagram showing the average excess in mean egg production of flocks kept in 50-bird pens over flocks kept in 150-bird pens for the first eleven months of the laying year, November to September......... 117

FIG. 15. Diagram showing the average excess in mean egg production of flocks kept in 100-bird pens over flocks kept in 150-bird pens for the first eleven months of the laying year, November to September........ 117

FIG. 16. Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 50 birds over flocks of 100 birds for the first eleven months of the laying year, November to September.. 119

FIG. 17. Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 50 birds over flocks of 150 birds for the first eleven months of the laying year, November to September.. 120

Page.
FIG. 18.—Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 100 birds over flocks of 150 birds for the first eleven months of the laying year, November to September 121
FIG. 19.—Frequency polygon of variation in November and December egg production 126
FIG. 20.—Frequency polygon of variation in January and February egg production 127
FIG. 21.—Frequency polygon of variation in March and April egg production.. 128
FIG. 22.—Frequency polygon of variation in May and June egg production.... 128
FIG. 23.—Frequency polygon of variation in July and August egg production.. 128
FIG. 24.—Frequency polygon of variation in September and October egg production 129
FIG. 25.—Diagram showing the general form or type of curve to which the frequency distributions of variation in monthly production apparently belong 133
FIG. 26.—Diagram to illustrate the nature of the hypothesis made in the text regarding the rate of egg production 137
FIG. 27.—Diagram illustrating cycles of fecundity in the hypothesis regarding egg production 137
FIG. 28.—Histogram and fitted curves for November egg production 145
FIG. 29.—Histogram and fitted curves for April egg production 150
FIG. 30.—Histogram and fitted curves for July egg production 154

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

II. SEASONAL DISTRIBUTION OF EGG PRODUCTION.[a]

INTRODUCTION.

In Part I of this study[b] the subject dealt with was variation in total annual egg production. This phase of egg production formed the logical starting point for a general investigation of the subject, and furnished an outline or programme of the underlying problems which must be solved in such a general investigation. Annual egg-production records furnish final mass results. There are involved in the attainment of these total annual results a large number of individual factors. The ultimate purpose of the study of egg production must be to separate and analyze each one of these factors individually so that it may be possible to control in greater or less degree total annual egg production through an intimate knowledge of the individual factors on which it depends.

In starting the analytical investigation of the problems outlined by the study of total annual production, the first question which presents itself is as to how egg production is, in general, distributed throughout the year. It is, of course, a perfectly well-known fact that on the average a flock of hens will not have just one-twelfth of their total yearly production in each month of the year. Certain seasons of the year have been recognized from the beginning of the domestication of poultry as primarily egg-producing seasons, whereas in certain other seasons it has always been particularly difficult to get eggs.

The whole matter of the seasonal distribution of egg production is extremely important from a practical as well as from a theoretical standpoint. The differences in the seasonal distribution of egg production cause corresponding differences in the price of eggs. In that season of the year when the poultryman finds it hardest to get eggs he is able to get the highest prices for such as he can send to market.

---

[a] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 20.

[b] A Biometrical Study of Egg Production in the Domestic Fowl. I. Variation in Annual Egg Production. U. S. Department of Agriculture, Bureau of Animal Industry, Bulletin 110, pp. 1-80. 1909.

The practical importance of a study of the seasonal distribution of egg production is well brought out if one compares in regard to this matter hens whose total annual production is the same. Such a comparison for a few individuals has been made in a recent paper by the present writers.[a] It is there shown that the money return is not by any means the same for all "200-egg" hens. A "200-egg" hen may bring in less money in a year than a bird laying very considerably fewer eggs, but producing them at a time when prices are high. The aim of the practical poultryman is to increase as much as possible his winter egg production. The matter of learning how to increase winter egg production is one of the primary problems which is under investigation at the present time in connection with the poultry work at the Maine Experiment Station.

From the theoretical side the investigation of the seasonal distribution of egg production is of the highest importance. Egg production is a part of the reproductive process in birds, and in the wild progenitors of domestic poultry it was only great enough for the perpetuation of the race. Around this primary, inherent egg-producing ability all the improvement in egg production shown by modern breeds of fowls has been built. In attempting to learn the underlying principles which have made this improvement possible lies the only hope that we may be able to continue the improvement in the future. It is important to know with as great exactness and detail as possible how the added egg production of modern races relates quantitatively and seasonally to the egg production of the wild representatives of the genus *Gallus*.

It is the purpose of this part of the paper to analyze the total annual egg-production records of the Maine station in respect to the seasonal distribution of production. In this analysis the month will be taken as the unit of discussion. The reason for taking the month as the unit is not primarily a theoretical but a practical one. It is obvious enough that a division of the year into calendar months is an arbitrary one from the standpoint of the biology of egg production. There is, for example, no natural or inherent reason for including an egg laid on Tuesday, February 1, with a different group from that in which an egg laid on Monday, January 31, is placed. The practical reason which makes it necessary to use the month as the first unit for the finer analysis of the total annual production is that, previous to the laying year, beginning November 1, 1907, there are, in the egg-production records of the station, no data available for smaller units of grouping than calendar months. From that date to the present time a daily egg record for each individual hen has been preserved. In the older records of the station the month is the smallest unit of which account was kept in the permanent records.

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[a] Poultry Notes—1908. Maine Agricultural Experiment Station, Bulletin 165, pp. 29-48. 1909.

While at first thought there may seem to be serious theoretical objections to the use of the month as a unit in the study of egg production, because of its somewhat arbitrary and unnatural character, yet a little consideration will show that any fixed unit of egg production whatsoever, except the day, is an arbitrary and in some degree unnatural one. Calendar months are not more open to criticism in this regard than are any other time units. In spite of the fact that the month is a more or less arbitrary division of the year from the standpoint of the biology of the hen, it yet remains that much of importance may be learned regarding the seasonal distribution of egg production by the analytical study of monthly records, and this unit is on the whole probably more satisfactory for the purpose than any other fixed time unit which could in actual practice be taken. The truly natural seasonal divisions of the year in respect to egg production are, we believe, considerably larger than the month. Evidence in our possession, a part of which will be presented in this paper, has led to this conclusion. At the present time the egg records of the station are being discussed on the basis of the division of the year into these larger seasonal periods.[a] Before this could be done, however, it was necessary to know the facts regarding the monthly distribution. To make any adequate analytical study of the subject of seasonal distribution a unit is necessary which is, on the one hand, longer than the day (otherwise the true seasonal changes will be obscured by the minor fluctuations of daily records) and on the other hand shorter than the natural seasonal divisions of the year. The month as a unit stands between these extremes.

With the month as a unit for a discussion of the seasonal distribution of egg production, a whole series of problems suggest themselves, as, for instance: What is the mean or average production for each month of the year with large flocks of birds when uniform methods of feeding and management are followed throughout the year? What is the exact form of the mean monthly egg production curve for the year? What is the comparative variability in respect to egg production in the different months of the year? Is the variability greatest in those months where the mean production is highest, or vice versa? Or is there no relation between mean production and variability in production? In general, what is the form of a monthly egg production variability curve for the year? These questions again suggest others. Is the seasonal distribution curve of egg production of the same character in different years; and Is the seasonal curve of variability in egg production of the same type in different years? How nearly alike are these curves for different breeds of fowls and under different climatic conditions or methods of management?

---

[a] Cf., for example, Maine Experiment Station, Bulletins 165 and 166, 1909.

In the previous part of this study an account was given of the experiment carried on at the Maine station for nine years to ascertain the effect of selection on average annual egg production. It was shown in that paper that the general trend of the average annual egg production had been downward throughout this time, although a rigid selection of high-producing parents had been maintained during the period. In the present part of the investigation we have to consider what was the effect of this method of breeding, so far as the monthly distribution of egg production was concerned. Is the general trend of the average production for each month of the year, when considered separately, downward or not during these years? It is conceivable that winter egg production, for example, might have been increased during these years that the total production was decreasing. Was this, as a matter of fact, the case? It is of great importance from the standpoint of the study of methods of breeding for egg production to know what was the effect of the method of selection practiced on the monthly averages throughout the period.

A further set of problems has to do with the correlations which exist in regard to egg production in the different months of the year. On the average, will hens that are high producers in December, let us say, be also high producers in July? It is of the greatest importance, practically as well as theoretically, to know just how closely each month's production is, on the average, associated or correlated with the production in every other month in the year. If a poultry keeper is only able on account of financial or other reasons to test the production of his hens with trap nests during a part of the year, which month or months will give him the best estimate of what the total yearly production will probably be for each hen?

These statements will give some idea of the general standpoint from which this second part of the investigation is undertaken and of the character of the problems with which it deals. It is not desirable to take further space here to set forth in detail the specific problems which are discussed in this portion of the work. These details will be considered in connection with the sections of the paper where the data bearing upon the several problems are discussed.

MATERIAL AND METHODS.

The statistical material which forms the basis of this part of the work is the same as that which was used in Part I. It comprises the trap-nest records of egg production during the pullet years from November, 1899, to November, 1907, at the Maine Agricultural Experiment Station. As was set forth in Part I of this study, the number of hens making complete yearly records in this period was 2,407. A full account was also given of the way in which these records were collected, so it will not be necessary to repeat this here.

Instead, whenever it becomes necessary in the discussion of monthly egg production reference will be made to the account given in Part I.

In this part of the study we are dealing with the monthly production in each month of the years above enumerated. All the statistics are for pullet-year production. The data of the present paper analyze the total annual production into its monthly components. It will be recalled (see Part I, pp. 16–19) that in certain of the years one or another sort of accident occurred to interfere with the egg production of certain months. Due account will be taken of this fact in the discussion of the records in this part of the study.

The detailed data on which this part of the work is based are given in tabular form in Appendix II at the end of this paper. In these tables are given for each month of the period discussed the ungrouped frequency distributions of egg production. It will be seen by referring to the tables that the unit classes of egg production ranging from zero to 31 are arranged horizontally while the vertical units are years and flocks. The first line of Table I may be taken as an illustration of how these tables in general are to be read. From this it is seen that 70 birds were available for record in November, 1899. Of these 70 birds 39 failed to lay at all in November and, therefore, have a zero record. One bird laid one egg during the month; three laid two eggs, and so on. This table gives the ultimate data on which this whole study is based. From these raw data it will be possible for anyone who cares to do so to check up at every point the computed constants.

The biometric methods used in the analysis of the statistical material in this part of the work are in general not different from those which were used in Part I (pp. 20–24). We have made use here, as before, of the ordinary biometric constants and their probable errors. A few words should be said, however, regarding some methods of computation which have been used in the present paper. The constants for monthly egg production have been calculated directly from the ungrouped frequency distributions of the appendix. Since ungrouped distributions were used, and since, further, as will be apparent to anyone studying these distributions, there is no approach to high contact at one end of the distributions at least, Sheppard's correction of the second moment has not been used in calculating the standard deviations. Instead the raw second moment has been used throughout. The computations have been very carefully checked, this being done in most cases independently by two different persons. In nearly every case the tabled constants have been checked independently three or four times. It is hoped that there are no errors in the computations remaining in the published work, although it is impossible in so extensive a piece of statistical work as this to be absolutely sure that minor arithmetical slips have not gone by unnoticed.

MEAN EGG PRODUCTION IN EACH MONTH OF THE YEAR.

In undertaking the discussion of the monthly distribution of egg production it is obvious that the most logical point at which to begin is with the mean or average monthly production for each month of the year. These monthly averages give the fundamental data around which the remainder of the discussion clearly must center.

The values of the mean or average monthly egg production of the Maine station's Barred Plymouth Rock flocks for the period from November, 1899, to November, 1907, are given in Table 1. In this table is given for each month, year, and pen the actual mean number of eggs per bird produced during that month and the probable error of this mean. There is also included for each month, year, and pen the percentage which the actual monthly average for that month is of the total production for the year. Thus, from the first row and column of Table 1 it appears that in November, 1899, the average egg production per bird was 4.31 ± .48, and that this egg production was 3.1 per cent of the total annual average production for that year. It was pointed out in Part I that in the years 1904 to 1907 the birds were kept in pens of three different sizes, containing, respectively, 50 100, and 150. For these years averages are given in the table for each of these flock sizes. In the last three lines of the table are given the weighted means of egg production for each of these three years. These weighted means were calculated in exactly the same way as were the weighted means for annual production for the same years in Part I (p. 27). The number of birds finishing records in each year is given in the last column of the table.

TABLE 1.—*Average monthly egg production and percentage of total yearly production of Barred Plymouth Rock flocks at Maine Experiment Station, 1899 to 1907.*

Year and pen.	November actual mean.	Per cent of annual.	December actual mean.	Per cent of annual.	January actual mean.	Per cent of annual.	February actual mean.	Per cent of annual.
1899-1900	4.31±.48	3.1	10.29±.73	7.5	16.60±.53	12.1	9.83±.55	7.2
1900-1901	3.08±.40	2.1	8.19±.65	5.7	12.34±.58	8.6	14.27±.45	9.9
1901-2	6.33±.81	4.1	12.02±.82	7.7	15.21±.56	9.8	11.67±.58	7.5
1902-3	5.93±.42	4.4	4.52±.35	3.4	5.94±.43	4.4	9.62±.38	7.2
1903-4	......	......	5.56±.24	4.7	8.72±.32	7.4	12.27±.29	10.4
1904-5:								
50-bird pens	4.64±.19	3.5	5.63±.30	4.2	10.03±.34	7.5	13.48±.26	10.1
100-bird pens	6.45±.55	5.0	10.67±.52	8.3	11.11±.51	8.6	11.28±.43	8.7
150-bird pens	5.64±.41	5.0	9.17±.46	8.1	10.72±.45	9.5	9.11±.39	8.1
1905-6:								
50-bird pen	6.29±.35	4.5	9.86±.46	7.1	13.83±.38	9.9	14.46±.29	10.4
100-bird pens	5.38±.32	4.2	9.91±.44	7.8	13.27±.38	10.5	13.39±.33	10.5
150-bird pens	3.67±.22	3.1	9.60±.38	8.1	12.44±.34	10.5	11.64±.28	9.8
1906-7:								
50-bird pen	3.25±.25	......	4.70±.33	......	5.70±.33	......	10.49±.32	......
100-bird pens	3.29±.30	......	6.16±.39	......	4.32±.33	......	5.18±.30	......
150-bird pens	4.28±.28	......	7.84±.32	......	4.67±.27	......	6.82±.28	......
1904-5, weighted mean	5.24	4.1	7.49	5.9	10.41	8.2	11.90	9.4
1905-6, weighted mean	4.89	3.9	9.76	7.7	13.07	10.3	12.93	10.2
1906-7, weighted mean	3.70	......	6.46	......	4.87	......	7.41	......

TABLE 1.—*Average monthly egg production and percentage of total yearly production of Barred Plymouth Rock flocks at Maine Experimental Station, 1899 to 1907*—Cont'd.

Year and pen.	March actual mean.	Per cent of annual.	April actual mean.	Per cent of annual.	May actual mean.	Per cent of annual.	June actual mean.	Per cent of annual.
1899-1900	13.04±.47	9.5	16.09±.44	11.8	15.29±.46	11.2	12.53±.51	9.2
1900-1901	17.14±.45	12.0	18.04±.42	12.6	15.45±.45	10.8	14.33±.49	10.0
1901-2	17.17±:47	11.1	18.50±.50	11.9	16.25±.58	10.5	16.88±.40	10.9
1902-3	14.96±.33	11.2	17.71±.32	13.3	17.02±.31	12.7	15.83±.33	11.8
1903-4	17.27±.28	14.6	14.78±.24	12.5	11.63±.23	9.9	12.27±.26	10.4
1904-5:								
50-bird pens	17.98±.23	13.5	15.51±.24	11.7	13.79±.27	10.4	12.59±.26	9.5
100-bird pens	12.23±.42	9.5	15.68±.42	8.7	14.36±.41	11.1	13.34±.46	10.3
150-bird pens	12.02±.41	10.6	13.16±.36	11.6	11.96±.41	10.6	10.22±.42	9.0
1905-6:								
50-bird pen	18.29±.26	13.1	16.48±.23	11.8	11.42±.27	8.6	13.47±.24	9.7
100-bird pens	17.33±.29	13.7	15.08±.26	11.9	10.74±.29	8.5	12.76±.30	10.1
150-bird pens	17.02±.20	14.3	15.20±.20	12.8	10.67±.23	9.0	11.65±.23	9.8
1906-7:								
50-bird pen	17.72±.30	......	17.12±.28	......	14.43±.31	......	11.13±.30	......
100-bird pens	13.96±.31	......	16.41±.28	......	15.23±.30	......	13.11±.30	......
150-bird pens	14.65±.26	......	15.21±.24	......	12.93±.24	......	10.62±.26	......
1904-5, weighted mean	15.33	12.1	14.90	11.1	13.39	10.5	12.08	9.5
1905-6, weighted mean	17.46	13.8	15.52	12.2	10.90	8.6	12.48	9.8
1906-7, weighted mean	15.33	......	16.10	......	14.01	......	11.47	......

Year and pen.	July actual mean.	Per cent of annual.	August actual mean.	Per cent of annual.	September actual mean.	Per cent of annual.	October actual mean.	Per cent of annual.	Total average production.[a]	Number of birds.
1899-1900	11.90±.46	8.7	11.54±.52	8.4	9.30±.52	6.8	6.14±.50	4.5	136.86	70
1900-1901	13.18±.47	9.2	11.66±.52	8.1	10.34±.52	7.2	5.40±.48	3.8	143.42	85
1901-2	14.90±.59	9.6	13.21±.64	8.5	9.29±.75	6.0	3.75±.60	2.4	155.18	48
1902-3	13.57±.33	10.2	12.58±.40	9.4	10.57±.41	7.9	5.40±.39	4.0	133.65	147
1903-4	10.18±.24	8.6	10.03±.27	8.5	9.93±.28	8.4	5.33±.25	4.5	117.97	254
1904-5:										
50-bird pens	10.87±.26	8.2	10.62±.27	8.0	10.82±.31	8.1	7.16±.30	5.4	133.12	283
100-bird pens	11.64±.50	9.0	10.75±.49	8.3	10.62±.55	8.2	5.49±.48	4.2	133.62	92
150-bird pens	9.81±.44	8.7	8.10±.38	7.2	7.49±.41	6.6	5.59±.36	4.9	112.99	140
1905-6:										
50-bird pen	10.49±.27	7.5	9.37±.28	6.7	9.73±.33	7.0	5.80±.33	4.2	139.49	178
100-bird pens	9.84±.31	7.8	7.05±.28	5.6	7.42±.33	5.8	4.66±.31	3.8	126.83	182
150-bird pens	9.09±.25	7.7	6.69±.24	5.6	6.79±.27	5.7	4.29±.24	3.6	118.75	275
1906-7:										
50-bird pen	12.27±.30	......	11.69±.27	......	4.52±.24	......	......	......	113.02	187
100-bird pens	11.48±.32	......	11.66±.33	......	7.99±.32	......	......	......	108.79	185
150-bird pens	10.03±.27	......	9.26±.25	......	4.67±.23	......	......	......	100.98	281
1904-5, weighted mean	10.72	8.4	9.96	7.8	9.88	7.8	6.43	5.1	126.95	515
1905-6, weighted mean	9.70	7.6	7.54	5.9	7.79	6.1	4.81	3.8	126.86	635
1906-7, weighted mean	11.08	......	10.64	......	5.57	......	......	......	106.64	653

[a] Obtained by adding the averages for the several months of the year. The figures in this column do not agree absolutely with those given in Table 2 of Part I, owing to difference in the arithmetical procedures involved. The differences are insignificant in amount and do not affect the results in any way.

Before discussing in detail the individual monthly averages for the several years as given in Table 1, it will be well to have at hand a summarized statement of these data. It is desirable to have the weighted mean production for each month of the year, as based on the whole period covered by our statistics. By taking the data from the whole period in this way and obtaining the weighted mean for each month, it will be possible to get rid of the disturbing effects of the chance fluctuations coming in from year to year to influence the egg production. The weighted mean monthly curve will give us a more just general idea of the normal distribution of monthly production than can be obtained directly from the detailed figures. In calculating such weighted mean monthly productions the proper basis of the weighting is obviously that which has been used previously in this study, namely, to weight each month's production for a given year with the number of birds which made the record.

In calculating the weighted mean monthly figures it will be proper, of course, to include only months in which the egg production was normal. In those months where an accident of any kind interfered with the egg production the average for that month of that particular year obviously should not be included with the records from the other years. Consequently, in determining the weighted means we have omitted all months in which accidents occurred as described in Part I (loc. cit.).

The weighted mean monthly productions obtained in the manner indicated are given in Table 2. Column 1 of this table gives the absolute weighted mean egg production per bird for each month of the year. The second column gives what may be termed the integral curve of mean monthly egg production. The figures in this column are obtained by adding successively to the sum already obtained the average for each particular month as given in column 1. The figures so obtained show what is the average number of eggs produced per bird up to the end of the designated month on the basis of the statistics here under consideration. An example will make the meaning of this column of the table clear. From the entry in column 2 it appears that in the eight years from 1900 to 1907 the average number of eggs per bird laid up to the end of March in the pullet year is 52.23.

In columns 3 and 4 of the table are given the data of columns 1 and 2 in relative or percentage form. Column 3 gives for each month the percentage which the weighted mean production per bird of that month (as given in column 1) is to the total yearly production. Column 4 gives the integral curve of the data of column 3. The significance of the figures in column 4 may again be illustrated by an example. From the data in this column it appears that, on the average, taking the results for the whole period covered by the study, 40.52 per cent of the total number of eggs produced in the pullet year were laid before April 1. Column 5 of the table gives the years from which records were used to obtain the data in the preceding columns. This column shows what years had a particular monthly record omitted because of an accident in that month. Thus, for example, it may be seen that in determining the weighted mean production for the month of November the data from the laying year 1903–4 were not included. Reference to page 17 of Part I of this study shows that the reason for this was that the birds made no record in November, 1903, because they were not moved into the laying house that year until the 5th of December.

TABLE 2.—*Weighted mean monthly egg production, with monthly and periodical percentages.*

Month.	Absolute weighted mean egg production per bird for month.	Absolute weighted mean egg production from November 1 of pullet year to the end of the month.	Percentage of total yearly production during month.	Percentage of total annual yield between November 1 and end of month.	Years from which data were taken in computing weighted monthly means.[a]
	Number.	*Number.*	*Per cent.*	*Per cent.*	
November	4.63	4.63	3.59	3.59	1899, 1900, 1901, 1902, 1904, 1905, 1906.
December	8.91	13.54	6.91	10.50	1899, 1900, 1901, 1904, 1905.
January	11.71	25.25	9.08	19.58	1900, 1901, 1902, 1904, 1905, 1906.
February	10.87	36.12	8.44	28.02	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
March	16.11	52.23	12.50	40.52	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
April	15.85	68.08	12.30	52.82	1900, 1901, 1902, 1903, 1904, 1905, 1907.
May	13.92	82.00	10.80	63.62	1900, 1901, 1902, 1903, 1904, 1905, 1907.
June	12.46	94.46	9.67	73.29	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
July	10.87	105.33	8.44	81.73	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
August	9.84	115.17	7.64	89.37	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
September	8.19	123.36	6.36	95.73	1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907.
October	5.50	128.86	4.27	100.00	1900, 1901, 1902, 1903, 1904, 1905, 1906.
Total	128.86	....	100.00	....	

[a] In the years for which data are not included in any particular month either an accident interfered with the egg production in that month or there was no record for that month in those years.

With the fundamental data given in Table I of the appendix and Tables 1 and 2 of this section in hand, we may proceed to the discussion of the specific problems with which this part of the work has to do.

THE MONTHLY DISTRIBUTION OF EGG PRODUCTION.

The first question which needs to be considered is as to the general form of the monthly egg-production curve. In the discussion of this matter here use will be made only of the Barred Plymouth Rock data already presented. The data given in column 1 of Table 2 may be taken as the basis from which to gain a general idea of the form of the monthly egg distribution polygon. In figure 1 these data are shown graphically.

It may be noted from the data given in Table 2 and figure 1 that—

1. The form of the monthly egg-production polygon is in general somewhat like that of a frequency curve of variation. The polygon has a single peak, and from that point slopes down toward the base line on both sides. The peak comes at a point not far from the middle of the laying year with the limits here used.

2. Considering the form of the polygon somewhat more in detail, we note that the line starts from a low point in November and rises rather rapidly and in almost a straight line to January. The slope of the line from January to February is downward. In other words, there is an indentation in the ascending limb of the egg-production polygon in the month of February. This is a very characteristic feature of the distribution of egg production, not only observed with the birds here under discussion but also in published records from other sources.

A study of Table 1 shows that this is generally true for every year covered by the investigation. While the February mean production is not necessarily actually lower than the January, though this is true in many cases, there is a perceptible slowing of the rate of increase in egg production which has obtained up to that time. The most probable interpretation of this appears to us to be that the February indentation in the egg-production curve represents a rest or reaction after the winter laying and in anticipation of the heavy March and April production. It marks the completion of a laying cycle on the part of those birds which have been laying during the winter months. After the maximum is reached in March, the descending limb of the egg-production polygon begins. This limb is very nearly a straight line over its whole extent. There is, however, a slight but characteristic indentation to be noted in May. This, like the February falling off in production, is probably to be regarded as the result of a reaction after a period of heavy laying. It marks the completion of the spring laying cycle. The May depression is, further, an expression of the normal brooding season following the heavy laying of March and April. From May on the line descends in a nearly straight course to a low point in the last month of the laying year, October.

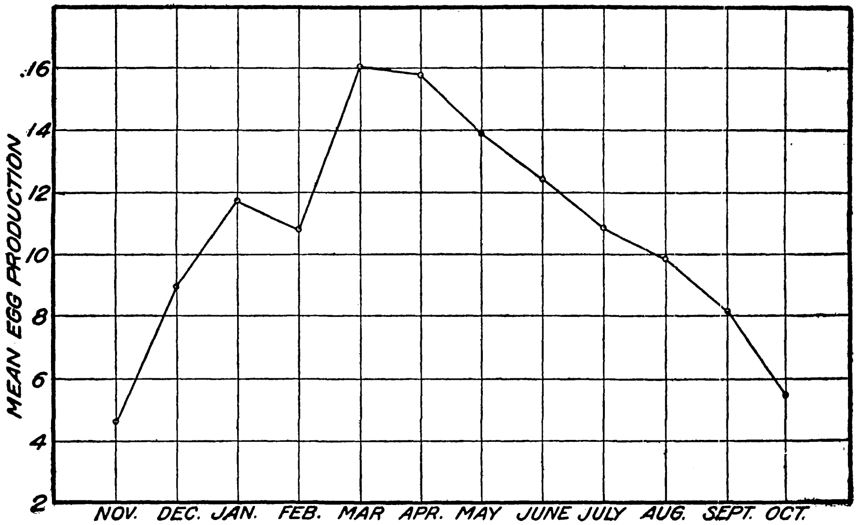

FIG. 1.—Diagram showing the weighted mean monthly egg production for each month of the pullet year.

3. The slope of the ascending limb of the production curve is steeper than that of the descending limb. The rise in egg production to the maximum is more rapid than is the descent from that point.

The curve of mean monthly egg production is shown in integral form in figure 2. This method of dealing with the data brings out some additional points. The curve shown in this figure is the graph of columns 2 and 4 of Table 2.

From figure 2 and the data in Table 2 we note—

1. The average egg production for the whole pullet year, taking the whole period covered by the statistics into consideration, was 128.86 eggs.

2. Of this total annual production, 28.02 per cent (or 36.12 eggs) is produced in the winter months (November 1 to March 1). In this third of the year only about two-sevenths of the total number of eggs are laid.

3. In the period from November 1 to July 1, 73.29 per cent of the total number of eggs is produced. Or, in other words, in the first two-thirds of the laying year nearly three-fourths of the year's eggs are laid.

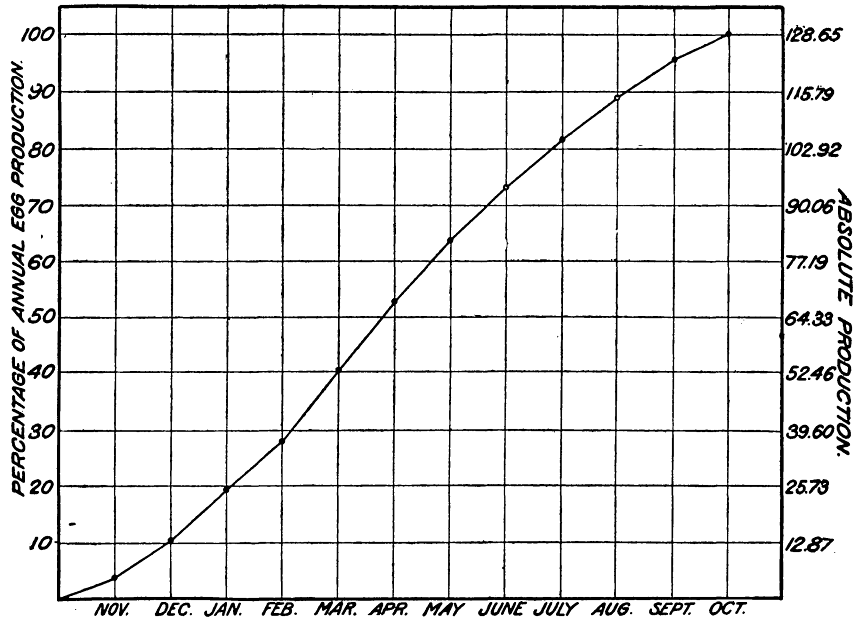

FIG. 2.—Integral curve of weighted mean monthly egg production, showing the absolute and relative number of eggs laid from the beginning of the laying year up to the end of each month.

4. From March 1 to June 1, 35.60 per cent of the total number of eggs is produced. In this particular quarter of the laying year a little more than a third of the eggs are laid.

5. The figures given in Table 2 are likely to prove useful in future discussions of egg-production statistics. They furnish an accurate and trustworthy datum with which other figures may be compared. The need for such a trustworthy basis for comparative discussion has been keenly felt in studies on egg production.

A question which needs more particular discussion than has so far been given it in this paper is as to exactly where in the laying year the maximum of productivity per unit of time occurs. Does this maximum point occur at the same time each year, or does it vary irregularly from year to year?

It is apparent from Table 2 that if the weighted mean figures for the whole period covered by the investigation be taken, the month of maximum production is March. The weighted mean for April, however, falls below that for March by only 0.26 egg—a very small amount. The facts regarding the individual years may most clearly be shown graphically. In figure 3 are platted the mean productions for the months of March and April of each year. Since the maximum production falls in one or other of these months in the present statistics, it will not be necessary to consider other months in this connection.

From this diagram it is clear that neither March nor April has uniformly held the place as the month of maximum production during the period covered by the investigation. In the earlier years (1899–1900 to 1902–3) April had a larger average than March and was the month of maximum production. From 1903–4 to 1905–6, inclusive, March had a larger mean production than April. The relation is reversed again in the last year covered by the statistics (1906–7), though in this year the averages for March and April are very nearly the same.

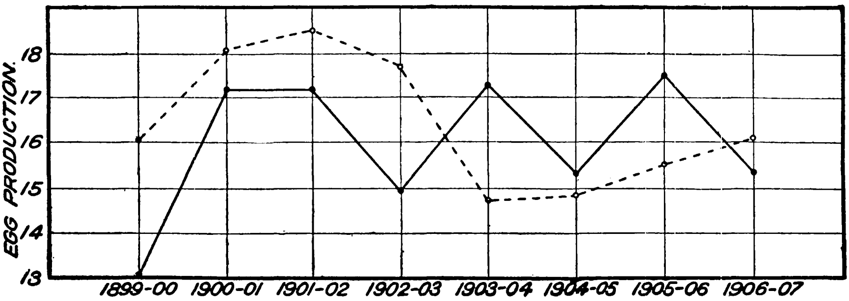

FIG. 3.—Diagram showing the fluctuations of mean monthly egg production in the two months of maximum production, March and April.

It further appears that the change in the month of maximum production which occurs as we pass from the laying year 1902–3 to 1903–4 is entirely due to a marked drop in the average April production for 1903–4 and subsequent years. The mean March production holds at about the same general level throughout. This change appears to have been due to the fact that in the fall of 1903 the birds were for the first time put into a "curtain-front" house. In the earlier years of the experiment the birds had been housed in tight, warmed houses.

Putting all the data together the following statement may be made: So long as the birds were kept in warmed, tight houses the month of maximum average egg production was April. When the change to the curtain-front system of housing was made the April average immediately dropped below that of March, the latter then becoming the month of maximum production. The highest weighted mean monthly average is that for March. This arises because of the larger number of birds and consequently greater weight given to the averages in the later years.

Another question to be considered is as to which is the month of minimum average production. This clearly is either November (at the beginning of the laying year) or October (the end of the year). The weighted mean figures of Table 2 and figure 1 show the month of minimum average production over the whole period to have been November. The weighted mean production for that month is very nearly one egg lower than that for October.

Turning to the records for individual years, the facts are shown graphically in figure 4. In this diagram are platted the October and November means for each year, except the November record for 1903–4 and the October record for 1906–7. As has been pointed out already, there were no records for these months.

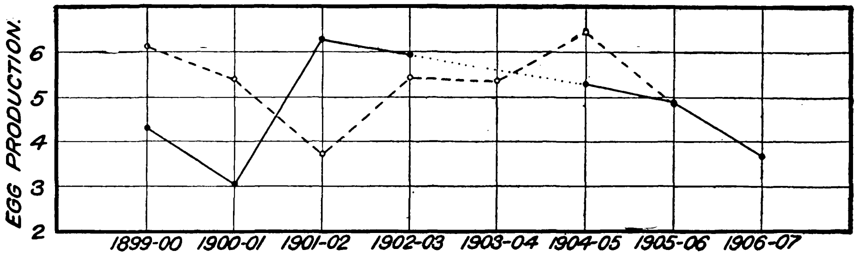

FIG. 4.—Diagram showing the fluctuation of mean monthly egg production in the two months of minimum production, October and November.

From this diagram it appears that there is no steady tendency for the November mean production to be lower than the October, or vice versa. In different years either month may show the lower production. The only conclusion that can be safely drawn from the data is that either one or other of these two months shows the minimum production for the year—which one depending upon circumstances.

VARIATION IN EGG PRODUCTION IN EACH MONTH OF THE YEAR.

In discussing the variability in egg production in the different months of the year use will be made of the same variation constants as in Part I of this paper, namely, the standard deviation and the coefficient of variation.[a] It will be found to be the case with the monthly distributions, just as with the annual, that the coefficient of variation is the more trustworthy constant on which to base one's judgment as to the actual degree of existing variability in this character. The reason for this is that the absolute average production is so different in the different months. In consequence of this the absolute amount of "scatter" in the distributions is changed.[a]

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[a] A description of the significance of these two variation-measuring constants—standard deviation and coefficient of variation—is given in Part I, page 27, and need not be repeated here. It suffices merely to recall that the standard deviation is the measure of absolute variation in terms of eggs, whereas the coefficient gives the relative variation in per cent.

In calculating the variation constants for the monthly distributions the same methods were used as have been described in Part I. Just as in the case of the means (cf. p. 87, supra), so in the case of the variation-measuring constants, weighted average figures have been taken for the three laying years, 1904–5, 1905–6, 1906–7, where the birds were kept in flocks of different sizes. The weighting was done here, as before, on the basis of the number of birds in each pen.

On account of the fact that the standard deviation has comparatively little significance in discussions of variation in egg production, it has not seemed necessary or advisable to take the time to calculate the probable errors for all the standard deviations of the monthly distributions. The probable errors have been determined, however, for the coefficients of variation.

The data for variation in monthly egg production are given in Tables 3 and 4. Table 3 contains the standard deviation in egg production for each month covered by our statistics and Table 4 the coefficient of variation.

TABLE 3.—*Standard deviation in egg production of Barred Plymouth Rocks for each month of the laying year.*[b]

Year and flock.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.
1899–1900	6.00	9.05	6.59	6.79	5.81	5.43	5.72	6.28	5.68	6.44	6.39	6.26
1900–1901	5.40	8.83	7.96	6.13	6.16	5.68	6.18	6.75	6.38	7.05	7.16	6.62
1901–2	8.35	8.46	5.79	5.92	4.81	5.13	5.99	4.07	6.08	6.55	7.75	6.15
1902–3	7.60	6.32	7.67	6.76	5.86	5.74	5.65	5.96	5.96	7.28	7.31	7.06
1903–4	......	5.60	7.46	6.93	6.54	5.67	5.53	6.16	5.75	6.31	6.72	5.81
1904–5:												
50-bird pens	4.64	7.59	8.41	6.50	5.80	5.97	6.85	6.49	6.53	6.77	7.66	7.40
100-bird pens	7.77	7.39	7.22	6.11	5.93	5.99	5.76	6.49	7.12	6.91	7.86	6.78
150-bird pens	7.23	8.10	7.89	6.78	7.14	6.37	7.24	7.44	7.80	6.71	7.17	6.25
1905–6:												
50-bird pens	6.83	9.10	7.46	5.64	5.15	4.61	5.28	4.83	5.25	5.57	6.53	6.58
100-bird pens	6.48	8.87	7.65	6.52	5.75	5.15	5.76	5.92	6.23	5.58	6.63	6.19
150-bird pens	5.47	9.26	8.44	6.80	4.89	4.86	5.64	5.62	6.14	5.92	6.57	6.02
1906–7:												
50-bird pens	5.06	6.77	6.75	6.55	6.01	5.73	6.23	6.05	6.12	5.40	4.85	......
100-bird pens	6.06	7.90	6.57	6.08	6.17	5.63	5.98	5.99	6.46	6.57	6.45	......
150-bird pens	6.92	8.01	6.72	6.89	6.58	5.94	5.95	6.45	6.63	6.24	5.61	......
1904–5, weighted mean	5.90	7.69	8.06	6.51	6.19	6.08	6.76	6.75	6.98	6.78	7.56	6.98
1905–6, weighted mean	6.14	9.10	7.94	6.39	5.21	4.87	5.57	5.48	5.92	5.72	6.58	6.23
1906–7, weighted mean	6.14	7.62	6.69	6.56	6.30	5.79	6.04	6.21	6.44	6.09	5.63	......

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[a] For a discussion of this point see Part I of this study, page 27.

[b] Because of the fact that the standard deviation is not the significant measure of variation in egg-production statistics, probable errors have not been calculated for the constants in this table. From the data given in this table and in Table 1 anyone can easily calculate the probable error of any standard deviation in this table. In general, the probable error of the standard deviation will be approximately 71 per cent of the probable error of the corresponding mean as given in Table 1.

TABLE 4.—*Coefficients of variation for the egg production of Barred Plymouth Rocks for each month of the laying year.*

Year and flock.	November.	December.	January.	February.
1899–1900	139.10±17.50	87.99± 8.01	39.70± 2.60	69.05±5.50
1900–1901	175.18±22.04	107.83±10.17	64.53± 4.53	42.93±2.60
1901–2	131.84±19.20	70.37± 6.83	38.08± 2.98	50.72±4.30
1902–3	128.17±10.44	139.60±12.15	129.13±10.58	70.25±3.90
1903–4	....	100.71± 5.25	85.63± 4.03	56.48±2.16
1904–5:				
50-bird pens	160.92±11.34	134.82± 8.23	83.85± 3.69	48.23±1.66
100-bird pens	120.57±10.62	69.24± 4.82	64.99± 4.39	54.17±3.39
150-bird pens	128.21±10.70	88.27± 5.69	73.63± 4.28	74.46±4.36
1905–6:				
50-bird pens	108.59± 7.11	92.33± 5.43	53.93± 2.42	39.02±1.59
100-bird pens	120.25± 8.39	89.48± 5.11	57.64± 2.63	48.72±2.09
150-bird pens	148.96± 9.99	96.37± 4.69	67.83± 2.70	58.38±2.18
1906–7:				
50-bird pens	155.77±13.14	144.08±11.41	118.52± 8.07	62.44±2.91
100-bird pens	183.96±17.98	128.28± 9.32	152.01±12.64	117.33±7.97
150-bird pens	161.87±11.50	102.15± 5.11	143.93± 9.29	100.99±5.01
1904–5, weighted mean	144.82	110.45	77.70	56.42
1905–6, weighted mean	129.41	93.26	61.01	50.18
1906–7, weighted mean	166.38	121.56	138.94	94.58

Year and flock.	March.	April.	May.	June.
1899–1900	44.58±3.00	33.79±2.13	37.39±2.41	50.13±3.50
1900–1901	35.96±2.09	31.49±1.78	40.00±2.38	47.11±2.93
1901–2	28.02±2.07	27.74±2.05	36.87±2.86	24.12±1.75
1902–3	39.14±1.76	32.39±1.40	33.21±1.44	37.66±1.68
1903–4	37.88±1.29	38.34±1.31	47.53±1.71	50.22±1.84
1904–5:				
50-bird pens	32.25±1.00	38.50±1.24	49.68±1.72	51.52±1.81
100-bird pens	48.46±2.92	38.16±2.16	40.09±2.29	48.65±2.94
150-bird pens	59.42±3.13	48.38±2.36	60.55±3.21	72.76±4.03
1905–6:				
50-bird pens	28.17±1.08	27.96±1.07	46.20±1.98	35.85±1.44
100-bird pens	33.17±1.30	34.17±1.34	53.63±2.38	46.41±1.96
150-bird pens	28.76±0.89	31.98±1.01	52.88±1.90	48.25±1.68
1906–7:				
50-bird pens	33.93±1.31	33.45±1.29	43.18±1.76	54.37±2.39
100-bird pens	44.19±1.83	34.31±1.34	39.28±1.58	45.72±1.91
150-bird pens	44.90±1.51	39.02±1.28	46.02±1.57	60.74±2.28
1904–5, weighted mean	42.53	41.13	50.92	56.78
1905–6, weighted mean	29.86	31.48	51.22	44.25
1906–7, weighted mean	41.56	36.09	43.30	54.66

Year and flock.	July.	August.	September.	October.
1899–1900	47.73±3.27	55.82±4.05	68.69±5.46	101.92±10.19
1900–1901	48.39±3.03	60.46±3.49	69.25±5.01	122.68±12.71
1901–2	40.79±3.24	49.57±4.17	83.39±8.88	163.98±28.51
1902–3	38.27±1.70	57.87±2.94	69.19±3.81	130.67±10.80
1903–4	56.50±2.16	62.86±2.52	67.68±2.80	109.03± 7.13
1904–5:				
50-bird pens	60.07±2.23	63.75±2.43	70.81±2.84	103.37± 5.19
100-bird pens	61.16±4.02	64.28±4.32	74.05±5.33	123.52±13.82
150-bird pens	79.51±4.82	82.81±5.14	95.64±6.48	111.86± 8.43
1905–6:				
50-bird pens	50.06±2.19	59.40±2.77	67.13±3.31	113.45± 7.67
100-bird pens	63.28±3.00	79.10±4.20	89.44±5.10	132.77± 9.98
150-bird pens	67.56±2.69	88.54±4.08	96.83±4.72	140.27± 8.96
1906–7:				
50-bird pens	49.88±2.13	46.20±2.50	107.40±6.12	....
100-bird pens	56.28±2.52	56.36±2.53	80.75±4.30	....
150-bird pens	66.07±2.57	67.37±2.65	120.29±6.75	....
1904–5, weighted mean	65.55	69.03	78.14	109.28
1905–6, weighted mean	61.43	77.67	86.39	130.60
1906–7, weighted mean	58.66	58.19	105.40	....

The first point which it is necessary to take up is in regard to the general trend of the variability in egg production in the different months of the year. In order to do this satisfactorily it is desirable to have in hand the weighted average figures for each month based on the whole experience covered by the statistics (1899 to 1907). It is as necessary to have these weighted figures in the case of variation constants as it was in the case of the means. They smooth out the insignificant fluctuations of individual years. Such weighted average values for the standard deviations and coefficients of variation are given in Table 5. The weighting is again made on the basis of the number of individuals involved.

TABLE 5.—*Variation in monthly egg production: Weighted mean standard deviations and coefficients of variation based on total period 1899–1907.*[a]

Month.	Weighted mean standard deviation.	Weighted mean coefficient of variation.
	Eggs.	*Per cent.*
November	6. 20	146. 40
December	8. 52	99. 63
January	7. 78	68. 82
February	6. 53	65. 75
March	5. 94	37. 76
April	5. 82	37. 03
May	6. 17	44. 72
June	6. 09	49. 85
July	6. 28	57. 94
August	6. 29	65. 96
September	6. 63	85. 58
October	6. 48	121. 30

[a] In calculating these mean figures no month was included which was in any respect abnormal. By reference to column 5 of Table 2, the years which contributed data to the averages for each month may be determined.

The data given in Table 5 are shown graphically in figures 5 and 6. With these data in hand we may proceed to discuss some of the problems regarding variability in monthly egg production. It has been shown in Part I (pp. 28, 29) that a certain degree of variation in annual production is fairly constant and characteristic for the flocks here under consideration. Is this same degree of relative variability characteristic for each month of year? Or is some other definite degree of variability more or less characteristic for the monthly distribution? Or is it the case that the flocks show greater variability in certain seasons of the year than in others? If the last is true, how are these differences distributed?

We may in this connection consider first the standard deviations. An examination of the data in Table 5 and figure 5 shows that starting at a point of maximum absolute variation in December there is a steady and rather rapid decrease until the minimum is reached in April. From that point on to the end of the laying year in October there is a very much more gradual but still distinct increase in the value of the standard deviation. The standard deviation of egg production in the month of November does not fall into this scheme shown by the remaining months of the laying year. If it did it should have a higher value than that for December, whereas, as a matter of fact, the standard deviation for November is just about the same as that for May and is very much below that for December. Leaving this single month out of account for the moment, however, it is plain that the curve of absolute variability in egg production is in a general way similar to the curve of mean monthly egg production turned upside down. Where the curve of mean monthly production increases rapidly the curve of absolute variability in egg production decreases rapidly (December to April). On the other hand, where the curve of mean production decreases slowly the curve of absolute variability in egg production increases also slowly. The exception afforded by the month of November to this rule is due to the fact that there is a very high frequency of zero production in the November distributions. (See Appendix II, Table I.) There are many hens which do not begin to lay in November. Hence there is very little absolute

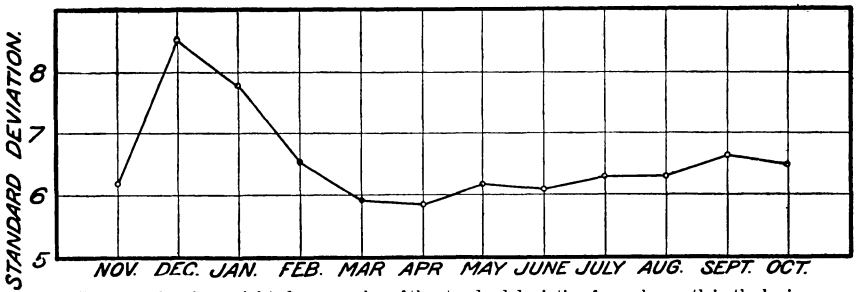

FIG. 5.—Diagram showing weighted mean value of the standard deviation for each month in the laying year.

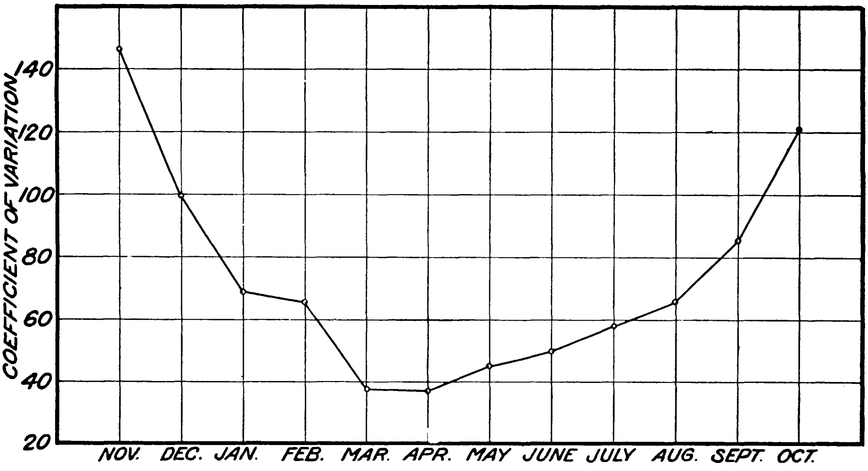

FIG. 6.—Diagram showing weighted mean value of the coefficient of variation for each month in the laying year.

"scatter" to the distribution for this month. Consequently it may be said that while as a general rule the absolute variation in egg production is greater when the average production per bird is low, this relation fails to hold when the laying gets so poor that the major part of the flock lays no eggs at all, thus putting a large frequency on one base element, viz, zero. This point will appear more clearly when the form of distribution for the individual months is taken up (p. 126).

Turning to a consideration of the curve of relative variability in egg production in the different months as given by the data in the second column of Table 5 and in figure 6, we see the same relation as is given by the standard deviations, but in a much more pronounced and distinct form. The curve starts with a maximum relative variability in November and falls steadily and rapidly to a minimum in March and April. It then rises gradually and in almost a straight line to August. From August to the end of the year in October the line of the coefficient of variation rises at a still sharper angle to the base. The data furnished by both absolute and relative measures of variation lead to the general result that when flocks are laying well there is a great deal of uniformity in regard to their laying (i. e., little variation), whereas when they are laying poorly they exhibit a great deal of variation.

There are one or two matters of detail in regard to these curves of variation in monthly egg production which it is desirable to discuss briefly. First, in respect to the time of the year in which the maximum degree of variability in egg production is exhibited, it is of interest to note that this time is at the beginning of the pullet-laying year rather than at the end. Thus, in figure 5 November and December show higher values of the coefficient of variation than do September and October. The same is true for the standard deviations save only for the exception made by November.

The month of minimum variability, both absolute and relative, is April. The difference, however, in the variability in production between March and April is so small as to be insignificant in the weighted figures or in any single year. It is of some interest to note, though, that with the single exception of the year 1901–2 the standard deviation is lower for April than for March (see Table 3). In two years—1903–4 and 1905–6—the coefficient of variation is greater in April than in March. There is clearly no definite change after the year 1902–3 in the degree of absolute or relative variability, such as was seen to be the case with the mean production (see p. 92).

From the data given in Table 2 of Part I (p. 28) it is possible to determine the following two values based on the whole experience covered by our statistics (1899–1907):

Weighted mean standard deviation for annual egg production = 41.74 eggs.

Weighted mean coefficient of variation for annual egg production = 34.21 per cent.

Comparing these with the monthly variabilities it is seen that, as would be expected, (*a*) the absolute variation or "scatter" is much greater for the annual distributions than for any of the monthly. (*b*) Relatively, however, there is no month which on the average shows so low a variability in proportion to its mean as does the production of the whole year. April comes nearest to this, but it falls short nearly 3 per cent.

THE NATURAL DIVISIONS OF THE LAYING YEAR.

It will undeniably be advantageous, in studying certain phases of the problem of egg production, to endeavor to use a time unit which conforms to the natural periodicity displayed by hens. In recent years it has been the custom in the discussion of egg production to make the unit one year. This custom has been followed in the work of the Maine station, in the egg-laying competitions in South Australia, which have excited world-wide interest, and by many other institutions and experimenters. It is safe to say, however, that all students of the subject of egg production have felt that the year was not in all respects an ideal unit for such studies. A serious objection to it is immediately apparent if one makes a close study of individual egg records. The total number of eggs laid by a bird in a year gives in reality an inadequate and incomplete idea of her egg-producing ability. Birds which make the same yearly total records are by no means always equivalent as egg producers.

The plan followed at the present time[a] in the investigations in progress at the Maine station breaks the laying year up into four parts. The first of these includes the months of November, December, January, and February. Broadly speaking, this is the period of winter laying and is so designated. The second period includes the months of March, April, and May. This, broadly speaking, obviously corresponds to the breeding season. The third period includes the months of June, July, and August, and is called the summer period. Finally, the fourth period includes the months of September and October, and is the period in which molting and its associated drop in egg production commonly occur. In future discussions of egg production by the present writers it is proposed, wherever necessary or desirable, to consider separately the egg production in each of these periods. By this method it will be possible to compare, for example, the production in the winter-laying period of different lots of birds, or their ability to lay during the breeding season, and so on.

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[a] See, for example, Maine Agricultural Experiment Station Bulletin 166.

The justification for the conclusion that this division of the year is in general a natural one and corresponds to a real cyclical periodicity in egg production is in considerable measure to be found in the facts regarding mean monthly egg production and variation in this character set forth in this and the preceding section of the paper. The winter-laying period is a period characterized by rapid increase in mean production associated with a relatively equally rapid decrease in variability, both absolute and relative. In this period a large part of the flock falls in the A component of the monthly distributions (see p. 142). This laying period is, strictly speaking, not a part of the natural or normal reproductive cycle of the hen. Egg laying in this part of the year is something which during domestication has been added on, as it were, to the natural reproductive activity of the wild *Gallus*. It is a result of "forcing" or special stimulation. From the evolution standpoint, egg production in these months is a comparatively recent acquisition. Such being the case, the greater variability observed in winter laying is only what would be expected.

The limits of this artificial winter cycle of egg production are fairly well defined. It begins with the beginning of the laying year. Its other limit is marked by the slacking up in egg production which occurs in February (see fig. 1). This slacking up in February, which appears to be a characteristic of egg production generally, is to be explained, we believe, chiefly if not entirely as the result of the ending of the winter cycle by the majority of birds which have laid during the early winter. Such birds rest for a period at about this time before beginning the spring laying cycle. Of course it must be understood that these statements are made only with reference to what might be called the general or average course of events. Particular birds may form exceptions in their laying. Many birds, of course, have no proper winter cycle of laying at all. They begin to lay for the first time in January or February, and keep on laying without any large break straight through the spring cycle.

The next period (March, April, and May) is the natural laying season. It corresponds to the egg-laying part of the natural reproductive cycle exhibited by the wild *Gallus*. This being the case, we should expect the egg production in the domesticated birds also to be at a maximum at this time. They ought further to show a minimum degree of variation in egg production at this time, because nearly every individual which is physiologically capable of ovulation is laying, or has laid very recently, and is broody. Naturally, therefore, a high mean and a low variability in production are exactly what we find characterizing the laying in each of the months in this period.

The third period (June, July, and August) is characterized by a gradually falling mean production and a variability gradually increasing, but still much lower than that for the winter period. This is the period in which rearing of the chickens naturally occurs, and it also represents an extension of the breeding season. It is a period in which laying naturally (i. e., without the forcing or stimulation of domestication) occurs in some degree. We are informed[a] by Mr. J. H. Burkill, of the Indian Museum in Calcutta, that in the central provinces of India the breeding season of *Gallus ferrugineus* Gmel. is said to last until November, beginning in the spring. Similarly the breeding season of *Gallus sonneratii* Temminck in the central provinces lasts from March to November.

The fourth period (September and October) is not easily separated from the third in respect to laying, but in general it is the period of molting, or, if the process has not actually begun, of preparation for molting. It is characterized by reduced laying and markedly increased variability. The variability in egg production in this period is not so great, however, as in the winter laying period, which falls entirely outside the natural reproductive cycle.

CHANGES IN MONTHLY EGG PRODUCTION DUE TO SELECTIVE BREEDING.

As has been stated earlier in the paper (p. 84), it is a matter of prime importance to examine the constants of monthly egg production with reference to their bearing on the problem of selection. It is the purpose of this section to discuss this matter. Specifically the attempt will be made to determine whether during the period in which selection for egg production was practiced[b] there was any definite steady trend of (*a*) the mean monthly production, or (*b*) the absolute variability, or (*c*) the relative variability in monthly production either upward or downward, each single month being considered separately.

The most direct and simple way to approach this problem is, without any doubt, to plat the values of each constant (mean, standard deviation, or coefficient of variation) for each month of the years covered by the statistics as given in Tables 1, 3, and 4, using weighted mean values for the last three years, 1904–1907. To obtain a just idea of the general trend of the zigzag lines so obtained it will be necessary to fit to each of them a line by some adequate method of curve fitting. Then by inspection of the fitted line, keeping in mind the values of the probable errors of the constants under discussion, it will be possible to conclude whether there has been any steady, definite change in that constant as selective breeding has proceeded. This is the method which will be followed. All the lines to be given were fitted by the method of least squares.

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[a] In litt.

[b] For a detailed account of the method of selective breeding followed, see Part I, p. 12.

EFFECT ON AVERAGE MONTHLY PRODUCTION.

We may first examine the mean production for each month with reference to whether it increased or decreased with continued selection for higher annual production. The fundamental data on this point are given in Table 1.

In figures 7 and 8 are given the data of Table 1 in diagrammatic form. Figure 7 includes the observed means and best fitting straight lines for the egg production of the months of November to April, inclusive. In the same way figure 8 shows the observed means and their fitted straight lines for the months of the second half of the laying year, May to October, inclusive. In all cases the weighted mean figures are used for the last three years, 1904–1907.

The equations to the straight lines in figures 7 and 8 are as follows, y denoting mean monthly production and x number of years since the laying year 1898–99:

November	$y=4.68+0.026x$	May	$y=16.69-0.544x$
December	$y=9.72-.375x$	June	$y=15.34-.412x$
January	$y=15.72-1.072x$	July	$y=13.997-.465x$
February	$y=12.33-.242x$	August	$y=13.05-.479x$
March	$y=15.19+.172x$	September	$y=11.11-.449x$
April	$y=17.86-.313x$	October	$y=5.37-.011x$

From these diagrams (figs. 7, 8) the following points are to be noted:

1. With the exception of the months of November and March, there is clearly observable in each month a downward tendency or trend of the line of mean monthly egg production. In other words, the mean egg production in ten months out of the twelve shows exactly the same phenomenon as does the total annual production during the period of continued selection (see Pt. I, pp. 38–43). The amount or degree of this downward trend differs in different months, being statistically insignificant in some months and quite certainly significant in others. January shows the greatest decrease in mean production. October exhibits the smallest decrease of any of the months showing any decline whatever. The line for this month is sensibly horizontal.

2. Of the two months which form apparent exceptions to the rule that the mean monthly production decreased during the period of investigation, neither can justly be regarded as affording a real exception. The line for November is certainly horizontal within the limits of the probable errors involved. There was neither a decrease nor an increase in the mean production in this month during the years studied. In the case of March the line slopes upward a little more definitely than does that for November, but still it can hardly be considered as deviating significantly from horizontality when the

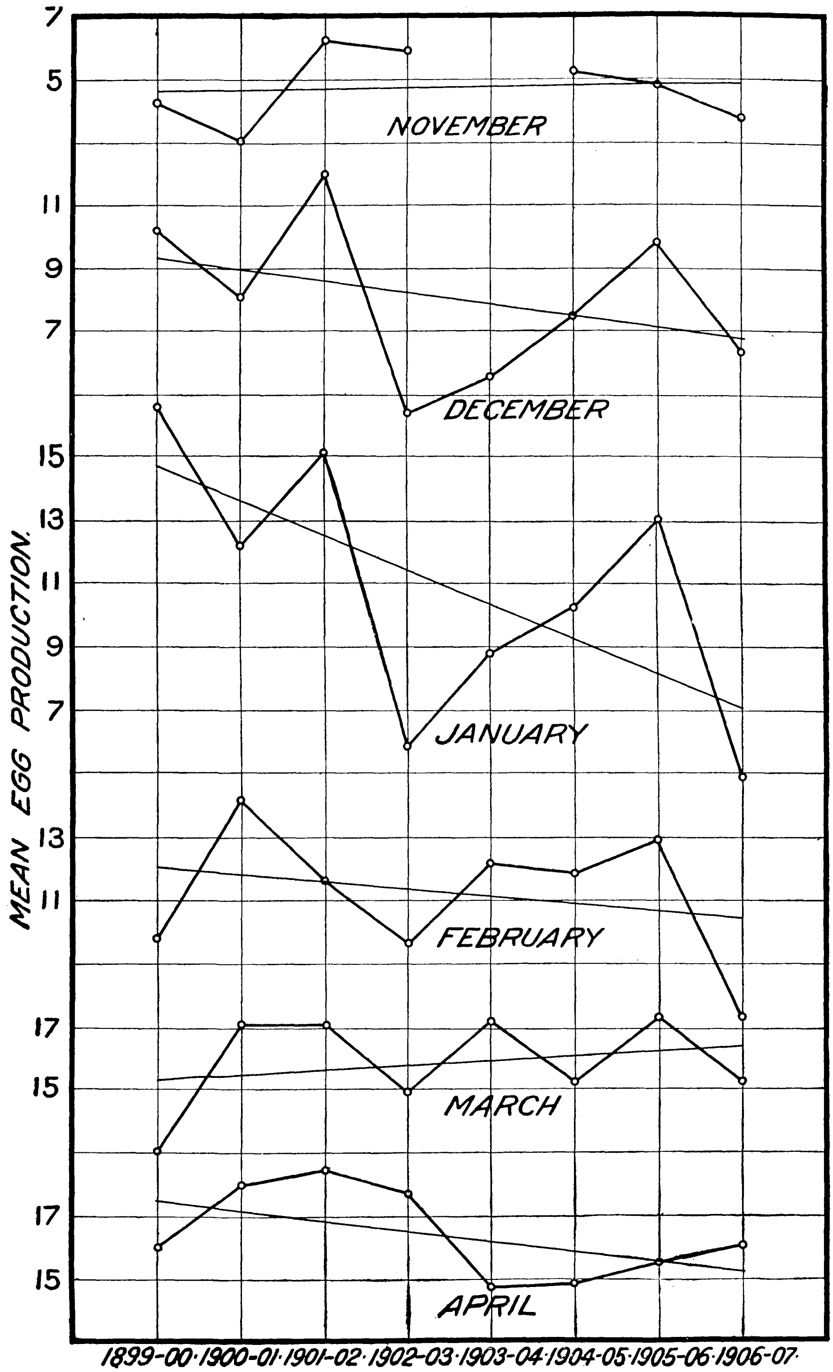

FIG. 7.—Diagram showing the course of the observed mean egg production in each of the months of November to April, inclusive, 1899-1907.

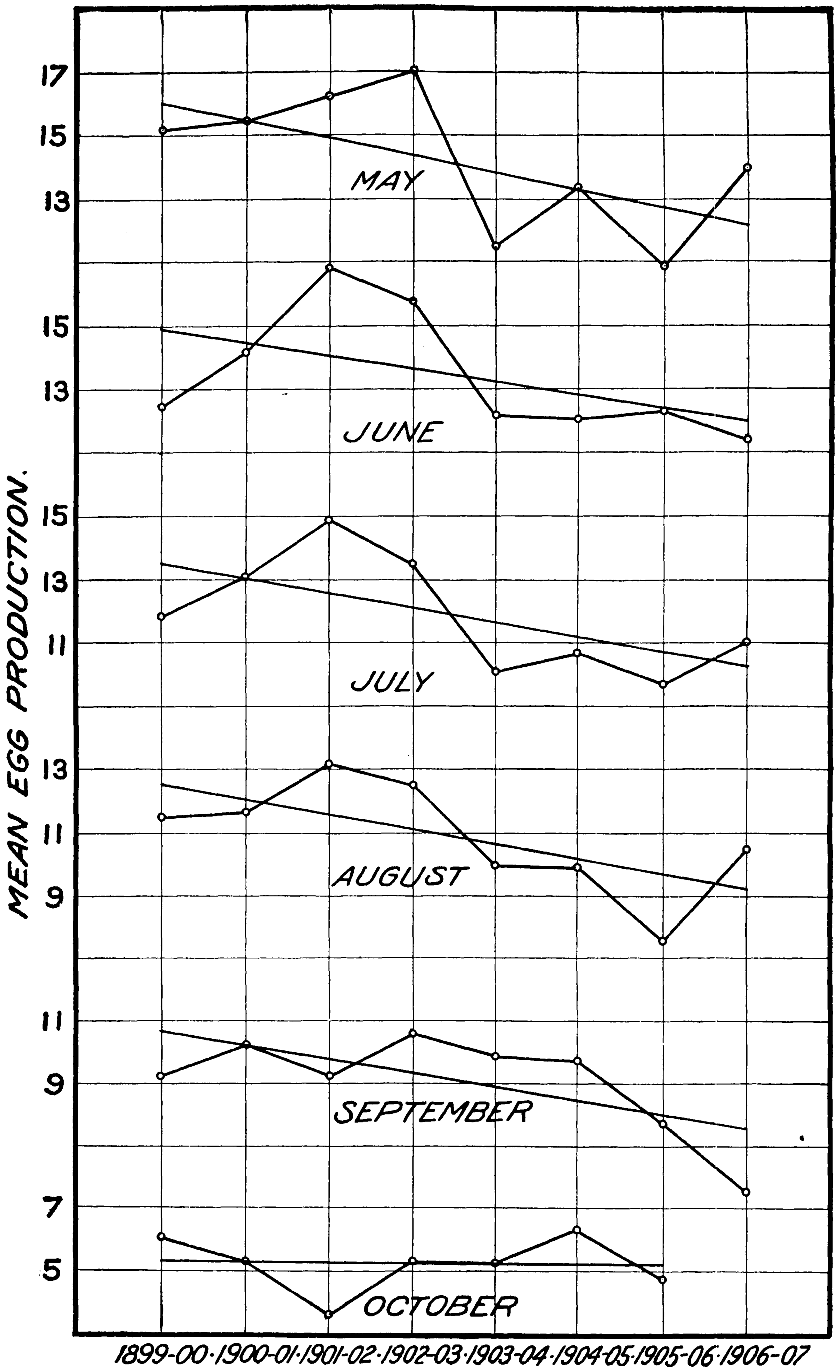

FIG. 8.—Diagram showing the course of the observed mean egg production in each of the months of May to October, inclusive, 1899–1907.

probable errors of the several means are taken into account. That there should have been an insignificant change in the mean March production is exactly what would be expected when it is remembered that this is the natural season of heaviest laying. In this period the egg production is very little affected either by breeding or by environmental influences.

3. In Part I of this study (pp. 40, 41), in discussing the general trend of average annual egg production under the influence of selective breeding, modifications or corrections of the observed means were made to allow for the influence of unavoidable accidents which were held to have affected adversely the egg production in particular months of certain years. Regarding these modifications the following general statement was made (loc. cit., p. 38):

> We may next proceed to modify the constants, for the influence of accidents and the like, from the actual values they show. In every case we can be sure of the general direction in which such modification should be applied. As to what its exact amount in each case should be there are, of course, no criteria. But if we make the modifications relatively large, we shall establish what may be regarded as a series of upper limiting values for the constants. We may then fairly conclude that had all the conditions remained normal throughout the period covered by the data the values exhibited by the constants would have fallen somewhere between the limits set by the actual and modified values. While such a method can not give us precise values, it is capable of showing what the general trend of the data is.

It is apparent that the significance of this reasoning ultimately rests on the question of whether the modifications made are sufficiently large to establish what are in reality upper limiting values. In Part I the attempt was made to show that the modifications used were sufficiently large and that if they were in error at all it was in being too large. It was, however, impossible to demonstrate that this was the case without the detailed monthly data in hand. These data we now have, and they are shown graphically in figures 7 and 8.

It is neither necessary nor desirable to take the space to show in detail that each of the modifications used was sufficiently large, for if anyone interested will, for himself, add to the averages for the individual months as shown in figures 7 and 8 the modifications given in Table 5 of Part I, platting the resulting points on the diagrams of figures 7 and 8, he will see at once that the modifying factors used were very much larger than strict fairness demanded. This may be shown specifically by taking up in detail a single example. In the year 1905–6, 50 per cent of the recorded April production (16.48 eggs) and of the recorded May production (11.42 eggs) in the 50-bird pens was added to allow for an accident (see Pt. I, p. 41). This results in a modified April mean of 24.72 and a May mean of 17.13. Now Table 1 and figures 7 and 8 show that the highest April mean production ever reached in the eight years' trap nesting was only 18.50 eggs (1901–2) and the highest May mean was 17.02 (1902–3). In other words, the modified April mean is some 6 eggs higher than the highest April mean ever observed for the station's flocks, while the modified May mean was equal to the highest May mean observed. There can be no doubt that the modifications used were in every case sufficiently large.

EFFECT ON ABSOLUTE VARIATION AS MEASURED BY THE STANDARD DEVIATION.

We may turn next to the examination of the changes in absolute variability in egg production as measured by the standard deviation which occurred in the period covered by the statistics. It has been shown in Part I (p. 45) that neither the absolute nor the relative variability in total annual egg production sensibly decreased with continued selection. We have now to determine whether the variability in egg production in any months of the year showed the effect of selection by becoming less as the selective breeding was continued. We will first discuss absolute variability as measured by the standard deviation, and later relative variability as measured by the coefficient of variation. The fundamental data for the treatment of absolute variability are given in Table 3, which records the standard deviation for each month. These data are platted in figures 9 and 10 and the observed values are fitted by the method of least squares with straight lines just as was done in the case of mean egg production. As before, weighted mean values were used for the years 1904 to 1907.

The equations to the straight lines in these figures are as follows, y denoting standard deviation and x number of years since the laying year, 1898–99:

November	$y=6.58-0.020\,x$	May	$y=5.86+0.016\,x$
December	$y=8.46-.139\,x$	June	$y=5.88+.017\,x$
January	$y=6.88+.086\,x$	July	$y=5.85+.066\,x$
February	$y=6.41+.019\,x$	August	$y=7.03-.112\,x$
March	$y=5.67+.042\,x$	September	$y=7.39-.112\,x$
April	$y=5.48+.015\,x$	October	$y=6.40-.010\,x$

From these diagrams (figs. 9, 10) the following points are to be noted:

1. There is less regularity in the trend of the variability curves than was observed in the case of the curves for the means. While in the majority of cases the standard deviation increases more or less during the course of the experiment, there are five cases in which it decreases by a greater or less amount. It should be said, however, that in nearly all of these cases the change, whether up or down, is very small in amount. The lines for the standard deviation run in all cases not far from the horizontal. The fair conclusion would seem to be that, during the period covered by the statistics when selective breeding for total annual egg production was practiced, it had prac-

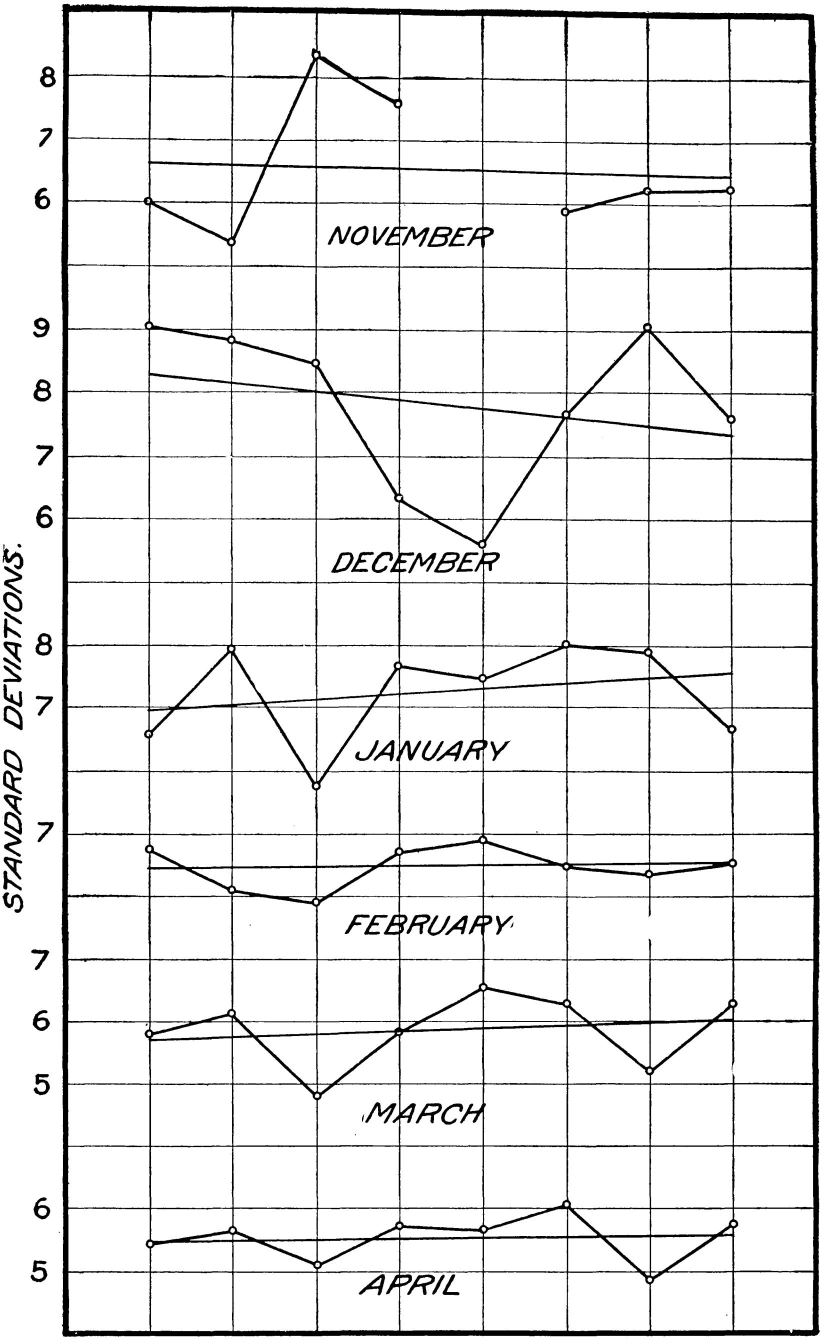

FIG. 9.—Diagram showing the trend of the observed absolute variability in monthly egg production as measured by the standard deviation in each of the months of November to April, inclusive, 1899-1907.

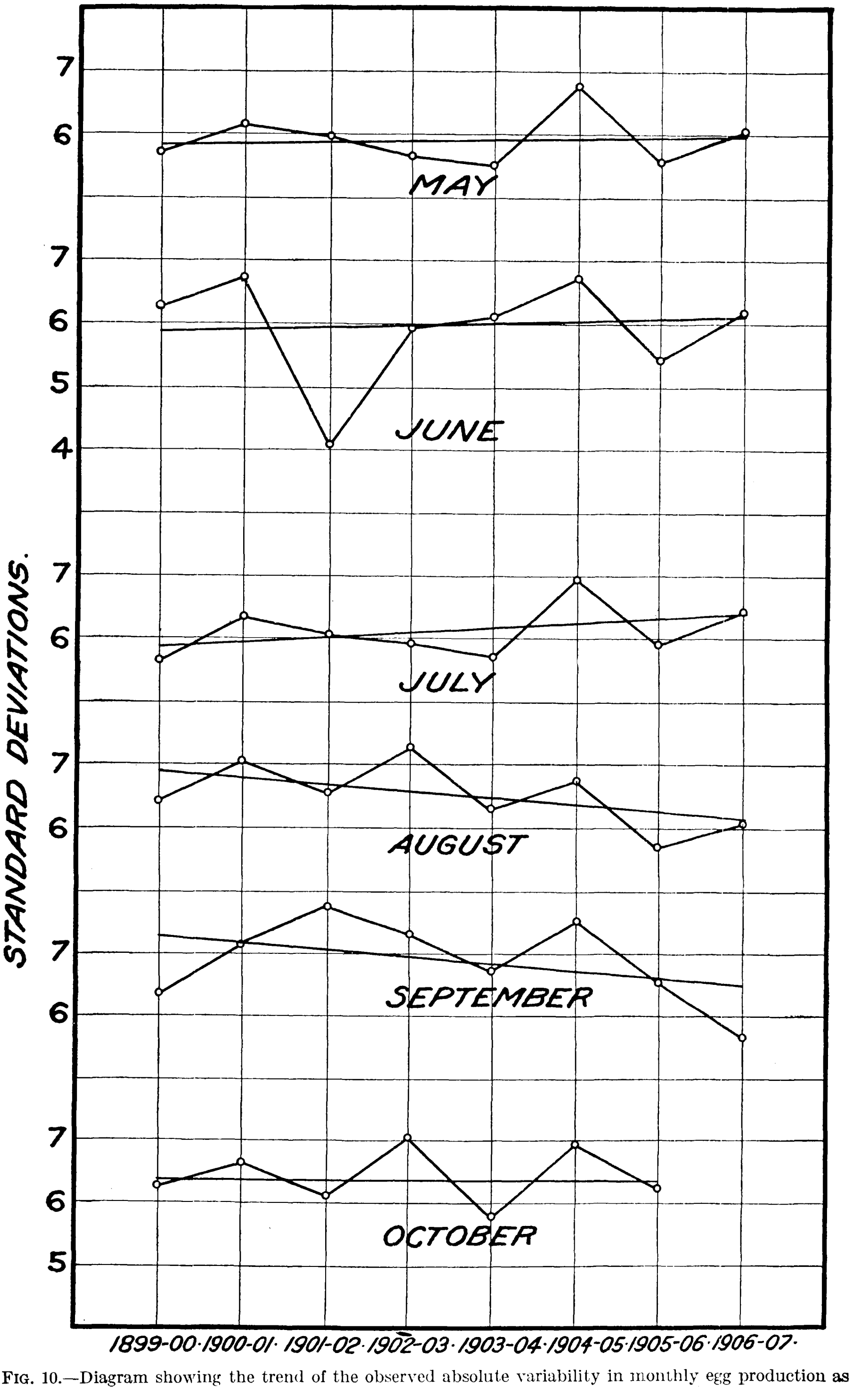

FIG. 10.—Diagram showing the trend of the observed absolute variability in monthly egg production as measured by the standard deviation in each of the months of May to October, inclusive, 1899–1907.

tically no effect either to decrease or to increase the absolute variability of the egg production of any of the single months.

2. The months showing the greatest decrease in the standard deviation are December and August. In both of these cases the decrease during the eight years is possibly significant statistically. Of the months showing an increase in the standard deviation the most marked are January and July. Comparison with figure 7 shows that January was the month in which the mean egg production showed the most pronounced drop during the period covered. We have here again an illustration of the rule brought out earlier in the paper that as mean egg production decreases the variability in this character tends to increase.

EFFECT ON RELATIVE VARIABILITY.

It has been seen throughout this study that the relative variability as given by the coefficient of variation is the more significant measure by which to arrive at a true conception of the actual existing degree of variability in egg production. Accordingly it is the more important to examine the trend of this constant with reference to the effect of selection. In making such an examination the same methods will be followed as in the case of the means and the standard deviations. The fundamental data are given in Table 4. These data are shown graphically in figures 11 and 12.

The equations to the straight lines in these figures are as follows, y denoting the coefficient of variation and x the number of years since the laying year 1898–99:

November . . . . $y=143.09+0.260\,x$
December . . . . $y=92.18+2.899\,x$
January . . . . $y=39.03+8.958\,x$
February . . . . $y=49.63+2.599\,x$
March . . . . $y=37.39-.112\,x$
April . . . . $y=30.73+.740\,x$
May . . . . $y=34.31+1.833\,x$
June . . . . $y=38.76+1.523\,x$
July . . . . $y=39.62+2.788\,x$
August . . . . $y=52.54+1.976\,x$
September . . . . $y=61.08+3.874\,x$
October . . . . $y=123.41+.153\,x$

From these diagrams (figs. 11, 12) the following points are to be noted:

1. With a single exception (March) the relative variability as measured by the coefficient of variation increased during the period in which selective breeding was practiced. The zigzag lines show that this increase was fairly steady and regular in nearly all cases, though it was very much more pronounced in some months than in others. In particular the variability of the January production increased very noticeably. This result indicates that the egg production of the single months follows the same rule as was shown to hold for the total annual production. So far as there was any change whatever in variability in monthly egg production during the period when selective breeding was practiced this change was not in the direction of

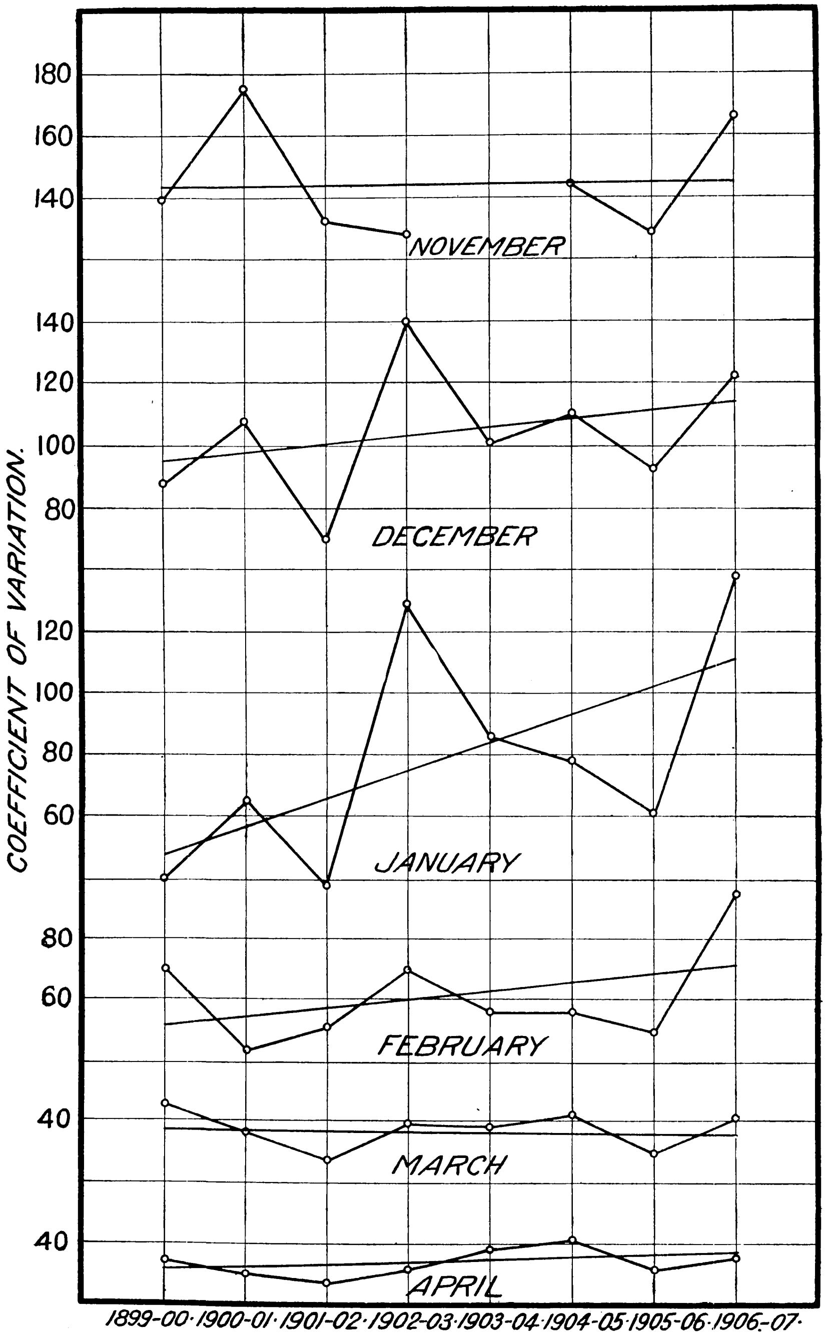

FIG. 11.—Diagram showing the course of the observed relative variability in egg production as measured by the coefficient of variation in each of the months of November to April, inclusive, 1899–1907.

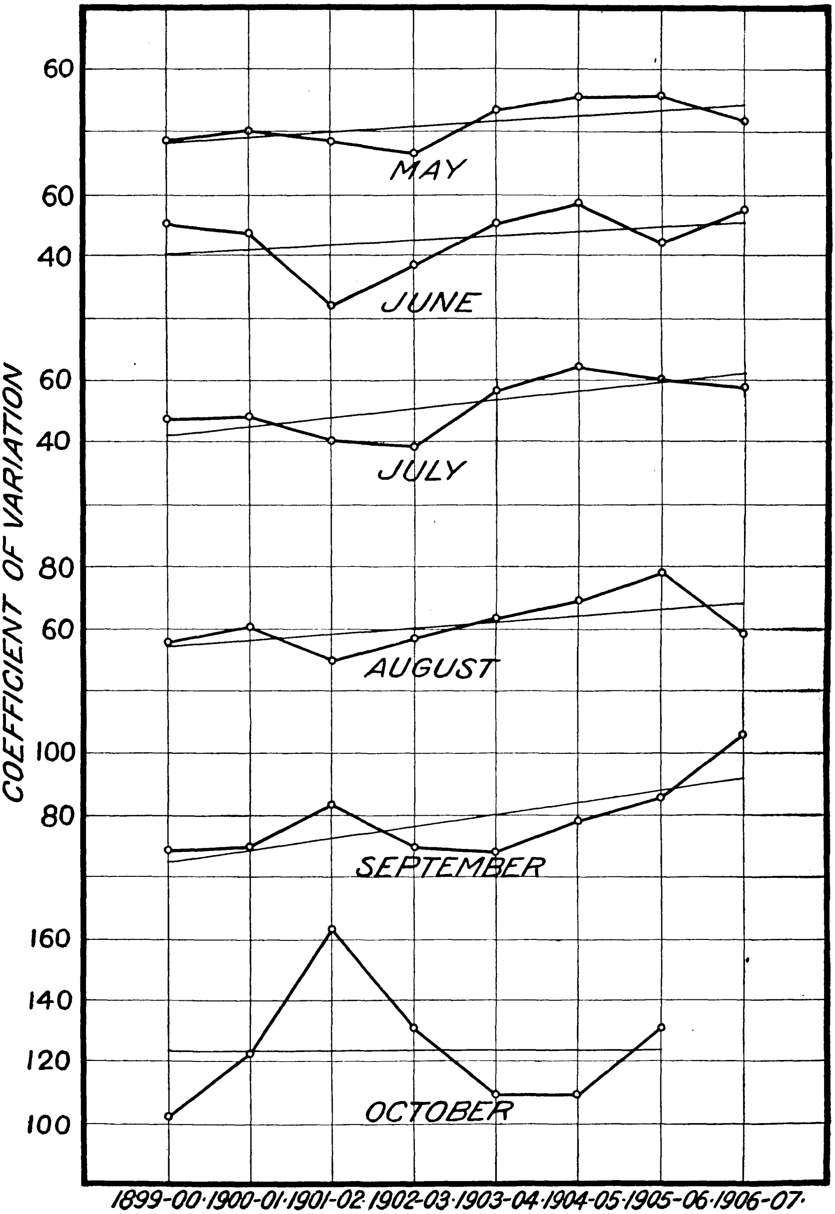

FIG. 12.—Diagram showing the course of the observed relative variability in egg production as measured by the coefficient of variation in each of the months of May to October, inclusive, 1899-1907.

a reduction, but, on the contrary, there was an actual increase in variability.

2. In the single month which forms the exception to the rule just stated, namely, March, the trend of the line of relative variability does not deviate sensibly from the horizontal. The explanation for this result is probably the same as that which was given for the behavior for the March mean production (see p. 102). In the months of March and April, which are the natural breeding season, laying is at its maximum and variability in production is at its minimum, and neither the mean production nor the variability in production is easily affected by selective breeding or by environmental influences. The egg production in these months is a fundamental and basic part of the reproductive cycle in the domestic fowl.

Putting all the results of this section together, it is seen that they are essentially the same as those which were reached in Part I from the study of total annual egg production. It might perhaps have been expected on a priori grounds that mass selection for increased annual production would at least have had a beneficial effect on winter egg production in the flocks. This, however, clearly was not the case. The general result was that the mean production for each month of the laying year either remained sensibly unchanged or showed a tendency to decrease during the period included in this experiment in selective breeding. At the same time the variability in monthly egg production, whether absolute or relative, either did not change at all or increased by a greater or less amount during the period through which the selection was practiced. In other words, it is seen that finer analysis of the total annual egg production returns does not tend in any way to change the conclusions drawn from the study of those returns, but, on the contrary, tends greatly to strengthen those conclusions. There can be no doubt whatever that in this experiment mass selection for increased egg production failed to attain its desired end.

INFLUENCE OF CERTAIN HOUSING CONDITIONS ON MONTHLY EGG PRODUCTION.

In Part I of this study (p. 58) there is described an experiment which was begun in the laying year 1904–5 and has been continued since that time to determine the influence on egg production of size of flock and amount of floor space per bird. Inasmuch as the general plan of this experiment has been fully described in the place cited, it will be unnecessary to repeat these details here. It need only be said that the birds were kept in flocks of 50, 100, and 150 each. The number of square feet of floor space per bird (4.8 square feet) was the same in the case of the 50-bird pens and the 100-bird pens, but in the 150-bird pens the floor space per bird (3.2 square feet) was about 1½ square feet less. From this arrangement of the flocks it is clear that data are furnished on two questions, namely, first, as to the effect on egg production of the number of birds kept together when floor space remains the same, and, second, as to the influence on egg production of the amount of floor space in itself.

In Part I the annual egg production of the flocks kept under these conditions during the years 1904–1907 was discussed. It was there shown that—

> When the laying hens were kept in flocks of 100 birds each, the average annual production per bird was distinctly and significantly lower than when they were kept in flocks of 50 birds each, though the number of square feet of floor space per bird was the same in the two cases and all other environmental conditions were made as nearly as possible identical. Laying birds kept in flocks of 150 birds each and with somewhat less floor space per bird than those kept in flocks of 50 and 100 birds each have in every case an average annual egg production significantly smaller than that of the birds kept in small flocks.

In discussing these results it was pointed out that caution must be shown in drawing practical conclusions from them, since they were based on annual returns only. In this section we shall deal with the effect of these housing conditions on monthly egg production. We may consider first the mean or average monthly production for each month in the period from November, 1904, to January, 1908.

INFLUENCE OF HOUSING CONDITIONS ON AVERAGE MONTHLY PRODUCTION.

The data for discussing the influence of housing conditions on mean production are given in Table 6. This table gives the results of a comparison of 50-bird with 100-bird pens, of 50-bird with 150-bird pens, and of 100-bird with 150-bird pens. The fundamental figures on which this table is based are the mean monthly productions and their probable errors given in Table 1 (pp. 86, 87). The best way in which to make a comparison between these different pens in the different years appears to be to find the differences between the mean monthly production of the flocks kept in the different sized pens for each month. In order that these differences may be expressed always in a uniform manner, the table is arranged in such a way as to show in its three columns, first, the amount by which the mean production of flocks kept in 50-bird pens exceeds that of flocks kept in 100-bird pens for each month of the laying year; second, the amount by which the mean production of flocks kept in 50-bird pens exceeds that of flocks kept in 150-bird pens; and, finally, the amount by which the mean production of flocks kept in 100-bird pens exceeds that of flocks kept in 150-bird pens. Under this arrangement, when the first designated flock of a pair is in excess the sign of the difference is plus, and when the second designated flock of a pair is in excess the sign is minus. To take an example: It appears from the second line of Table 6 that in December, 1904, the birds kept in 150-bird pens had a mean production 3.54 eggs higher than the mean production of the birds kept in 50-bird pens in the same month and year. The probable errors of the differences in Table 6 are calculated in the usual way.

It will be noted that data for only eight months of the laying year 1907–8 are given. The reason for this is that owing to the necessity for a thorough overhauling of the poultry plant in the summer of 1908, trap-nest records had to be stopped on July 1 of that year, and no returns for that year were made after that date.

TABLE 6.—*Differences in mean monthly egg production between flocks of different sizes.*

Year and month.	Difference in egg laying between—		
	50 and 100 bird pens.	50 and 150 bird pens.	100 and 150 bird pens.
1904–5—November	−1.81±.58	−1.00±.45	+0.81±.68
December	−5.04±.60	−3.54±.55	+1.50±.69
January	−1.08±.61	− .69±.56	+ .39±.68
February	+2.20±.50	+4.37±.47	+2.17±.58
March	+5.75±.48	+5.96±.47	+ .21±.59
April	− .17±.48	+2.35±.43	+2.52±.55
May	− .57±.49	+1.83±.49	+2.40±.58
June	− .75±.53	+2.37±.49	+3.12±.62
July	− .77±.56	+1·06±.51	+1.83±.67
August	− .13±.56	+2.52±.47	+2.65±.62
September	+ .20±.63	+3.33±.51	+3.13±.69
October	+1.67±.57	+1.57±.47	− .10±.60
1905–6—November	+ .91±.47	+2.62±.41	+1.71±.39
December	− .05±.64	+ .26±.60	+ .31±.58
January	+ .56±.54	+1.39±.51	+ .83±.51
February	+1.07±.44	+2.82±.40	+1.75±.43
March	+ .96±.39	+1.27±.33	+ .31±.35
April	+1.40±.35	+1.28±.30	− .12±.33
May	+ .68±.40	+ .75±.35	+ .07±.37
June	+ .71±.38	+1.82±.33	+1.11±.38
July	+ .65±.41	+1.40±.37	+ .75±.40
August	+2.32±.39	+2.68±.37	+ .36±.37
September	+2.31±.47	+2.94±.43	+ .63±.43
October	+1.14±.45	+1.51±.41	+ .37±.39
1906–7—November	− .04±.39	−1.03±.38	− .99±.41
December	−1.46±.51	−3.14±.46	−1.68±.50
January	+1.38±.47	+1.03±.43	− .35±.43
February	+5.31±.44	+3.67±.43	−1.64±.41
March	+3.67±.43	+3.07±.40	− .69±.40
April	+ .71±.40	+1.91±.37	+1.20±.37
May	− .80±.43	+1.50±.39	+2.30±.38
June	−1.98±.42	+ .51±.40	+2.49±.40
July	+ .79±.44	+2.24±.40	+1.45±.42
August	+ .03±.43	+2.43±.37	+2.40±.41
September	−3.47±.40	− .15±.33	+3.32±.39
1907–8—November	+ .55±.27	+ .70±.51	+ .15±.49
December	+ .99±.74	+2.34±.63	+1.35±.60
January	+ .32±.74	+2.36±.65	+2.04±.69
February	−4.08±.69	−3.88±.60	+ .20±.66
March	+2.21±.73	+1.80±.62	− .41±.68
April	+2.25±.76	+ .46±.66	−1.79±.67
May	+2.88±.72	+2.51±.63	− .37±.65
June	+4.34±.63	+2.69±.57	−1.65±.59

The first thing which is apparent from an examination of this table is that while one certain size of flock and pen may always show a greater mean annual production than the others, this superiority is not uniformly present in each month of the laying year. On the contrary, it is clearly evident from the table that sometimes one size of flock is in excess in a particular month, sometimes another. This, together with the fact that a large amount of material is included in the table, make it as it stands rather confusing and difficult to draw any conclusions from. In order to get at the essential facts regarding the influence of housing conditions on mean monthly egg production, it is desirable to get the data in diagrammatic form. This has been done with the results shown in figures 13, 14, and 15. These figures show for each month of the laying year the average excess of the first designated flock of the pair over the second in mean egg production. The averages are taken for the four years covered in the case of the months of the laying year before July and for three years in the months after July. In calculating these averages the signs of the separate entries are, of course, observed. In platting the figures, there has been placed on either side of the zero line what may be called a probable error line. This line is at a distance from the zero line equal to the mean of the probable errors contained in the column on which the diagram is based. October is not included in these diagrams, because we have data on this month for but two years.

From these diagrams and the data on which they are based, the following points are to be noted:

1. The flocks kept in 50-bird pens do not, in general, show an excess in mean production over those kept in 100-bird and 150-bird pens in the first three winter months—November, December, and January. In those months the birds in the 100-bird or in the 150-bird pens show average productions either higher or at least sensibly equal to those of the flocks in the 50-bird pens. In other words, the figures obtained from the four years' experiment seem to indicate clearly that winter egg production is not adversely affected by keeping the birds in large flocks, and even by crowding them so that each bird has only a little over 3 square feet of floor space.

2. The greatest excess in mean production of flocks kept in 50-bird pens over those kept in either 100 or 150 bird pens is in the month of March. In this month the smaller flocks are very markedly superior to the larger ones in egg production.

3. Aside from the maximum excess in March of the smaller flocks over the larger, there is a secondary and smaller excess to be observed in the month of August. In this month the 50-bird pens show a decided superiority over the larger flocks. This would appear to indicate that, as would be expected, overcrowding shows its bad effect chiefly in the extremely hot weather of midsummer.

4. In all the months after and including February the excess of the 50-bird pens over the 150-bird pens in mean production is very marked. In the months after March this is probably to be explained as a result of the depressing effect of increasingly warmer weather on birds kept in large and crowded flocks. The reason for the excess in the months of February and March is not apparent.

5. Up until May there is no significant difference in mean monthly egg production between flocks kept in 100 and 150 bird pens. The line fluctuates up and down, but does not extend significantly above the probable error lines. After this month (that is, from May to September, inclusive) the 100-bird pens show marked and increasing superiority in mean monthly egg production over the 150-bird pens. In this we again see the bad effect on egg production of crowding in hot weather.

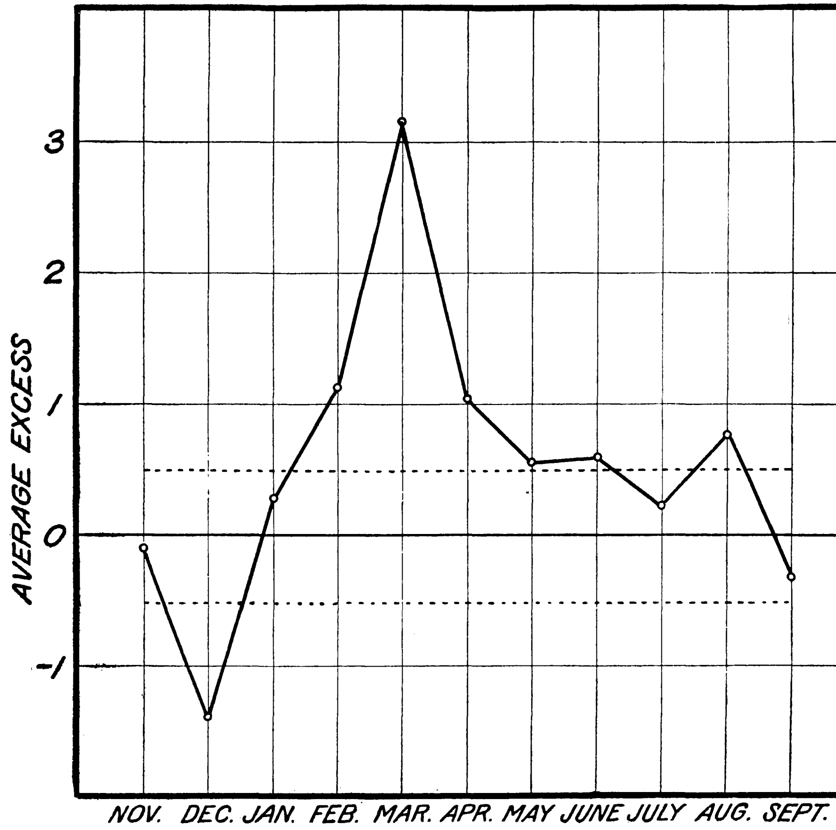

FIG. 13.—Diagram showing the average excess in mean egg production of flocks kept in 50-bird pens over flocks kept in 100-bird pens for the first eleven months of the laying year, November to September.

6. Summarizing, we may say that, so far as winter egg production is concerned, the present statistics show no bad effects of keeping birds in large and crowded flocks (up to the limits included in the present study). On the other hand, overcrowding tends distinctly to lower summer egg production. It is chiefly as a result of this effect upon summer production that the mean annual production is lower in the large flocks, as shown in Part I, pages 58–61.

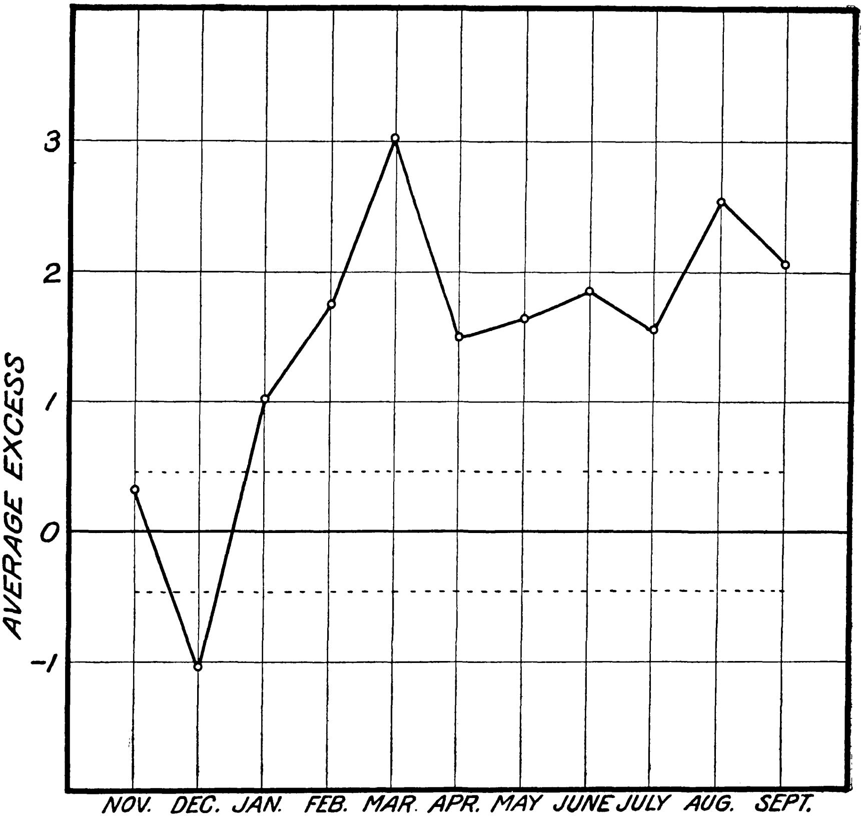

FIG. 14.—Diagram showing the average excess in mean egg production of flocks kept in 50-bird pens over flocks kept in 150-bird pens for the first eleven months of the laying year, November to September.

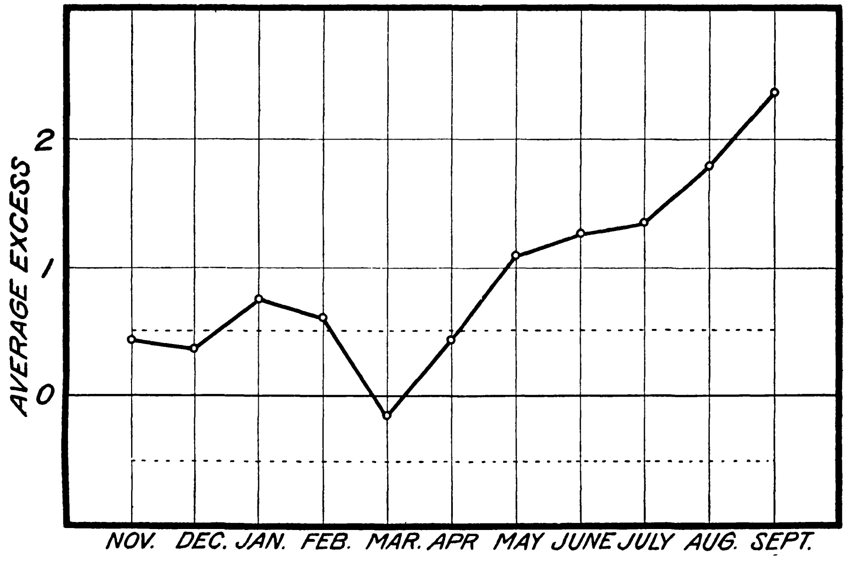

FIG. 15.—Diagram showing the average excess in mean egg production of flocks kept in 100-bird pens over flocks kept in 150-bird pens for the first eleven months of the laying year, November to September.

INFLUENCE OF FLOCK SIZE ON VARIABILITY IN EGG PRODUCTION.

We may turn next to the subject of the effect of flock size upon variability in egg production in the several months of the year. In connection with this question it will be necessary to discuss only the coefficients of variation for reasons already set forth (p. 94, and Pt. I, p. 44). The coefficient of variation gives us a measure of the relative or percentage variability. Such data may be used for purposes of comparison.

As in the case of the mean monthly production (p. 113), the most satisfactory way of dealing with the data is to put them in the form of a difference table for the coefficients of variation. This is done in Table 7, which is constructed in exactly the same way as was Table 6. The first column gives the difference in variability between 50 and 100 bird pens, the second column between 50 and 150 bird pens, and the last column between 100 and 150 bird pens. A difference is noted as plus when the first flock size of a designated pair is the more variable and as minus when the second flock size is the more variable.

TABLE 7.—*Differences in relative variability in egg production, between flocks of different sizes, as measured by the coefficient of variation.*

Year and month.	Difference between—		
	50 and 100 bird pens.	50 and 150 bird pens.	100 and 150 bird pens.
1904-5—November	+40.35±15.54	+32.71±15.59	−7.64±15.07
December	+65.58±9.53	+46.55±10.01	−19.03±7.46
January	+18.86±5.73	+10.22±5.65	−8.64±6.13
February	−5.94±3.76	−20.29±5.52	−26.23±4.67
March	−16.21±3.09	−27.17±3.29	−10.96±4.28
April	+.34±2.49	−9.88±2.67	−10.22±3.20
May	+9.59±2.86	−10.87±3.64	−20.46±3.94
June	+2.87±3.45	−21.24±4.42	−24.11±4.99
July	−1.09±4.60	−19.44±5.31	−18.35±6.28
August	−.53±4.96	−19.06±5.69	−18.53±6.71
September	−3.24±6.04	−24.83±7.08	−21.59±8.39
October	−20.15±14.76	−8.49±9.90	+11.66±16.19
1905-6—November	−11.66±11.00	−40.37±12.26	−28.71±13.04
December	+2.85±9.86	−4.04±9.65	−6.89±6.94
January	−3.71±3.57	−13.90±3.62	−10.19±3.77
February	−9.70±2.63	−19.36±2.70	−9.66±3.02
March	−5.00±1.69	−.59±3.84	+4.41±1.57
April	−6.21±1.71	−4.02±1.47	+2.19±1.68
May	−7.43±3.10	−6.68±2.74	+.75±3.04
June	−10.56±2.43	−12.40±2.21	−1.84±2.58
July	−13.22±3.71	−17.50±3.47	−4.28±4.03
August	−19.70±5.03	−29.14±4.93	−9.44±5.86
September	−22.31±6.08	−29.70±5.77	−7.39±6.95
October	−19.32±12.59	−26.82±11.79	−7.50±13.41
1906-7—November	−28.19±22.27	−6.10±17.46	+22.09±21.34
December	+15.80±14.73	+41.93±12.50	+26.13±10.63
January	−33.49±14.99	−25.41±12.31	+8.08±15.69
February	−4.89±8.49	−38.55±5.79	+16.34±9.41
March	−10.26±2.25	−10.97±2.00	−.71±2.37
April	−.91±1.86	−5.57±1.82	−4.71±1.85
May	+3.90±2.37	−2.84±2.36	−6.74±2.23
June	+8.65±3.06	−6.37±3.30	−15.02±2.97
July	−6.40±3.30	−16.19±3.34	−9.79±3.60
August	−10.16±3.56	−21.17±3.64	−11.01±3.66
September	+26.65±7.48	−12.89±9.11	−39.54±8.00
1907-8—November	−51.47±56.41	−112.80±65.85	−61.33±74.80
December	−23.64±29.39	−58.49±32.12	−34.85±35.28
January	−16.30±14.45	−52.31±17.28	−36.01±18.64
February	+48.09±14.79	+51.29±14.24	+3.20±7.82
March	−12.65±3.89	−4.18±3.03	+8.47±3.79
April	−7.93±3.92	+2.15±3.07	+10.08±3.55
May	−11.67±4.08	−6.19±3.32	+5.48±4.00
June	−19.98±4.77	−11.76±3.60	+8.22±4.91

Just as in the case of the means, the mass of data presented in the table is so great as to be confusing. It is difficult to draw any conclusions from the detailed constants. As an aid in the interpretation of the table we may again resort to the use of graphical methods. Figures 16, 17, and 18 are diagrams showing the average excess in relative variability as measured by the coefficient of variation of, respectively, 50-bird pens over 100-bird pens, 50-bird pens over 150-bird pens, and 100-bird pens over 150-bird pens. These diagrams are constructed on exactly the same plan as figures 13, 14, and 15. If in the description of the plan of those diagrams on page 115 the words "coefficient of variation" be substituted for "mean" whenever it occurs, that account will describe the diagrams here, and need not be repeated.

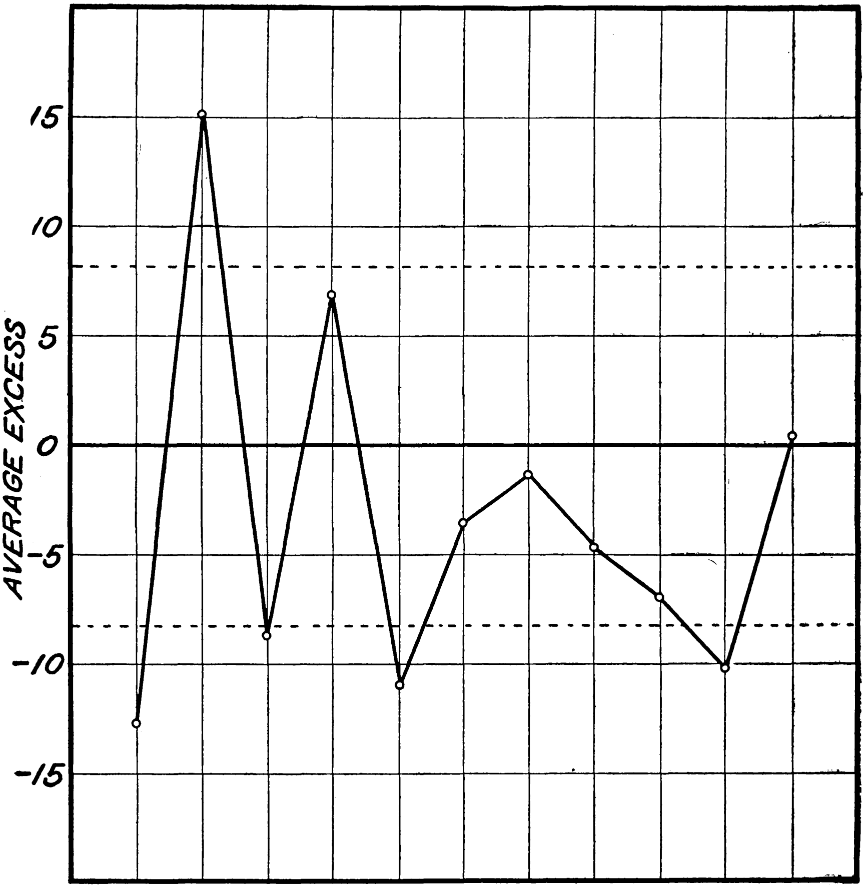

FIG. 16.—Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 50 birds over flocks of 100 birds for the first eleven months of the laying year, November to September.

From these diagrams and the data on which they are based we note the following points:

1. The excess in relative variability of egg production of the larger flocks (100 and 150 birds) over the smaller (50 and 100 birds) is on the whole fairly evenly distributed over the whole year. In only one case (December, 50 and 100 bird pens) do we find any considerable + excess, denoting for that month a greater variability of the smaller

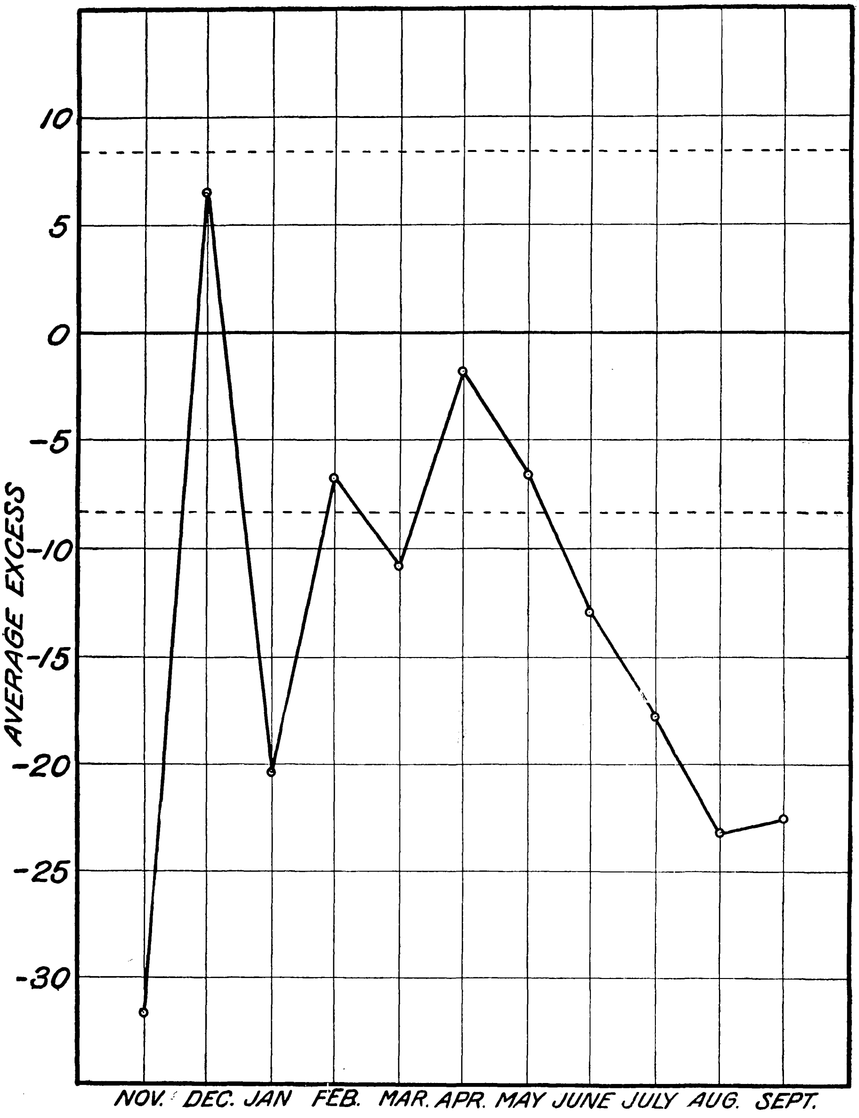

FIG. 17.—Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 50 birds over flocks of 150 birds for the first eleven months of the laying year, November to September.

flock. In this case, further, the average result very largely depends upon the very great difference observed in the single year 1904–5, which in turn results from the very high relative and absolute variability shown by the 50-bird flocks in their December production of that year.

2. In the period comprised within the months February to July, inclusive, there is no significant difference in relative variability

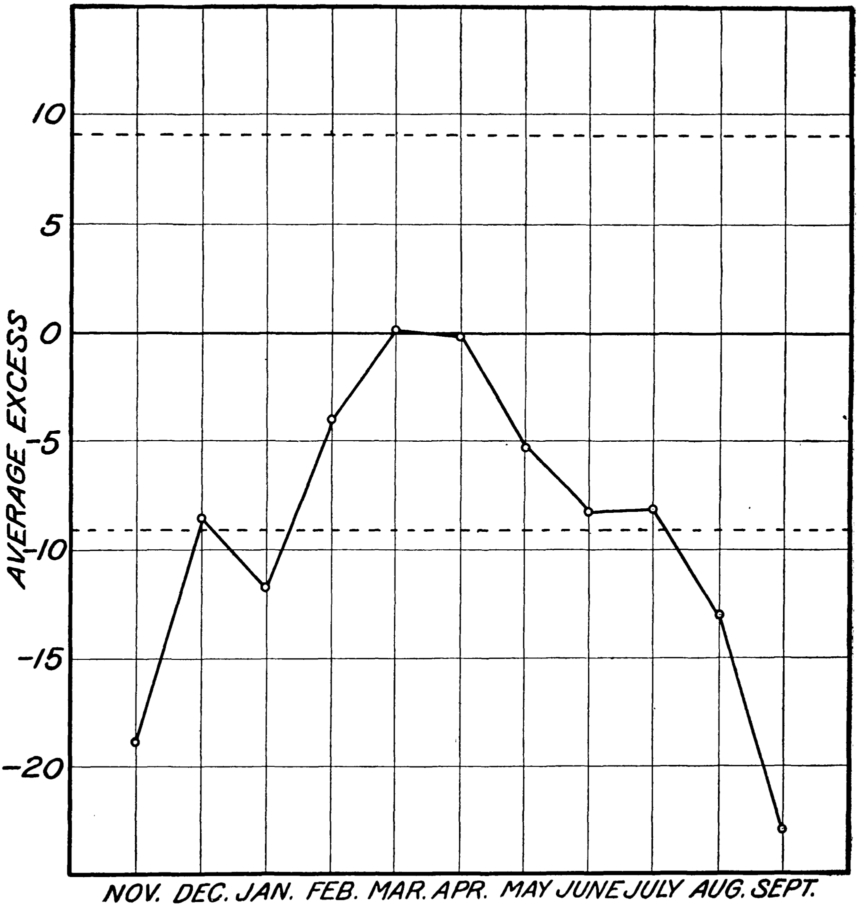

FIG. 18.—Diagram showing the average excess in relative variability, as measured by the coefficient of variation, of flocks of 100 birds over flocks of 150 birds, for the first eleven months of the laying year, November to September.

between any of the flocks. In other words, it appears that during that period of the year in which there is the heaviest production such environmental differences as are implied in the different flock sizes in this experiment do not appreciably affect the relative variability of production.

3. The greatest effect of flock-size and floor-space factors on relative variability of production is to be seen in the months of

November and September—at the beginning and the end of the laying year.

In the discussion of the influence of floor space and flock size on mean annual production in Part I (p. 63) it was pointed out that caution must be exercised in drawing practical conclusions from the annual data. The reason for such caution is apparent from the facts set forth in the present section. It has been shown that while it is true that the average annual production of large and relatively crowded flocks is not so great as that of smaller flocks with or without more floor space per bird, yet the difference is, for practical purposes, almost entirely in the economically unimportant summer egg production. Winter egg production is, speaking generally, in no wise adversely affected by keeping the birds in 100 and 150 bird flocks and allowing as little as 3.2 square feet of floor space per bird. The most probable reason for such a result as this would appear to lie in temperature relations. The large, crowded flocks presumably get a greater benefit from their animal heat in the cold winter months, particularly at night on the roosts. This means a smaller strain on the metabolic mechanism solely for the purpose of maintaining body temperature. In the hot summer months the same factors operate in the opposite way. That hens suffer greatly in very hot weather everyone who has observed them knows. That a crowded condition, particularly on the roosts, is likely to increase the discomfort from heat seems very probable.

All of these considerations have led the writers to the tentative opinion that the number of cubic feet of space in a house is a much more important and significant factor in the poultry-housing problem than is the number of square feet of floor space. And particularly the number of cubic feet per bird in the so-called "roosting closet" of houses of the curtain-front type would seem to be a significant factor, the possible influence of which on egg production and the general well-being of birds ought to be subjected to more thorough and careful experimental study than has ever been given to it so far as the writers are aware. It seems possible that along this line may be found the explanation of the good results obtained by Robinson[a] with his "low type" of house. Theoretically it would seem probable that there is some general relation—not absolutely fixed, but varying within narrow limits—between square feet of floor space per bird, cubic feet of house space per bird, and the well-being and productiveness of hens. The practical problem is to find out what this relation is.

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[a] Robinson, J. H. The Height of the Poultry House. Farm Poultry. Vol. XIX, p. 342, 1908.

THE FREQUENCY DISTRIBUTION OF VARIATION IN MONTHLY EGG PRODUCTION.

In Part I (pp. 30–38) of this study the frequency distributions for variation in annual production were discussed analytically, and it was shown that in general variation in egg production followed the same laws as hold for fluctuating variation in other characters. But these annual production curves present only the gross facts of variation in egg production. They present rather than solve a problem. It is necessary to analyze these annual curves into their components in order to arrive at an idea of the factors involved in their genesis. That is to say, while we know that all such curves showing the distribution of fluctuating variations arise through the operation of an indefinite number of small "chance" or "statistical" causes, there is involved in every frequency curve a specific and particular biological problem. The problem is this: What are the immediate biological factors which result in the genesis of the particular form of frequency curve shown by the given material rather than some other form? It was shown in Part I of this study that the curves for variation in annual production had certain definite characters peculiar to themselves. Thus there appeared to be a general tendency for such curves to have an unduly high frequency at the low-production end of the curve. Their skewness was always in the same direction. Now, how does it come about (i. e., what are the proximate causes) that these egg-production curves show these particular characteristics rather than some others? This is the problem presented by gross frequency curves. The first step toward answering it must be made by further analyzing these curves. Final analysis in terms of individuals must always be the goal aimed at.

It is evident that a study of the distribution of variation in monthly egg production may well serve as a beginning of the analysis of annual egg-production curves. It will be understood, of course, that this is in no sense an analysis in terms of individual variants, but rather in terms of a smaller time unit. It seems wise to begin this study of the monthly distributions of variation in egg production with a general graphical appreciation of their characteristics. Following this the detailed mathematical treatment will be undertaken. For the graphical treatment all available data have been used. The monthly frequency distributions so formed are given in Table 8. The absolute frequencies in this table are the totals of the fundamental Tables I to XII of Appendix II. From these absolute frequencies the percentage frequencies used in the diagrams were calculated. The manner in which the table is to be read may be illustrated with an example: The first entry in the table means that of the 2,153 hens for which we have November production records 1,189, or 55.22 per cent, laid no eggs in that month.

TABLE 8.—*Absolute and percentage frequency distributions for the egg production in each month of the laying year, based on the records of the years 1899–1900 to 1906–7, inclusive.*

Eggs laid.	November.		December.		January.		February.	
	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.
0	1,189	55.22	906	37.68	654	27.17	322	13.38
1	137	6.36	123	5.11	107	4.45	86	3.51
2	47	2.18	67	2.78	71	2.95	68	2.83
3	37	1.72	51	2.12	65	2.70	65	2.70
4	26	1.21	63	2.62	59	2.45	68	2.83
5	36	1.67	57	2.37	48	1.99	84	3.49
6	37	1.72	49	2.04	64	2.66	69	2.87
7	32	1.49	53	2.20	49	2.04	90	3.74
8	32	1.49	59	2.45	55	2.29	78	3.24
9	41	1.90	58	2.41	60	2.49	66	2.74
10	39	1.81	67	2.78	70	2.91	93	3.86
11	49	2.28	52	2.16	52	2.16	73	3.03
12	36	1.67	54	2.24	62	2.58	94	3.91
13	61	2.83	55	2.29	72	2.99	75	3.12
14	47	2.18	59	2.45	72	2.99	116	4.82
15	77	3.58	45	1.87	85	3.53	123	5.11
16	36	1.67	72	2.99	89	3.70	158	6.56
17	43	2.00	60	2.49	91	3.78	135	5.61
18	39	1.81	87	3.61	94	3.91	162	6.73
19	41	1.90	80	3.32	97	4.03	126	5.23
20	23	1.07	71	2.95	94	3.91	119	4.94
21	24	1.11	56	2.33	108	4.49	72	2.99
22	13	.60	56	2.33	90	3.74	41	1.70
23	5	.23	49	2.04	49	2.04	14	.58
24	6	.28	35	1.45	29	1.20	8	.33
25	....	....	15	.62	13	.54	1	.04
26	....	....	4	.17	5	.21	1	.04
27	....	....	3	.12	3	.12	....	....
28	....	....	1	.04	....	....	....	....
29	....	....	....	....	....	....	....	....
30	....	....	....	....	....	....	....	....
31	....	....	....	....	....	....	....	....
Total	2,153	100.00	2,407	100.03	2,407	100.02	2,407	99.99

Eggs laid.	March.		April.		May.		June.	
	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.
0	68	2.83	55	2.29	118	4.90	193	8.02
1	24	1.00	29	1.20	50	2.08	44	1.83
2	13	.54	16	.66	33	1.37	30	1.25
3	19	.79	19	.79	23	.96	27	1.12
4	26	1.08	18	.75	42	1.74	34	1.41
5	29	1.20	23	.96	47	1.95	31	1.29
6	25	1.04	25	1.04	55	2.29	45	1.87
7	44	1.83	33	1.37	58	2.41	54	2.24
8	64	2.66	28	1.16	100	4.15	100	4.15
9	69	2.87	58	2.41	91	3.78	108	4.49
10	63	2.62	76	3.16	126	5.23	129	5.36
11	94	3.91	79	3.28	170	7.06	172	7.15
12	109	4.53	119	4.94	149	6.19	181	7.52
13	98	4.07	125	5.19	158	6.56	151	6.27
14	101	4.20	150	6.23	156	6.48	161	6.69
15	103	4.28	168	6.98	150	6.23	145	6.02
16	132	5.48	162	6.73	121	5.03	151	6.27
17	150	6.23	186	7.73	130	5.40	116	4.82
18	143	5.94	185	7.69	111	4.61	97	4.03
19	178	7.40	171	7.10	108	4.49	97	4.03
20	171	7.10	184	7.64	102	4.24	107	4.45
21	158	6.56	164	6.81	93	3.86	90	3.74
22	178	7.40	124	5.15	78	3.24	60	2.49
23	147	6.11	95	3.95	60	2.49	44	1.83
24	111	4.61	63	2.62	33	1.37	16	.66
25	54	2.24	29	1.20	25	1.04	11	.46
26	22	.91	15	.62	11	.46	8	.33
27	8	.33	5	.21	5	.21	2	.08
28	5	.21	3	.12	2	.08	3	.12
29	1	.04	....	....	1	.04	....	....
30	0	....	....	....	0	....	....	....
31	0	....	....	....	1	.04	....	....
Total	2,407	100.01	2,407	99.98	2,407	99.98	2,407	99.99

TABLE 8.—*Absolute and percentage frequency distributions for the egg production in each month of laying year, based on records of years 1899–1900 to 1906–7, inclusive*—Cont'd.

Eggs laid.	July.		August.		September.		October.	
	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.	Absolute frequency.	Percentage frequency.
0	297	12.34	345	14.33	554	23.02	773	44.07
1	48	1.99	109	4.53	147	6.11	114	6.50
2	35	1.45	53	2.20	86	3.57	52	2.96
3	27	1.12	55	2.29	64	2.66	38	2.17
4	42	1.74	53	2.20	69	2.87	41	2.34
5	55	2.29	68	2.83	89	3.70	44	2.51
6	65	2.70	77	3.20	91	3.78	38	2.17
7	86	3.57	82	3.41	95	3.95	41	2.34
8	110	4.57	103	4.28	100	4.15	45	2.57
9	125	5.19	105	4.36	89	3.70	56	3.19
10	157	6.52	151	6.27	93	3.86	40	2.28
11	168	6.98	169	7.02	100	4.15	61	3.48
12	164	6.81	151	6.27	99	4.11	55	3.13
13	167	6.94	142	5.90	105	4.36	52	2.96
14	144	5.98	130	5.40	86	3.57	54	3.08
15	119	4.94	123	5.11	79	3.28	57	3.24
16	109	4.53	94	3.91	71	2.95	56	3.19
17	95	3.95	84	3.49	92	3.82	43	2.45
18	81	3.36	75	3.12	61	2.53	38	2.17
19	79	3.28	67	2.78	67	2.78	19	1.08
20	60	2.49	53	2.20	51	2.12	15	.86
21	59	2.45	40	1.66	55	2.29	9	.51
22	36	1.50	21	.87	34	1.41	3	.17
23	35	1.45	35	1.45	17	.71	4	.22
24	20	.83	11	.46	6	.25	4	.22
25	11	.46	7	.29	4	.17	1	.06
26	6	.25	3	.12	3	.12	0	....
27	4	.17	0	....	....	....	1	.06
28	1	.04	1	.04	....	....	....	....
29	0	....	....	....	....	....	....	....
30	0	....	....	....	....	....	....	....
31	2	.08	....	....	....	....	....	....
Total	2,407	99.97	2,407	99.99	2,407	99.99	1,754	99.98

The data of the foregoing table are given graphically in figures 19 to 24, inclusive. These figures are the graphs of the percentage distributions, the percentage rather than the absolute figures being used in order to make all the diagrams directly comparable one with another. The area of each is the same. To economize space the distributions for two successive months are put together in the same diagram. In all these diagrams the ordinates denote the percentage frequency of birds each laying the number of eggs designated by the abscissa.

The first point which is at once noticed on examining these frequency polygons is that we are clearly not dealing here with simple frequency distributions such as are usually found in variation studies in which the variants are distributed about a single center or mode.[a] The nearest approach among these polygons to this ordinary unimodal type of frequency curve is found in the distributions for March and April, and in particular that for April. But even in this case

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[a] The "mode" of a frequency curve is that particular value of the varying character (here "eggs produced per month") which is present in the largest number of the varying individuals making up the population. A modal egg production of 15 eggs for November would mean that more birds produced just 15 eggs in that month than produced any other one particular number of eggs, either greater or less than 15. For a further discussion of the significance of this constant see Part I, page 27. A curve which shows a single "peak" or maximum ordinate is said to be unimodal; one which has two, to be bimodal, and one which has more than two to be multimodal.

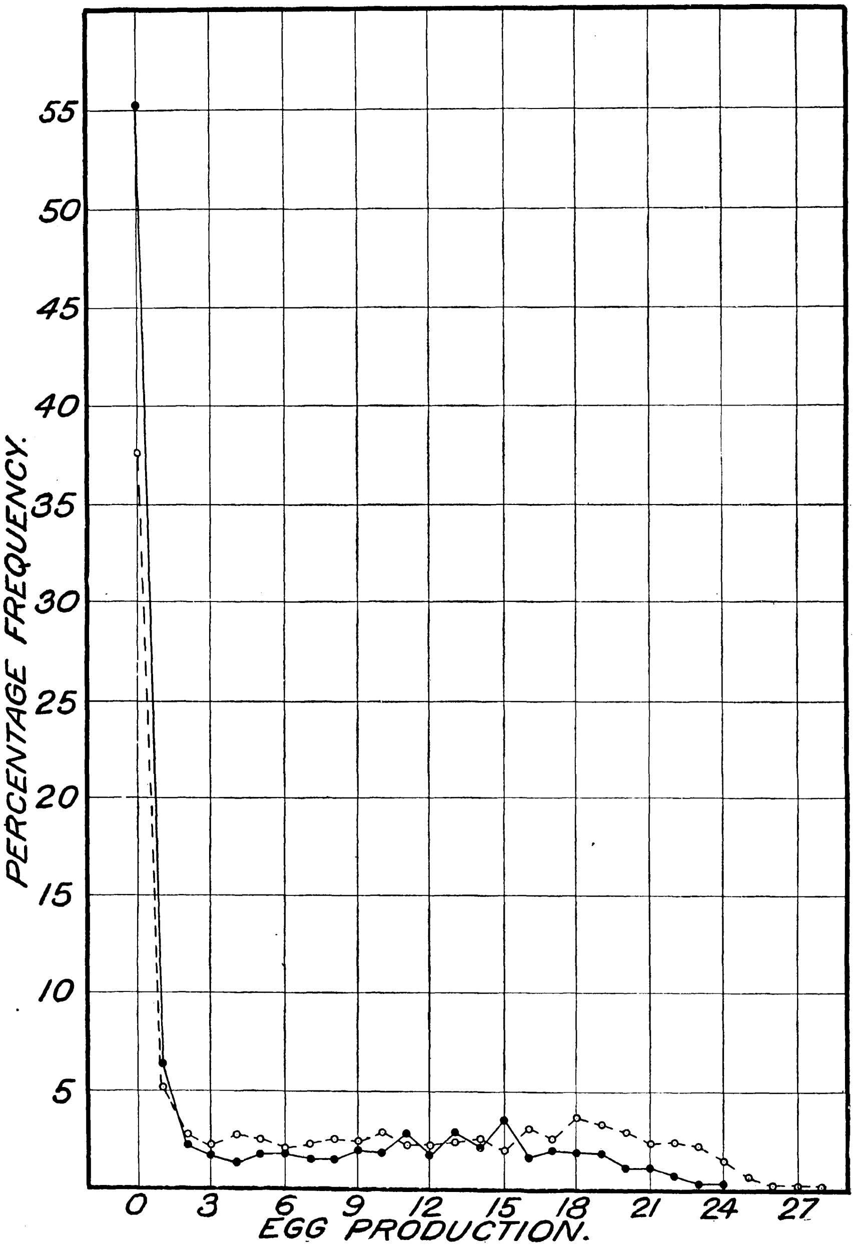

FIG. 19.—Frequency polygon of variation in November and December egg production.

the distribution clearly differs from the usual frequency curve. All of these polygons are distinctly bimodal in some degree. Each one shows one distinct mode at zero egg production. The fact that every single one of the monthly frequency distributions clearly and definitely shows this bimodal condition indicates that we are dealing with a real phenomenon of variation. In fact, as will presently appear, we apparently have to do in these cases with a definite and hitherto unrecognized type of frequency distribution. Simple bimodal distributions have, of course, been known since the beginning of the statistical study of variation. Such distributions, however, have practically only been noted hitherto for cases of dimorphism, where one deals with so-called "high" and "low" forms of the organism. Such distributions have, for example, among animal forms, been described by Bateson[a] for *Forficula* and for the rhinoceros beetle (*Xylotrupes gideon*), and by Graber[b] for *Blissus* and others.

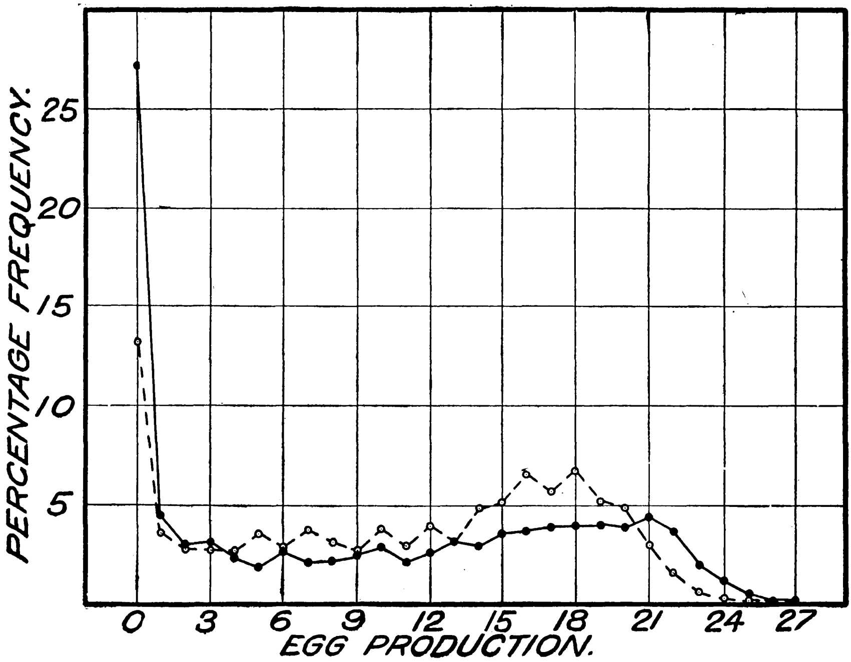

FIG. 20.—Frequency polygon of variation in January and February egg production.

Apart from these cases of dimorphism, variation leading to frequency distributions of anything other than the common unimodal type occurs very rarely. Even with all the work which has been done in recent years in the biometrical study of fluctuating variation there have been extremely few instances brought to light which do not conform to the unimodal type of distribution. Besides dimorphism already noted, practically the only instances of multimodal distributions which have been brought forward are those for variation in the number of ray flowers in certain of the Compositæ.[a] In these cases there is some evidence of a tendency toward the occurrence of definite peaks or modes on the Fibonacci numbers. It is the fact, however, that the statistical significance of these observational modes has never yet been adequately settled.

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[a] Bateson, W. Materials for the Study of Variation. London, 1894, pp. 36–42.

[b] Graber, J. F. Dimorphism in *Blissus leucopterus*. Biological Bulletin, vol. 5, No. 6, pp. 330–335. 1903.

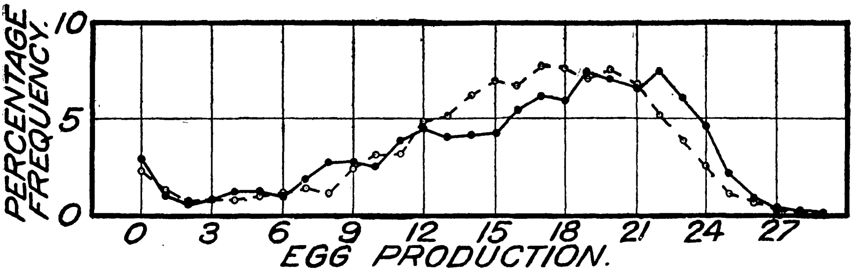

FIG. 21.—Frequency polygon of variation in March and April egg production.

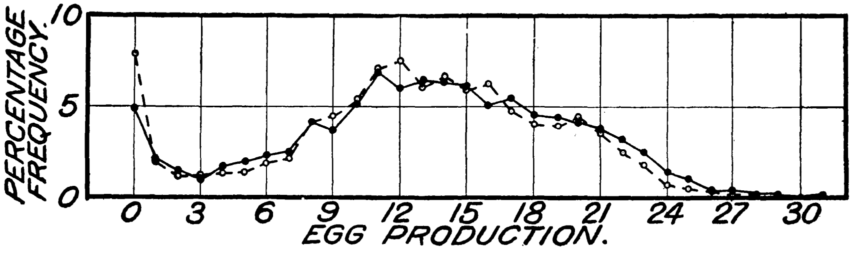

FIG. 22.—Frequency polygon of variation in May and June egg production.

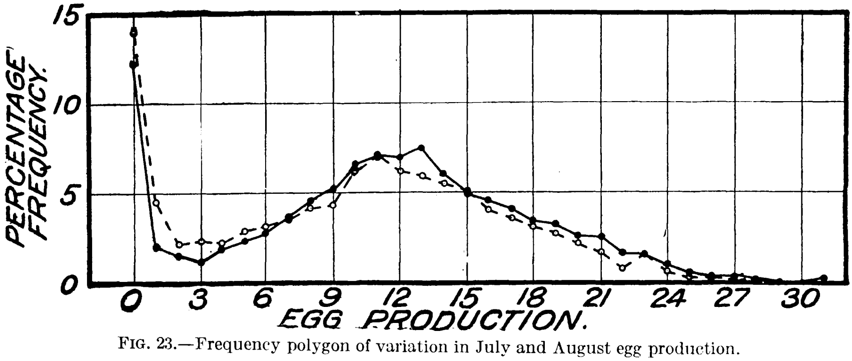

FIG. 23.—Frequency polygon of variation in July and August egg production.

In view of the rarity of types of frequency distribution differing from the common unimodal one, it is of importance to examine with considerable care the nature of these monthly fecundity distributions. To facilitate the discussion of this matter it is desirable to make some preliminary definition of terms necessary to be used in the description of bimodal curves. In the following discussion the terms "upper component" and "lower component" will be used to designate, respectively, the portion of the variation polygons (or, in other words, the groups of variants) centering about the two modes of higher and lower monthly productivity. These terms are here used only in a very general sense and without any attempt to limit precisely the range or the area of either of the components. The two modes about which the components center will be known as the "upper" (i. e., of higher production) and the "lower" (i. e., of lower production) modes, respectively. In discussing bimodal curves it is

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[a] For a list of the literature on this subject see the bibliography in Davenport's Statistical Methods (New York, 1904), and the bibliographical lists appearing currently in Biometrika.

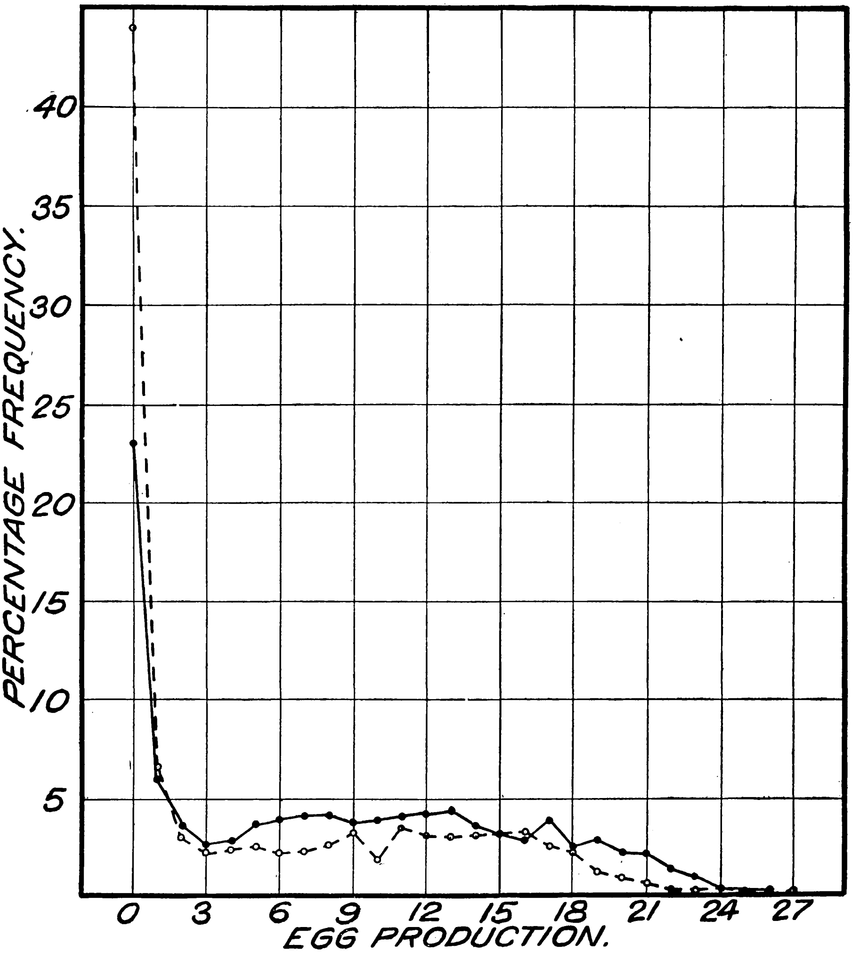

FIG. 24.—Frequency polygon of variation in September and October egg production.

very desirable to have some simple brief term to designate the point on the curve between the two modes where the frequency is at a minimum. It is in this region that the two components of such curves overlap. After careful consideration we have decided to use the term "antimode" to designate the point in any bimodal frequency distribution where the frequency over a unit base element is at a minimum.

POSITION OF MODES AND ANTIMODE IN MONTHLY DISTRIBUTIONS.

Proceeding to the discussion of the monthly distributions, it is to be noted in the first place that the position of the upper mode changes in a definite manner as we pass from the beginning to the end of the laying year. The nature of this change as well as certain other significant facts will be brought out if the position of the observational modes and of the antimode be tabled. These data are given in Table 9. It should be kept in mind that in this table we are only dealing with the highest observed peaks of the distributions, and not with the true modes, the positions of which can only be determined by detailed mathematical treatment of the data. Observational peaks only serve to locate the position of true modes roughly.[a] These observed values will, however, serve for the purposes of the present discussion. In this table the observed mode which has the higher modal ordinate in each of the curves is printed in bold-faced type.

TABLE 9.—*The position (abscissa) of the observational modes and the antimode in the frequency distributions of monthly production.*

[The mode with the higher ordinate is in black-faced type.]

Month.	Position of lower observed mode.	Position of upper observed mode.	Position of antimode.
November	**0**	15	4
December	**0**	18	[b] 15
January	**0**	21	5
February	**0**	18	3
March	0	[c] **20.5**	2
April	0	**17**	2
May	0	**11**	3
June	0	**12**	3
July	**0**	11	3
August	**0**	11	[d] 3
September	**0**	13	3
October	**0**	11	3

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[a] Cf. Pearson, K. Biometrika, Vol. I, pp. 260, 261. 1902.

[b] If this curve were to be smoothed by any accurate process of graduation the antimode would fall at a point considerably below the observed actually smallest ordinate, probably at about 5.

[c] In this distribution there are two observational peaks of equal height in the modal region, one at 19 and one at 22. Rather than take either of these points as the mode it seems fairer from inspection of the polygon to locate the observational mode halfway between the two, viz, at 20.5.

[d] While the ordinates at 2 and 4 in this distribution are slightly smaller than that at 3, inspection shows that in a fitted curve to these observations (say if a parabola were struck through the ordinate at 3 and the four ordinates on either side of 3) the minimum ordinate would fall at about 3.

From this table it is apparent that the upper mode moves steadily upward (i. e., toward a greater egg production) during the first four months of the laying year. It reaches its actually highest point in the March distribution, although if we had available accurately calculated modes it is probable that there would be little difference in the values for March and April. After attaining this maximum point the upper mode moves again toward the zero end of the curve in a rather steady decline. The lower mode falls in every case at the zero abscissa; or, in other words, the "fashionable" production for this component is zero.

Associated with this change in the position of the upper of the two modes is a change in the direction of the skewness, or asymmetry, of the upper component of the distributions. In the distributions for November and December the total area comprised in the upper component is so small that it is difficult without the use of analytical methods to determine what the sign of the skewness is; but, beginning with the January distribution, the direction of the skewness is evident on inspection. The upper component in the January, February, and March distributions shows a definite and considerable negative skewness. This component of the April distribution is probably still negatively skew, though the degree of asymmetry is here very slight. The May distribution shows a marked difference. Here the skewness of the upper component is clearly positive. The mode of this component has now come to lie below (i. e., to the left in the diagram) the mean. Distinct positive skewness is further exhibited by the upper components of the June, July, and August distributions. It is probable that the September and October distributions also have a positive skewness of this component. Here, however, just as in the case of the November and December polygons, the total area comprised in the upper component is so small as to make it impossible to determine this point from mere inspection. Altogether we have here a very interesting case of a variation curve passing gradually and by successive stages from a condition of distinct negative skewness through a symmetrical condition to distinct positive skewness.

RELATIVE HEIGHT OF MODAL ORDINATES.

Another point to be considered is the relative height of the modal ordinates of the two modes in the several distributions. In the earlier months of the year the ordinate of the lower mode is many times higher than that of the upper one. The height of this lower modal ordinate steadily and rapidly decreases month by month until it reaches a minimum in the April distribution. In the months after April it increases again, until it reaches a maximum in the October distribution. In the case of the upper mode the height of the modal ordinate may be said, in general, to increase as the value of the mode increases; that is to say, as the mode of the upper component is located at a higher and higher production up to the maximum in March and April the height of the modal ordinate (i. e., the percentage of the flock laying the modal number of eggs) of this component also becomes greater. The highest observed modal ordinate occurs in the April distribution. After that point the height of the modal ordinate of this component steadily becomes less, until it reaches a minimum in the October distribution. In no one of the distributions, however, does the modal ordinate of the upper component even approximate in magnitude the values attained by that of the lower mode at the ends of the laying year. Or, in other words, the percentage of the flock which lays the most common or usual number of eggs in the natural laying season is always very much smaller than the percentage which lays no eggs at all in the seasons of poor production.

The effect of the change in position of the mode of the upper component and of the changes in the magnitude of the two modal ordinates is well brought out in the following table. The data show that—

One-half (50 per cent) of the flock lays in the month of—	November fewer than 1 egg apiece.
	December fewer than 2 eggs apiece.
	January fewer than 7 eggs apiece.
	February fewer than 10 eggs apiece.
	March fewer than 16 eggs apiece.
	April fewer than 16 eggs apiece.
	May fewer than 13 eggs apiece.
	June fewer than 12 eggs apiece.
	July fewer than 11 eggs apiece.
	August fewer than 10 eggs apiece.
	September fewer than 6 eggs apiece.
	October fewer than 6 eggs apiece.

In only three months of the year do one-half of the birds, according to these statistics, lay more than a dozen eggs apiece in the month.

The antimode in these curves appears to maintain a nearly constant position throughout the year. It is significant that this point of minimum frequency is not at an egg production of 1, but is definitely and clearly at a higher production. This indicates that we are dealing in these distributions with two definite component curves, and not merely with a unimodal distribution with an abnormal frequency on the single base element 0. The frequency over the base elements 1, 2, and probably also 3, in nearly every month, clearly belongs in the main to the lower component as well as that over the base element 0. There is a definite downward slope or "tailing off" of this lower component before the upper component begins. There is some indication that the antimode has a real and definite tendency to lie at a higher point in the earlier months of the laying year (November and December) than in the later ones. This can not be accurately determined, however, without graduation of the raw data.

It appears to be the case that all of these monthly distributions belong to the same general type of frequency curve. This type can be characterized in the following way: The curve is in general S-shaped, as shown in figure 25, presenting two distinct points of maximum frequency or modes.

One limb of the S-shaped curve tends to become asymptotic to the y axis. The curve appears to be made up of two distinct components, one of which (the upper) is of the usual "curve of error" type, while the other (the lower) is of the J-shape taken by Pearson's Type III and Type I curves under certain conditions.

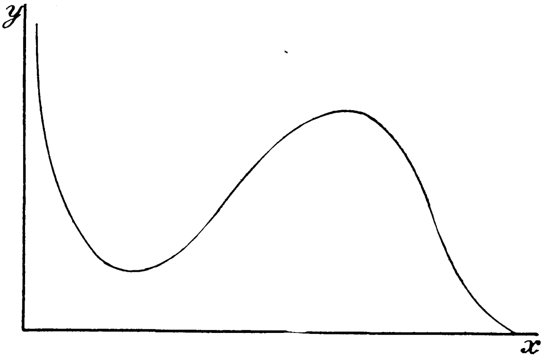

FIG. 25.—Diagram showing the general form or type of curve to which the frequency distributions of variation in monthly production apparently belong.

ANALYSIS OF THE MONTHLY EGG-PRODUCTION DISTRIBUTIONS.

The facts brought out in the last section present a series of problems of much interest. So far as we are aware, S-shaped frequency distributions have not hitherto come to light in any studies of variation in homogeneous material. The question is immediately raised as to what are the biological factors which lead to the development of these curious and unique distributions. It is believed that the results herein to be set forth will throw considerable light on this problem. Since these results are reached by a process of reasoning which has not commonly been employed in biological studies, though it is very often used in the investigation of physical problems by a combination of mathematical and experimental methods, the greatest pains will be taken to set forth as clearly and precisely as possible the steps taken. It is hoped that the present case may serve in some measure to indicate to biological readers the possible utility of biometric methods in a direction not generally recognized.

At the outstart two sets of facts regarding these monthly fecundity distributions need to be recognized. These are:

1. The material in which these bimodal, S-shaped distributions appear is as homogeneous in respect to egg production as is possible in mass selected material. This positive assertion of biometrical

homogeneity for this material can be made because the data come from controlled conditions. The birds were "pure bred" and "line bred." That is, no "strange germ plasm" had been introduced into the race for many generations. Close selection within the race had been practiced in respect to egg production for a long period. The birds made their records in the same place and under essentially the same conditions within the year as far as it was possible to control them. In other words, the peculiarities in the monthly distributions are in no way to be explained as the result of different methods of handling the birds in different months of the year. That egg production is probably inherited in "pure lines" and that this material includes a number of such "pure lines" need not concern us here, since the problem now under discussion is one of the physiology of egg production and not at all one of its inheritance.

An objection may be made to the fact that in the discussion of these monthly distributions the records of eight years have been put together. In regard to this it should be said that this has been done only after the most careful study of the detailed single year data given in Appendix II (Tables I–XII). If anyone will take the trouble to study those detailed data carefully, as we have done, he will find that within the limits of error of random sampling the characteristic distribution for each month is faithfully reproduced every year. The November distribution, for example, is of essentially the same character in 1899 and in 1906. The same is true of the distributions for every month and all of the years. Adding the several years together does not in any way obscure or hide any facts. On the contrary, by providing more ample statistics, it gives smoother curves and enables us (as the outcome proves) to reach a clear understanding of the nature and biological genesis of these monthly fecundity distributions by a method of mathematical analysis which could not successfully be carried out on the much more meager data of a single year.

The writers are well aware of the importance of the statistical canon which says that different sets of data should not be mixed. They would, however, call attention to two facts. In the first place, the essential homogeneity of the present material in regard to the character here under discussion, and so far as concerns strictly the problem of physiology (and not of heredity) with which we are here and now dealing, is in itself a sufficient justification for the statistical procedure followed. In the second place devotion to the letter of a scientific canon should not be so blind as to lose sight of the main purpose of the use of the method involved. The purpose of the present investigation is to learn so far as possible from the available data the true facts regarding the nature and physiological genesis of the monthly fecundity distributions. Thorough study of all the detailed data has convinced us, as it will convince anyone else, we feel sure, who will take the trouble to carry it out, that combining the distributions for the several years greatly helps and in no way hinders us in arriving at a correct and adequate knowledge of the facts.

2. While the material which gives origin to these monthly fecundity distributions is biometrically homogeneous, it is not possible to fit any one of them with a simple unimodal frequency curve. There is no escape from the conclusion that they are compound distributions, even though statistically homogeneous, and must be dealt with as such. This point is emphasized because there is some tendency in biometric work to assert that if a distribution does not fall into the usual simple unimodal type the material is, by virtue of that fact, heterogeneous. This, of course, is not necessarily true. Proof that it is not, if any be needed, is afforded by the curves of monthly fecundity of the present paper or by the well-known curve of human mortality. It is perfectly certain, regardless of the homogeneity of the sample or the population from which it is drawn, that if the number of persons dying at each year of duration of life from 0 to the upper limit at 90+ out of, say, 100,000 born at the same time be platted, the resulting curve will not be a simple unimodal one, but, on the contrary, a compound one. It was shown a number of years ago by Pearson[a] that the human mortality curve is composed of a number of simple frequency curves. The existence of such compound curves in homogeneous material should make one cautious in the use of purely statistical tests of heterogeneity. Any statistical test of heterogeneity which depends upon the discovery of an incipient tendency to bimodality in the data would appear to be valid only in cases where it is a priori certain that the material does not actually follow a compound rather than a simple unimodal frequency curve, even when strictly homogeneous. And, on the other hand, an assertion of homogeneity on the basis of a statistical test alone, without previous knowledge of the biological facts, may be equally liable to be invalid. A very excellent example in this direction, indicating the entirely fallacious result to which a purely statistical test of homogeneity may lead in the absence of biological information as to the detailed facts underlying the statistics, is very well brought out in an example given by Johannsen[b] from some bean measurements.

With these two facts of (1) the homogeneity of the material and (2) the compound character of the distributions clearly in mind, we may turn to the discussion of their analysis.

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[a] Pearson, K. The Chances of Death, Vol. I.

[b] Johannsen, W. Elemente der exakten Erblichkeitslehre, Jena, 1909, p. 122.

At the outstart it is desirable to outline clearly the general method followed in this work. The reasoning developed in the rest of this section may be briefly outlined in the following way:

1. The statement of an hypothesis as to the biological nature and genesis of the S-shaped distributions which in various forms are found to be characteristic of variation in fecundity in the several months of the laying year.
2. The mathematical testing of this hypothesis.
3. The demonstration of its substantial accuracy and adequacy.

The method of procedure here outlined is made necessary by the nature of the problem. We have no a priori knowledge as to why these S-shaped distributions appear. It is not feasible to go further in the direction of experimentally producing them than has been done in getting the present statistics. But we can make a biological hypothesis regarding the distributions. Then it will be possible to test this hypothesis mathematically by determining what the degree of probability is that the actually observed system of frequencies in any given case follows that law of frequency which flows from the biological assumption made. In other words, by a mathematical comparison of observation and theory we can measure the worth of the theory. If the agreement is very close, it is legitimate to conclude that the hypothesis chosen is the correct one.

STATEMENT OF THE HYPOTHESIS.

Careful consideration of the various facts brought out in the preceding section led to the adoption of the following hypothesis as a working basis for further treatment of these curves.

It is assumed—

1. That variations or changes in the rate of fecundity in the hen are fundamentally or innately continuous (in the mathematical sense), though the objective manifestation of fecundity here is discontinuous, i. e., expressed in discrete units. That is to say, the assumption is that if an individual hen's daily egg record ran like this: (A) 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, (B), it would mean, first, that her rate of fecundity was increasing during this period AB, and second, that this increase in rate was continuous throughout the period, so that it might be graphically represented by some kind of a smooth curve, as in the accompanying diagram (fig. 26).

Mathematically expressed, the assumption is that if y denotes rate of fecundity, then

$$y=\phi\ (x, t, \ldots\ldots z)$$

where $\phi\ (x, t, \ldots\ldots z)$ is a continuous function of the several variables, such that any change in the independent variables, however small, will be accompanied by a corresponding change in the dependent variable.

2. That actual, visible egg production in each individual bird tends to occur in definite cycles or periods of varying lengths, alternating with nonproductive periods.

3. That the rate of fecundity (amount of egg production per unit of time conceived in the sense of the differential calculus) is in any bird at a minimum at the beginning of a cycle of production, increases to a maximum at what may be termed the height of the cycle, and decreases to a minimum as the end of the cycle is approached.

4. That each of the monthly fecundity distributions is compound and made up of two parts. It is assumed, in other words, that at any given instant of time in the laying year a flock of laying hens could be separated into two groups if it were possible to know exactly the physiological condition of their reproductive organs. In one group would be included all birds which were at the moment of examination well along in a period of laying activity (or cycle of fecundity). These would be birds whose rate of fecundity was at or above that in the region of B in figure 27. In the other group would be included those birds not laying at all (that is, in a nonproductive condition or period) and those that had just emerged from this condition of zero fecundity and started on a laying cycle. These would be birds whose rate of fecundity was anything from zero up to about A in figure 27.

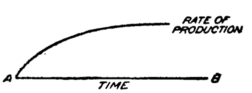

FIG. 26.—Diagram to illustrate the nature of the hypothesis made in the text regarding the rate of egg production.

5. That (*a*) the proportion of the whole flock which falls into each of these two classes, and (*b*) the particular rate of fecundity which marks the boundary between the two classes (the line AB in fig. 27) are not constant, but, on the contrary, change in a definite and orderly manner in the different parts of the laying year.

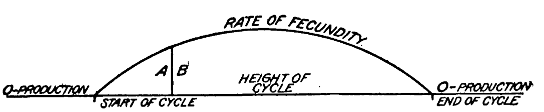

FIG. 27.—Diagram illustrating cycles of fecundity in the hypothesis regarding egg production.

It is plain that on the hypothesis so far set forth a bird's observed egg production in any given month does not immediately determine in which of the two component classes of the flock it belongs. Let it be supposed, for example, that a particular hen lays 7 eggs in the month of July. The mere knowledge of this fact alone does not tell us whether she laid this small number of eggs because she was just starting on a new cycle of production, or on the other hand because she was just ending a cycle in the previous course of which her rate of fecundity had been much higher than 7 eggs in a thirty-day period. Yet this is what must be known in order to assign her to her proper class.

6. That the distribution of frequency within each of the two hypothetical components of the monthly fecundity distributions

follows a simple, unimodal skew frequency curve. In other words, it is assumed that the curve describing the entire monthly fecundity distribution is in each case the sum of two skew frequency curves.

The first of these assumptions needs some further discussion. The idea that egg production is fundamentally a continuous process (in a mathematical sense) may at first thought be difficult to grasp because the objective expression of egg production is in discrete units. But if the various biological steps involved in the production of an egg be once viewed as a whole, there will then be no difficulty in realizing the continuity of the process. Beginning in the young embryo there is the appearance of the primary germ cells in the germinal epithelium. Later, but still in a continuous process of development, the definitive ovary forms. Then the primary oocytes begin to grow. Presently yolk begins to be deposited more or less rapidly, and, finally, at the end of a long but perfectly continuous process of development, the ovum is, by rupture of its inclosing follicle, detached from the ovary, passes down through the oviduct, receiving albumen, membranes, and shell, and is laid as a complete egg. But in the meantime many other oocytes are going through this same process of development and growth. So far as the organism as a whole is concerned, egg production viewed as a whole is a continuous process. The "rate of egg production" clearly means when looked at in a broad way, the rate at which all phases of this complex biological process are going forward at a particular time. This rate varies at different times in the life history of a bird. There may be a definite and positive "rate of fecundity" when a bird is not laying at all. Of course, under such circumstances this rate can not be directly measured, although it can indirectly be estimated. It is the rate at which development of oocytes preparatory to laying is taking place.

The second assumption is really not an assumption at all, but a well-known fact. The third to the sixth assumptions inclusive will be tested in the present work.

The clue to the way in which to make this test is given in the last assumption, which asserts that the observed distributions represent the sum of two frequency curves. If this is in reality the fact it should be possible to graduate or "fit" the observed data satisfactorily with two skew curves, and a successful outcome of the trial would be practically conclusive evidence of the validity of the hypothesis made. If, on the contrary, trial should show that it was impossible to graduate these monthly distributions satisfactorily as the sum of two skew distributions, it would be necessary to conclude that the hypothesis made was either inadequate or entirely incorrect. It is plain that in the practical working out of this matter the decision as to which of these two conclusions shall be drawn will depend on the goodness of fit of the theoretical skew frequency curves to the observed distributions; that is, it will turn on the mathematical criterion of the "satisfactory" or "successful" nature of the graduation. Fortunately in the case of frequency curves we have an effective criterion of the goodness of fit. It was first shown by Pearson[a] that the probability P that a complex system of n errors will occur with a frequency as great or greater than that of the observed system is given by the expression:

$$P=\frac{\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\ \chi^{n-1}\ d\chi}{\int_{0}^{\infty} e^{-\frac{1}{2}\chi^2}\ \chi^{n-1}\ d\chi},$$

wherein χ is such a quantity that, if m_r and m'_r be respectively the observed and theoretical frequencies in each of the n frequency groups, then

$$\chi^2=S\left\{\frac{(m_r-m'_r)^2}{m_r}\right\}$$

S denoting summation as usual. In the paper cited the complex integrals in the expression for P were evaluated, and the proper equations for P with n odd or even were given. Later Elderton[b] published tables giving the value of P for a rather wide range of values of χ^2 and n. Regarding the significance of this test of the goodness of fit Pearson (*loc. cit.*, p. 166) says:

> Thus I think we can conclude, when we are dealing with a sufficiently long series to give small probable errors to the constants of the series, that—
>
> (*i*) If $\chi^2{}_s$ be so small as to warrant us in speaking of the distribution as a random variation on the frequency distribution determined from itself, then we may also speak of it as a random sample from a general population whose theoretical distribution differs only by quantities of the order of the probable errors of the constants, from the distribution deduced from the observed sample.
>
> (*ii*) If $\chi^2{}_s$ be so large as to make it impossible for us to regard the observed distribution as a sample from a general population following the law of distribution deduced from the sample itself, it will be impossible to consider it as a sample from any general population following a distribution differing only by quantities of the order of the probable errors of the sample distribution constants from that sample distribution. In other words, if a curve is a good fit to a sample, to the same fineness of grouping it may be used to describe other samples from the same general population. If it is a bad fit, then this curve can not serve to the same fineness of grouping to describe other samples from the same population.

What we propose to do, then, is to determine mathematically the probability that, as indicated by our samples, the general population from which these samples may be supposed drawn (i. e., Plymouth Rock pullets in general) follows the system of distribution of fecundity which on biological grounds we have proposed as an hypothesis. The probability determined indicates the degree of probability that the hypothesis is true. Especial effort is made to make this point perfectly clear because at first thought it might appear that the procedure here used is of the same character as that which one of the writers has criticized on a previous occasion.[a] The cases are, however, entirely different. The point which was criticized in the paper referred to was the attempt to prove an essentially *qualitative* hypothesis by the use of *quantitative* methods alone. In the present instance we have made an hypothesis which has to do solely with quantitative characteristics of the physiological process of egg production (fecundity). The proper and strictly logical procedure is to test the validity of this quantitative hypothesis by the use of appropriate quantitative (mathematical) methods.

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[a] Pearson, K. On a criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling. Philosophical Magazine, 1900, pp. 158–175.

[b] Elderton, W. P. Tables for testing the goodness of fit of theory to observation. Biometrika, Vol. I, pp. 155–163. 1901.

THE FITTING OF THE CURVES.

Let us turn now to the fitting of the curves. Inasmuch as the graduation of a frequency distribution as the sum of two skew curves has not hitherto been developed,[b] it was necessary to work out a method of approaching the problem. In doing this we have proceeded in a strictly empirical manner. The limitations of the writers' mathematical attainments forbid the undertaking of a general analytical discussion of the problem of resolving a compound distribution into two skew curves. Our experience, however, in dealing with these distributions in an empirical way has at a number of points suggested strongly that there must exist a general analytical solution of the problem. We would suggest this problem to any unoccupied mathematician as one of considerable theoretical interest, and, if solved, of much practical importance.

The method of approach which has been followed in the present investigation is as follows:

1. To take the area of the sum of the two component skew curves to be equal to the area of the observed distribution.

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[a] Pearl, R. Some recent studies on growth. American Naturalist, Vol. XLIII, pp. 302-316. 1909.

[b] The treatment of a skew curve as the sum of two normal curves was fully worked out by Pearson a number of years ago. See Philosophical Transactions, Vol. 185 A, pp. 71-110. 1894. While the present paper was passing through the press a method of treating certain bimodal curves similar in form to the S-shaped curves here dealt with was published by G. Udny Yule, under the title "On the Distribution of Deaths with Age When the Causes of Death act Cumulatively, and Similar Frequency Distributions." Jour. Roy Stat. Soc. Vol. 73, Part I, pp. (of reprint) 1-13, 1910.

2. To make, by graphical methods, a series of assumptions as to the range, midpoint, and shape of each of the two component distributions.

3. To test the goodness of these assumptions by fitting skew curves to a given pair of assumed components, determining areas, summing the two curves obtained, and then testing how good a fit had been obtained to the whole distribution, using Elderton's tables. If a given assumption gave a high value for P it was concluded that the component curves had been determined adequately enough.

It will be seen that this is a strictly empirical, "cut-and-try" method, but it is not more so than any kind of curve fitting where the appropriate curve to use is not known a priori. In ordinary work it is often impossible a priori to tell whether a third order or a fifth order parabola or a logarithmic curve will best graduate a given set of data. The only way to find out is to try each of the curves and see which gives the best result, taking all known facts into consideration. In all such curve fitting the only general principle which one has as a guide is that "the proof of the pudding is in the eating," not forgetting, of course, the distinction between true graduation methods on the one hand and interpolation methods of fitting or their equivalents on the other hand.

Since the amount of labor involved in dealing with these compound distributions by the method outlined is very great, and since the monthly distributions (with two possible exceptions to be presently considered) are of the same general type, it has seemed neither necessary nor advisable to undertake in detail here the analysis of the distribution for each month. Instead we have chosen for analysis several months which give typical illustrations of the different forms which the S-shaped distributions take in the course of the year. As has been previously shown (pp. 126–132), the distributions in the fall and winter months are characterized by having a very high zero frequency and a rather flat upper component. As an illustration of the extreme deviation of the monthly distributions in this direction November has been chosen for analysis. The other extreme in the shape of the distributions is found in the spring months where the lower component is very small and the high frequencies fall in the upper component. As a typical illustration of the spring type of distribution April has been chosen for analytical treatment. The summer laying leads to distributions which, as has been seen already, are intermediate in form between the extremes mentioned. As an illustration of this intermediate type the month of July has been chosen.

These three months—November, April, and July—fairly represent the different forms taken by the monthly distributions in the different parts of the laying year. Nothing essential to the present discussion will be gained by dealing in detail with the distributions for the other months.

At this point a word should be said in regard to the methods of calculation used in fitting these curves. The raw moments were used in all cases, Sheppard's corrections not being applied. The reason for this was the absence of high contact at one end of the lower component in all cases. Practically all computing was done on a large Millionaire calculating machine, and all possible checks were applied to insure arithmetical accuracy. In calculating the areas of the fitted curve Simpson's quadrature formula

$$\int_0^1 y_x dx = \frac{1}{6}\left\{ y_0 + 4y_{\frac{1}{2}} + y_1 \right\}$$

was used except for the end class in the case of the J-shaped lower component curves. In these curves the ordinate at the start of the range is infinite, but the area over any base element is finite. Here evidently no quadrature formula is adequate.

An equation for such a case where, as in the present examples, the J-curve is of Type I, is[a]—

$$\int_0^x y_0 x^{m_1}(b-x)^{m_2} dx = y_0 x^{m_1+1} b^{m_2} \left(\frac{1}{m_1+1} - \frac{m_2 x}{b(m_1+2)} + \ . \ . \ . \right)$$

where x is, of course, the distance from the start of the curve where the ordinate is infinite up to the upper limit of the end class to which this ordinate belongs. In using this formula y_0 must be calculated by the formula

$$y_0 = \frac{\mathrm{N}}{b^{m_1+m_2+1}} \cdot \frac{\Gamma(r)}{\Gamma(m_1+1)\ \Gamma\ (m_2+1)}$$

COMPONENTS AND FITTED CURVES FOR NOVEMBER DISTRIBUTION.

The November distribution may be discussed first, although it was not the first actually to be resolved into two skew curves. Experience with other and more tractable distributions made it possible to hit upon an adequate assumption as to the component curves in this case at the first trial. The assumed component distributions and the fitted curves are given in Table 10. In this table and all the discussion which follows the lower component (small egg production) will be designated as the "A component" and the upper (large production) as the "B component."

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[a] Elderton, W. P. Frequency Curves and Correlation. London, 1907. P. 104.

TABLE 10.—*November egg production, showing the assumed components and their fitted curves.*

Eggs laid in the month.	Assumed A component distribution.	Theoretical A component distribution (areas).	Assumed B component distribution.	Theoretical B component distribution (areas).
0	1,189	1,190	....	....
1	137	132	....	....
2	47	55	....	1
3	33	30	4	5
4	19	18	7	11
5	10	11	26	17
6	6	6	31	24
7	3	4	29	30
8	2	2	30	36
9	1	1	40	41
10	....	....	39	45
11	....	....	49	48
12	....	....	36	51
13	....	....	61	52
14	....	....	47	51
15	....	....	77	50
16	....	....	36	48
17	....	....	43	44
18	....	....	39	40
19	....	....	41	34
20	....	....	23	28
21	....	....	24	22
22	....	....	13	15
23	....	....	5	9
24	....	....	6	3
Total	1,447	1,449	706	705

The analytical constants of these distributions are given in Table 11. Since the data were not grouped in the calculations, the unit of the constants throughout is one egg.

TABLE 11.—*Analytical constants of variation in November egg production.*

Constant.	A component curve.	B component curve.
N	1447	706
μ_2	1.0698	22.4391
μ_3	4.2481	−4.0612
μ_4	23.4654	1,132.1670
β_1	14.7403	0.0015
$\sqrt{\beta_1}$	3.8393	0.0382
β_2	20.5041	2.2485
β_2-3	17.5041	−0.7515
κ_1	−9.2128	−1.5072
κ_2	−5.8466	−0.0007
Pearson's type	I	II
a_1	−8.7732	
a_2	19.7364	11.5882
m_1	−0.8822	
m_2	1.9847	1.4922
Skewness	+8.8855	0
d	9.1903	0
Standard deviation	1.034±.013	4.737±.085
Mean	0.372±.018	13.347±.120
Mode	−8.8185[a]	13.347±.120
Coefficient of variation	278.184±14.158	35.491±.713
Range	10.9632	23.1764
+ end of range	10.9179	24.9352
− end of range	−0.0453	1.7588

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[a] In "J-shaped" Type I curves of this sort the mode ceases to have its usual physical meaning and becomes simply an algebraical constant of the curve. It, of course, still remains the point of origin of x.

From the table the following points are to be noted:

1. Both of the component curves prove to belong to limited range types of Pearson's skew curves. This must be regarded as a first point in favor of the general hypothesis on which we are working. The amount of egg production per bird in a given time interval is a physically limited quantity. Theoretically, then, there is good ground for expecting that the theoretical curves which are to describe the quantitative features of this process should be limited range curves. The estimation of the range given by the curves in the present case is in excellent accord with observed fact. The lower range end of the A component and hence of the whole curve is at —0.045 egg, or practically at 0 egg production. The upper range end of the B component and hence of the whole curve is at 24.93, or practically 25 eggs. The highest observed November production is 24 eggs, but the frequency is unduly high on this class and undoubtedly should be distributed over two or three higher classes.

2. The A component curve is of Pearson's Type I. It belongs in the class of J-shaped curves where one of the m's is negative. The B component is a symmetrical curve of Type II, and has the ordinary "cocked-hat" shape. The absence of skewness in this component is noteworthy.

3. From the figures given it appears that 67 per cent of the flock falls in the A component in the November laying, and 33 per cent in the B component. In other words, according to the hypothesis developed above, it appears that 67 per cent of the flock either are not laying at all in November or have just started to lay, while 33 per cent are well along in a laying cycle in the course of the month.

4. The centroid (mean) of the A component falls within the zero class (0.372 egg). The centroid of the B component is at 13.347, almost exactly 13 eggs higher. It thus appears that while some 33 per cent of the birds are well on in a laying cycle in November, the rate of production, even of these B component birds, is not as high as it becomes later in the year.

5. The B component shows the greater absolute variability (measured by standard deviation). But in proportion to the mean of the character varying (coefficient of variation) it exhibits a much lower variability than the A component. The coefficient of variation of the B component has about the same value as the average for annual production (see Part I, p. 29).

The equations to the curves are:

$$\text{A component } y=71.9022\left(1-\frac{x}{8.7732}\right)^{-.8822}\left(1-\frac{x}{19.7364}\right)^{1.9847}$$

$$\text{B component } y=51.6283\left(1-\frac{x^2}{134.2861}\right)^{1.4922}$$

The November distribution and its fitted curves are shown graphically in figure 28.

We have now to consider the question of the closeness of agreement between observation and theory for the November egg production. It is evident from figure 28 that in general the fit of the combined A and B component curves to the observations is close. The general sweep of the observations is satisfactorily accounted for by the theoretical curve. In applying Pearson's test of the goodness of fit here, however, there are some points which need preliminary consideration. In the first place it has to be recognized that, while our sample is fairly large as statistical data in biology go, yet over 50 per cent of the total frequency in this case falls in one class (zero egg production). Further, since the data are ungrouped, it results that the remaining frequency is spread over a large number of classes (24). This means that the frequency on any class except 0 will be small, and hence, with a character so highly variable absolutely and

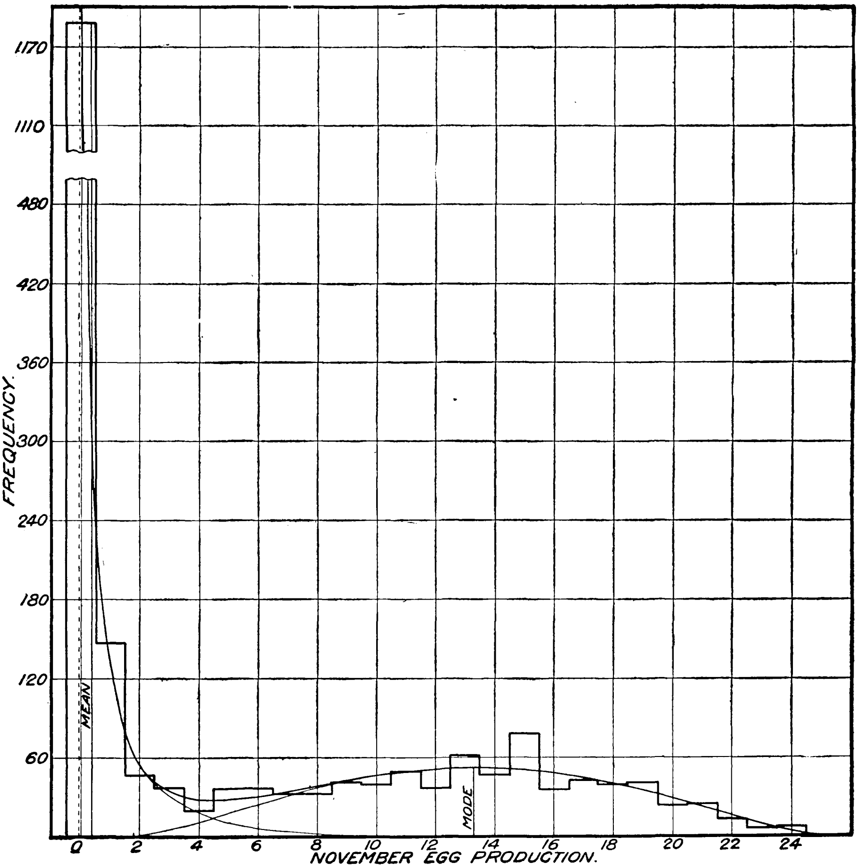

FIG. 28—Histogram and fitted curves for November egg production.

relatively as egg production is, it can not be expected that a sample of anything like the size of the present one will give a smooth distribution. Figure 28 plainly shows that there is a great deal of irregularity in individual frequencies in the B component part of the distribution. Such irregularity is only what is to be expected in a distribution of the shape of the present one, when the sample is no larger than this is. As a result of this irregularity due to the smallness of the sample in relation to the number of classes showing a frequency, the value of χ^2 may be made very large as the effect of one or two irregular classes, even though the theoretical curve represents very precisely the general sweep of the observations. In view of these facts it seems advisable to smooth the minor irregularities of the individual classes in the B component, which have no relation to the general form of the curve. This will help to make the application of Pearson's test a fair measure of the goodness of fit of the whole curve to the observations. We have accordingly made a graphical "first smooth" of the observations from class 10 on to the end of the curve. The smoothing is started at 10 because the A component curve ends at 9. In other words, no smoothing is done in the critical region of the curve where both A and B components contribute to the final result.

In Table 12 Pearson's test of the goodness of fit is applied successively to the rough and the smoothed observations.

TABLE 12.—*Comparison of observed and theoretical November egg production distributions.*

Eggs laid in month.	Observed total distribution.	"First smooth" of observations beyond end of range of A component.	Theoretical total distribution (areas).	$\frac{(m_r - m'_r)^2}{m_r}$ Raw data.	$\frac{(m_r - m'_r)^2}{m_r}$ "First smooth."
0	1,189	1,189	1,190	0.001	0.001
1	137	137	132	.189	.189
2	47	47	56	1.446	1.446
3	37	37	35	.114	.114
4	26	26	29	.310	.310
5	36	36	28	2.286	2.286
6	37	37	30	1.633	1.633
7	32	32	34	.118	.118
8	32	32	38	.947	.947
9	41	41	42	.024	.024
10	39	42	45	.800	.200
11	49	43	48	.021	.521
12	36	45	51	4.412	.706
13	61	51	52	1.558	.019
14	47	58	51	.314	.961
15	77	59	50	14.580	1.620
16	36	47	48	6.750	.021
17	43	40	44	.023	.364
18	39	40	40	.025	.400
19	41	37	34	1.441	.265
20	23	27	28	.893	.036
21	24	20	22	.182	.182
22	13	13	15	.267	.267
23	5	7	9	1.778	.444
24	6	4	3	3.000	.333
Total	2,153	2,147	2,154	43.112	13.407

From the table the following points are to be noted:

1. As the raw observations stand, Pearson's test would indicate that the theoretical curve gives an extremely poor graduation. For $n=25$ and $\chi^2=40$, Elderton's table gives $P=0.021387$, or on this result it would be expected that 98 or 99 times in every 100 trials we should be likely to get a better agreement between observation and theory than that here shown if samples were taken from a general population following the law of frequency given by the theoretical curve.

2. But it is clear from figure 28 that the curve is by no means so bad a fit as this would indicate. On the contrary, it is a plain and obvious fact that the curve gives a very excellent representation of the general sweep of the observations. In other words, the curve gives a satisfactory graduation of the data as a whole, though in so doing it necessarily quite fails to hit a few (actually 4) very irregular class frequencies.

3. An analysis of the χ^2 column for the raw data shows that this value of 43.112 owes its extreme magnitude chiefly to the contributions from 4 classes, viz, 12, 15, 16, and 24. Class 15 alone adds 14.580 to χ^2. A study of the detailed figures in this column in connection with figure 28 will serve to convince anyone that the value of P obtained from the raw data in this case merely signifies that the curve fails entirely to fit 4 out of the 25 observed class frequencies. It gives no proper measure at all of the validity of the graduation of the data as a whole.

4. Such a measure is much more accurately given by the figures in the last column of the table. Here the theoretical curve and the observations graphically smoothed (in a nonarbitrary manner) in the part of the distribution beyond the end of the A component range are compared. Here we get $\chi^2=13.407$. Elderton's tables show that for $n=25$, $P=0.966121$ when $\chi^2=13$ and 0.946650 when $\chi^2=14$. In other words, if samples of the size of the present one were drawn from a population obeying the law of frequency given by the theoretical curve, only about 5 times in every 100 trials should we expect to get as close or closer agreement between observation and theory than the present figures show.

5. Taking all the facts into consideration, it appears that the theoretical compound frequency curve obtained as the sum of two skew curves so closely fits the observations that it may be inferred with a high degree of probability that what differences exist between observation and theory arise only as a result of the errors of random sampling. From this the conclusion would seem to be justified (according to the principles set forth in the quotation from Pearson, p. 139) that the distribution of November egg production is, in general, compound in character, as postulated in our original hypothesis.

COMPONENTS AND FITTED CURVES FOR APRIL DISTRIBUTION.

Let us turn now to the consideration of the April distribution. The assumed components and their fitted curves are given in Table 13.

TABLE 13.—*April egg production, showing the assumed components and their fitted curves.*

Eggs laid in the month.	Assumed A component distribution.	Theoretical A component distribution (areas).	Assumed B component distribution.	Theoretical B component distribution (areas).
0	55	48	....	1
1	28	29	1	2
2	13	18	3	3
3	11	12	8	6
4	8	8	10	9
5	4	5	19	15
6	2	3	23	21
7	1	1	32	31
8	....	....	28	42
9	....	....	58	56
10	....	....	76	72
11	....	....	79	90
12	....	....	119	108
13	....	....	125	127
14	....	....	150	146
15	....	....	168	162
16	....	....	162	175
17	....	....	186	183
18	....	....	185	186
19	....	....	171	181
20	....	....	184	169
21	....	....	164	151
22	....	....	124	126
23	....	....	95	97
24	....	....	63	66
25	....	....	29	38
26	....	....	15	16
27	....	....	5	3
28	....	....	3	....
Total	122	124	2,285	2,282

The analytical constants for these distributions are given in Table 14.

TABLE 14.—*Analytical constants of variation in April egg production.*

Constant.	A component curve.	B component curve.
N	122	2285
μ_2	2.6506	22.2406
μ_3	5.6934	−41.7310
μ_4	28.4613	1387.6466
β_1	1.7406	0.1583
$\sqrt{\beta_1}$	1.3193	0.3977
β_2	4.0509	2.8053
β_2-3	1.0509	−0.1947
κ_1	−3.1199	−0.8642
κ_2	−0.6314	−0.1436
Pearson's type	I	I
a_1	−7.8617	25.4224
a_2	15.6646	10.1462
m_1	−0.5239	6.7437
m_2	1.0438	2.6914
Skewness	5.7344	−0.2832
d	9.3360	−1.3355
Standard deviation	1.628±.070	4.716±.047
Mean	1.295±.099	16.539±.067
Mode	−8.0409	17.8747
Coefficient of variation	125.712±11.072	28.514±.307
Range	7.8029	35.5686
+end of range	7.6237	28.0208
−end of range	−0.1792	−7.5478

From this table we note the following points:

1. As in the case of the November distribution, both of the component curves are of limited range. In this case both are of Type I, the A component curve being, as before, of the J-shape. The estimation of the range ends is in close accord with the facts in the A component curve, giving the lower at -0.1792 or practically zero and the upper at 7.6237. The upper end of the B component (28.0208) falls a little short of the observational range, but not badly so. The lower end of the B curve is at -7.5 roughly, which is, of course, a physically impossible value. However, no particular importance is to be attached to this fact, because the frequency in this component below the zero class is negligible.[a]

2. The values of the areas show that in the April laying only a trifle more than 5 per cent of the total flock falls in the A component of the curve, while 95 per cent falls in the B component. This result agrees very well with common experience that in April, which represents about the height of the natural breeding season, practically all of the birds are well along in a laying cycle.

3. The mean of the A component falls at 1.295, about 1 egg higher than in the case of the November distribution. The mean of the B component is at approximately 16.5 eggs, showing an increase of about 3 eggs over the November B component.

4. The absolute variabilities of the two components stand in about the same relation to each other as in the November curve. The A component is, however, somewhat more variable absolutely in this curve than in the other. Relatively (as measured by the coefficient of variation) both components have a lower variability in the April than in the November curve.

5. The B component in this curve is skew in the negative direction (mode larger than mean), though the asymmetry is not great in amount.

The equations of the curves are—

$$\text{A component } y=25.93\left(1-\frac{x}{7.8617}\right)^{-0.5239}\left(1-\frac{x}{15.6646}\right)^{1.0438}$$

$$\text{B component } y=185.91\left(1+\frac{x}{25.4224}\right)^{6.7437}\left(1-\frac{10.1462}{x}\right)^{2.6914}$$

The distribution and fitted curves are shown graphically in figure 29.

It is apparent from the figure that the agreement between observation and theory in this case is, in general, excellent. Here the small area comprised in the A component leaves relatively large frequencies for the remaining portion of the curve. The detailed figures for the Pearson test of the goodness of fit are given in Table 15.

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[a] See Part I, pp. 32–38, for a discussion of this matter of the extension of the range into negative values.

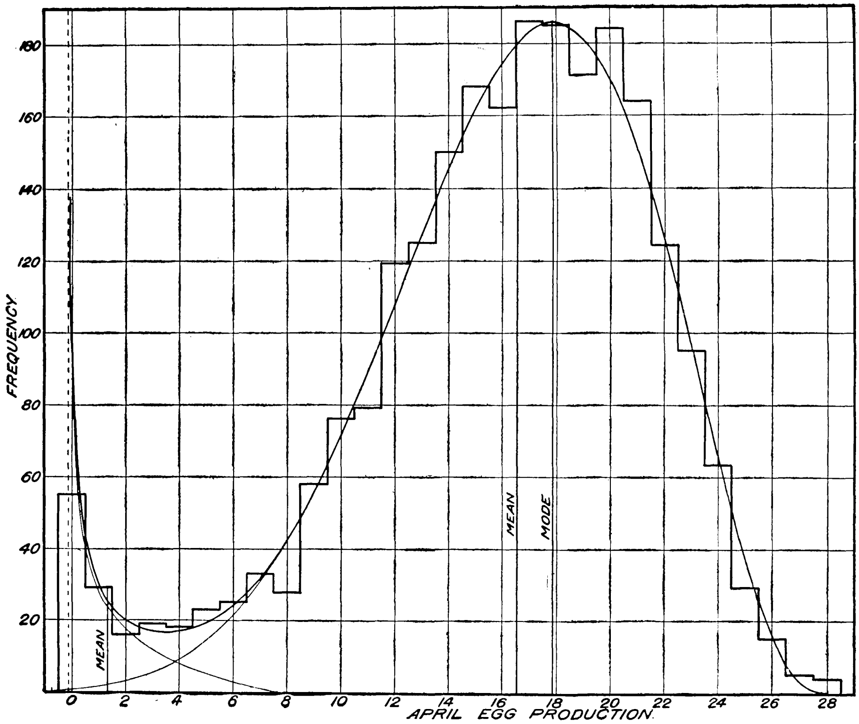

FIG. 29.—Histogram and fitted curves for April egg production.

TABLE 15.—*Comparison of observed and theoretical April egg-production distributions.*

Eggs laid in the month.	Observed total distribution.	Theoretical total distribution (areas).	$\frac{(m_r - m'_r)^2}{m_r}$
0	55	49	0.735
1	29	31	.129
2	16	21	1.190
3	19	17	.235
4	18	17	.459
5	23	20	.450
6	25	24	.042
7	33	32	.031
8	28	42	4.667
9	58	56	.071
10	76	72	.222
11	79	90	1.344
12	119	108	1.120
13	125	127	.032
14	150	146	.110
15	168	162	.222
16	162	175	.966
17	186	183	.049
18	185	186	.005
19	171	181	.552
20	184	169	1.331
21	164	151	1.119
22	124	126	.032
23	95	97	.041
24	63	66	.136
25	29	38	2.132
26	15	16	.063
27	5	3	1.333
28	3	....	....
Totals	2,407	2,405	18,418

The figures in this table support the conclusion reached from an examination of the curve. From Elderton's table it appears that when $n=28$ and $\chi^2=18$, $P=0.903519$, and when $\chi^2=19$, $P=0.870001$. Or, in other words, the figures indicate that if the distribution of April egg production in Barred Plymouth Rock pullets followed exactly the law given by the combined theoretical curves we should expect to get a better agreement between observation and theory when tested by random samples of the size of the present one in only about 11 or 12 out of every 100 trials. The graduation is excellent, and may be taken to indicate with the degree of probability implied in the above figures that the general hypothesis regarding the quantitative characteristics of egg production is true for April production.

COMPONENTS AND FITTED CURVES FOR JULY DISTRIBUTION.

Let us turn next to the July distribution. The final assumed component distributions and their fitted curves are given in Table 16.

TABLE 16.—*July egg production, showing the assumed components and their fitted curves.*

Eggs laid in the month.	Assumed A component distribution.	Theoretical A component distribution (areas).	Assumed B component distribution.	Theoretical B component distribution (areas).
0	297	296		0.2
1	42	45	6	2
2	26	23	9	7
3	14	15	13	17
4	10	11	32	32
5	7	6	48	50
6			65	71
7			86	93
8			110	113
9			125	130
10			157	143
11			168	151
12			164	154
13			167	152
14			144	145
15			119	135
16			109	122
17			95	107
18			81	91
19			79	75
20			60	61
21			59	48
22			36	36
23			35	25
24			20	18
25			11	12
26			6	8
27			4	5
28			1	3
29			0	1
30			0	1
31			2	0.3
Totals	396	396	2,011	2,008.5

The analytical constants of these distributions are given in Table 17.

TABLE 17.—*Analytical constants of variation in July egg production.*

Constant.	A component curve.	B component curve.
N	396	2011
μ_2	1.2489	24.8575
μ_3	3.2640	37.1164
μ_4	12.3682	1703.9598
β_1	5.4687	0.0897
$\sqrt{\beta_1}$	2.3385	.2995
β_2	7.9293	2.7577
β_2-3	+4.9293	−.2423
κ_1	−6.5474	−.7537
κ_2	−1.6290	−.0916
Pearson's Type	I	I
a_1	7.1849	12.9692
a_2	−1.6429	26.3983
m_1	−0.8576	3.7155
m_2	0.1961	7.5628
Skewness	−5.9014	0.2029
d	−6.5951	1.0114
Standard deviation	1.476±.035	4.986±.075
Mean	0.533±.050	13.053±.053
Mode	[a] 7.1279	12.0413
Coefficient of variation	276.922±26.830	38.197±.462
Range	5.5420	39.3675
+end of range	5.4851	38.4397
−end of range	−0.0570	−0.9278

[a] The "mode" here is, of course, merely an algebraical constant of the curve.

From these figures we note that—

1. Both component curves are again limited-range Type I curves. The A component curve is of the J-shaped type. The theoretical estimation of the range by these curves is, as in the other curves, in good accord with observed fact. The lower range end of the A component falls almost exactly at 0 production, while the same end of the B component is at −0.9, an insignificant overestimation. The upper end of the B component (and hence of the compound curve) is at about 38 eggs. This is higher than the observed range end, but the theoretical frequency (area of the curve) beyond the latter is negligible in amount.

2. In the July distribution approximately 16.5 per cent of the total frequency falls in the A component and 83.5 per cent in the B component. This indicates the decreasing rate of fecundity in July as compared with April.

3. The mean of each component is lower than the mean of the corresponding component of the April curve. The mean of the B component (13.053) is very nearly the same as the mean of the November B curve (13.347). In other words, the birds which are fairly well started on a laying cycle lay at about the same rate in November as in July.

4. The relative variability of the A component becomes again very high, practically equaling the November variability. The B component shows relatively more variability than in either of the other curves examined.

5. The skewness of the B component is here positive, the mode lying below the mean.

The equations of the curves are—

$$\text{A component } y = 0.8234\left(1+\frac{x}{7.1849}\right)^{-0.8576}\left(1+\frac{x}{1.6429}\right)^{0.1961}$$

$$\text{B component } y = 154.44\left((1+\frac{x}{12.9692}\right)^{3.7155}\left(1-\frac{x}{26.3983}\right)^{7.5628}$$

The fitted curve is shown graphically in figure 30.

It is evident from the diagram that the graduation is a good one. Applying Pearson's test of the goodness of fit, we have the results set forth in Table 18.

TABLE 18.—*Comparison of observed and theoretical July egg-production distributions.*

Eggs laid in the month.	Observed total distribution.	Theoretical total distribution.	$\frac{(m_r - m'_r)^2}{m_r}$
0	297	296	0.003
1	48	47	.021
2	35	30	.833
3	27	32	.781
4	42	43	.023
5	55	57	.070
6	65	71	.507
7	86	93	.527
8	110	113	.080
9	125	130	.192
10	157	143	1.371
11	168	151	1.914
12	164	154	.649
13	167	152	1.480
14	144	145	.007
15	119	135	1.896
16	109	122	1.385
17	95	107	1.346
18	81	91	1.099
19	79	75	.213
20	60	61	.016
21	59	48	2.521
22	36	36	.000
23	35	25	4.000
24	20	18	.222
25	11	12	.083
26	6	8	.500
27	4	5	.200
28	1	3	1.333
29	0	1	1.000
30	0	1	1.000
31	2	[a] 1	1.000
Total	2,407	2,406	26.272

[a] Total area of theoretical curve for 30 and beyond.

Elderton's tables extend only to $n=30$. By graphical extrapolation from these tables we find that for $n=32$ and $\chi^2=26$, P is roughly 0.72. In other words, it appears that while the graduation of the July data is not quite so good as that of the April, it still can only be regarded as excellent. If egg production in July followed exactly the law of frequency given by the present theoretical curve, we should expect to get a better agreement between observation and theory when tested by random samples of the size of the present one in only one sample out of every three or four tried.

It appears, taking all the evidence together, that there is a very high degree of probability that the hypothesis made above (p. 136 et seq.) regarding the quantitative relations of egg production is true. We have found a very close accordance between observation and theory in the case of three months chosen as examples to represent the different types of distribution which appear in the different parts of the laying year. There is an accumulative probability, arising from the fact that all three of these distributions show such excellent agreement between observation and theory, that the hypothesis adopted represents in essential features the actual facts. So great is this probability that the evidence presented may be said practically to demonstrate the validity of the hypothesis advanced, so far at least as the months specially discussed are concerned.

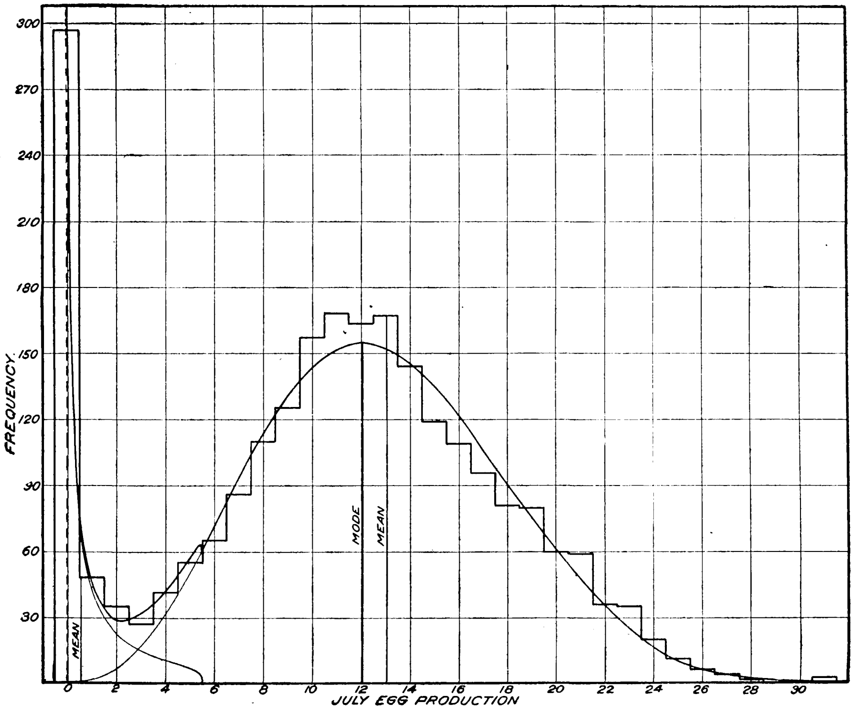

FIG. 30.—Histogram and fitted curves for July egg production.

CONFORMITY OF REMAINING MONTHS TO GENERAL TYPE.

It now remains to consider briefly the question of the extent to which the other months of the year conform to the general type of S-shaped compound distribution of egg production which have been analyzed for November, April, and July. The facts brought out in the preceding section leave little doubt that all the months are of essentially this same type. The only months regarding which there can be any doubt on this point are December and January, and here the doubt is not as to whether these curves are compound and S-shaped, but merely as to whether it may not be necessary in graduating them to make a subsidiary hypothesis in addition to that which suffices for the treatment of the other months. In the observed distributions for these months there is an unduly high frequency on the classes lying in the boundary region where the two components overlap. An analytical study of the January distribution shows that the unsmoothed figures are not so readily to be graduated satisfactorily as the sum of two curves as the other months have proved to be. However, this result by no means proves that another law of frequency obtains for December and January. This will be evident if the following facts are taken into consideration. December and January are both months showing the extreme type of distribution with a very high frequency on zero, and consequently, with samples of the present size, have an entirely inadequate remaining frequency to give a smooth distribution over the other classes. In other words, we have the same difficulty as has been noted for the November distribution, though here possibly more pronounced. Any existing difficulty in regard to the graduation of the polygons of these months may arise simply because the existing sample is a particularly bad one from which to determine the true nature of the distribution. The conclusion to which the facts point is that the December and January distributions are in their general features like those for the other months of the year, which, as we have seen, represent the sum of two skew frequency curves. With our present data, however, we are not able to determine whether or not some minor additional subsidiary hypothesis may be necessary to graduate satisfactorily the distributions for these two months. This is a question of detail which must wait for further data. It does not influence the general result of this investigation.

A special summary of the points brought out in this section of the paper is not given here, but in order to save space, is presented in the general summary of the whole paper which immediately follows:

SUMMARY OF RESULTS AND CONCLUSIONS.

This paper constitutes Part II of a general study of egg production in the domestic fowl. It deals with the seasonal distribution of egg production. The distribution of egg production in each month of the pullet-laying year is particularly studied. The data on which this study is based are the trap-nest records of Barred Plymouth Rocks collected at the Maine Experiment Station, involving detailed monthly egg records of more than 2,400 birds, collected in a period of nine consecutive years. The chief results of this analysis are summarily stated in the following paragraphs:

1. The mean or average monthly egg production exhibits the following characteristic changes in the course of the laying year:

(*a*) The lowest mean production of the year is in the month of November.

(*b*) The mean monthly production increases in December and January at a relatively very rapid rate.

(*c*) There is a slacking up in the rate of increase in February, which probably represents the point of the ending of the first, or winter, cycle of egg production. This February slacking up amounts in many cases to an actual decrease in productiveness as compared with the point attained in January.

(*d*) The mean production reaches a maximum in March.

(*e*) While the mean production for April is practically the same as that for March, there is a steady decline after April on to the end of the laying year in October.

(*f*) There is a tendency toward a slightly larger drop in mean production in May. This is the period of natural broodiness.

2. The present data indicate that only a trifle more than a quarter of the total eggs produced are laid in the winter third of the year (November 1 to March 1). In the first two-thirds of the laying year approximately three-fourths of the total eggs are produced.

3. The month of maximum productivity varied in the experiments furnishing the present data with the method of housing. In a closed, warmed house the month of maximum production was April; in a curtain-front house it was March.

4. The greatest relative variability in egg production is at the beginning of the laying year (month of November). The month of lowest variability, both absolute and relative, is April.

5. The laying year may be divided into four natural periods or cycles with reference to egg production. The first of these periods (roughly November 1 to March 1) is the winter period, wherein egg production is essentially a nonnatural (i. e., forced or stimulated) process. The second or spring period (roughly March 1 to June 1) is the natural laying period of the domestic fowl in its normal reproductive cycle. The third (roughly June 1 to September 1) and fourth (roughly September 1 to October 31) periods are not sharply separated from one another. The summer egg production represents in part a natural continuance of the normal breeding season (rearing of a second brood by the wild *Gallus*) and in part a stimulated process. This period is terminated by the molt, which is the characteristic feature of the fourth period.

6. There is no evidence that the continued selection for higher egg production practiced during the eight years covered by the experiment produced any increase whatever in the mean egg production

of any month in the year. On the contrary, the mean production in all but two of the months actually decreased during the period of selection.

7. So far as there was any change whatever in variability in monthly egg production during the period when selective breeding was practiced, this change was not in the direction of a reduction as a result of the selection; but, on the contrary, there was an actual increase in variability in all but one month of the year, and here the platted variability line did not sensibly deviate from the horizontal.

8. The present statistics show no bad effect on egg production in the winter months (November to March) of keeping birds in large and crowded flocks (up to the limits included in the present study). On the other hand, overcrowding tends distinctly to lower summer (and to a smaller extent spring) egg production. It is chiefly as a result of this effect on summer production that the mean annual production is lower in the large flocks, as shown in Part I of this work (Bureau of Animal Industry Bulletin 110, Part I, pp. 58–61).

9. The excess in relative variability of egg production of the larger flocks (100 and 150 birds) over the smaller (50 and 100 birds) observed in the annual records (see Part I) is found upon analysis to be on the whole fairly evenly distributed over the whole year. In the period of the year in which there is the heaviest production, such environmental differences as are implied in the different flock sizes in the experiment do not appreciably affect the relative variability of production.

10. The polygons of variation in monthly egg production are all bimodal, though homogeneous. The position of the mode changes in a regular manner during the course of the laying year, as does also the skewness of this portion of the distributions. These monthly distributions may be described as S-shaped, and are in general of the same type in the different months, though different in detail.

11. By a process of mathematical reasoning set forth in detail in the text it is shown that the observed facts regarding the character of the distributions of variation in monthly egg production are adequately accounted for by an hypothesis which includes the following points. These statements are therefore set down as conclusions of this study:

(*a*) That variation or changes in the rate of fecundity in the hen are fundamentally or innately continuous (in the mathematical sense), though the objective manifestation of fecundity is discontinuous, i. e., expressed in discrete units.

(*b*) That visible egg production in each individual bird tends to occur in definite cycles or periods of varying length which alternate with nonproductive periods.

(*c*) That the rate of fecundity (amount of egg production per unit of time, conceived in the sense of the differential calculus) is in any bird at a minimum at the beginning of a cycle of production, increases to a maximum at what may be termed the height of the cycle, and decreases to a minimum (usually quite rapidly) as the end of the cycle is approached.

(*d*) That each of the monthly fecundity distributions is compound, and made up of two parts. In one part are included all birds which are well along in a period of laying activity (or cycle of fecundity). The other part includes those birds not laying at all (that is, in a nonproductive condition or period) and those that have just emerged from this condition of zero fecundity and started on a laying cycle.

(*e*) That (1) the proportion of the whole flock which falls into each of these two classes, and (2) the particular rate of fecundity which marks the boundary between the two classes, are not constant, but, on the contrary, change in a definite and orderly manner in the different parts of the laying year.

(*f*) That the distribution of frequency within each of the two hypothetical components of the monthly fecundity distributions follows a simple, unimodal skew frequency curve, and that the curve describing the entire monthly fecundity distribution is in each case the sum of two skew frequency curves.

APPENDIX II.

In the following 12 tables are given the raw data on which the present paper is based:

TABLE I.—*Frequency distribution of egg production for the month of November.*

Laying year.	Number of hens laying number of eggs designated in box head.[a]																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1899–1900	39	1	3	0	2	1	3	1	4	2	0	2	2	1	3	1	0	2	2	0	0	1	0	0	0	0	0	0	0	0	0	70
1900–1901	54	5	1	1	2	4	1	2	1	3	1	1	0	1	0	2	2	2	1	0	0	1	0	0	0	0	0	0	0	0	0	85
1901–2	26	1	1	1	1	0	0	1	0	0	3	0	1	1	1	0	1	1	2	1	2	1	1	1	1	0	0	0	0	0	0	48
1902–3	76	5	0	1	2	3	4	4	5	5	4	2	3	3	0	5	1	2	6	4	1	6	3	2	0	0	0	0	0	0	0	147
1903–4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1904–5:																																
50-bird pens	180	12	7	2	1	2	3	1	0	3	3	0	3	5	6	16	6	2	5	5	7	9	2	1	2	0	0	0	0	0	0	283
100-bird pens	45	5	2	1	0	1	1	0	3	1	1	1	0	5	1	9	0	4	1	8	1	0	2	0	0	0	0	0	0	0	0	92
150-bird pens	72	9	4	3	0	0	1	0	1	1	2	3	0	12	0	15	1	8	1	6	0	0	0	1	0	0	0	0	0	0	0	140
1905–6:																																
50-bird pens	70	15	6	2	2	3	4	1	3	3	7	9	7	9	7	8	2	7	7	4	2	0	0	0	0	0	0	0	0	0	0	178
100-bird pens	75	17	4	8	6	5	2	4	3	4	4	8	4	4	11	4	3	6	4	3	2	1	0	0	0	0	0	0	0	0	0	182
150-bird pens	160	16	7	5	2	6	5	5	6	6	6	9	4	11	9	9	6	2	1	0	0	0	0	0	0	0	0	0	0	0	0	275
1906–7:																																
50-bird pens	107	14	1	7	3	6	10	4	2	6	3	3	5	4	5	1	1	1	2	1	0	0	1	0	0	0	0	0	0	0	0	187
100-bird pens	120	16	3	3	0	1	1	3	3	3	3	3	4	2	1	2	4	3	2	2	2	3	1	0	0	0	0	0	0	0	0	185
150-bird pens	165	21	8	3	5	4	2	6	1	4	2	8	3	3	3	5	9	3	5	7	6	2	3	0	3	0	0	0	0	0	0	281
Total birds	1,189	137	47	37	26	36	37	32	32	41	39	49	36	61	47	77	36	43	39	41	23	24	13	5	6	0	0	0	0	0	0	2,153

[a] The figures in the box head indicate the number of eggs laid respectively by the number of hens shown below in each column; e. g., in November, 1899, there were 39 hens that laid no eggs, 1 hen that laid 1 egg, 3 hens that laid 2 eggs, etc.

TABLE II.—*Frequency distribution of egg production for the month of December.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899-1900	24	1	3	0	0	1	1	1	2	0	0	0	1	1	3	3	4	4	5	2	4	2	5	1	1	0	0	0	1	0	0	0	70
1900-1901	37	1	2	2	2	0	2	1	0	2	2	1	1	2	2	4	3	1	3	4	3	2	3	1	3	1	0	0	0	0	0	0	85
1901-2	10	0	0	2	1	1	1	1	1	2	2	0	3	1	1	0	5	2	0	2	4	2	1	1	4	1	0	0	0	0	0	0	48
1902-3	75	3	4	7	5	6	5	4	5	6	3	4	2	1	1	0	3	2	3	0	3	1	3	1	0	0	0	0	0	0	0	0	147
1903-4	79	21	15	6	10	7	12	10	11	9	19	11	10	9	5	6	3	2	4	4	1	0	0	0	0	0	0	0	0	0	0	0	254
1904-5:																																	
50-bird pens	150	10	4	3	8	6	5	4	6	7	7	4	4	6	8	4	3	4	11	10	1	6	6	4	1	1	0	0	0	0	0	0	283
100-bird pens	15	4	1	5	2	2	1	3	2	4	2	3	1	5	8	4	6	5	6	1	6	2	1	1	2	0	0	0	0	0	0	0	92
150-bird pens	43	7	3	2	1	2	1	2	3	6	5	6	5	3	4	2	7	5	10	9	4	1	5	3	1	0	0	0	0	0	0	0	140
1905-6:																																	
50-bird pens	59	8	4	3	4	1	0	4	2	1	3	6	5	8	2	2	5	3	10	12	8	6	8	6	4	2	1	1	0	0	0	0	178
100-bird pens	43	11	6	8	9	4	3	3	5	4	4	4	5	6	2	2	4	4	6	7	8	8	7	10	5	3	1	0	0	0	0	0	182
150-bird pens	94	15	6	2	4	4	4	8	4	3	10	3	3	7	4	5	2	10	13	11	16	11	8	13	8	5	0	2	0	0	0	0	275
1906-7:																																	
50-bird pens	95	13	5	5	4	5	6	4	5	7	1	4	4	1	4	0	3	6	2	1	2	6	2	1	1	0	0	0	0	0	0	0	187
100-bird pens	94	10	0	3	3	3	5	2	4	1	3	2	5	2	5	2	12	3	6	7	4	2	3	0	3	0	1	0	0	0	0	0	185
150-bird pens	88	19	14	3	10	15	3	6	9	6	6	4	5	3	10	11	12	9	8	10	7	7	4	7	2	2	1	0	0	0	0	0	281
Total birds	906	123	67	51	63	57	49	53	59	58	67	52	54	55	59	45	72	60	87	80	71	56	56	49	35	15	4	3	1	0	0	0	2.407

TABLE III.—*Frequency distribution of egg production for the month of January.*

Laying year.	Number of hens laying number of eggs designated in box head.																																Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899–1900	3	0	2	1	0	0	0	1	1	2	2	2	2	2	4	4	4	2	4	5	1	9	8	3	5	3	0	0	0	0	0	0	70
1900–1901	9	3	2	3	2	4	2	2	2	5	4	1	1	5	1	3	4	1	4	5	6	3	5	5	2	0	0	1	0	0	0	0	85
1901–2	0	0	0	1	3	0	0	2	2	1	3	0	3	3	0	4	4	5	1	0	2	7	6	0	0	1	0	0	0	0	0	0	48
1902–3	76	6	1	4	3	0	3	1	3	4	2	2	2	6	6	4	2	5	0	6	3	1	5	2	0	0	0	0	0	0	0	0	147
1903–4	52	22	10	9	10	7	10	5	3	7	13	6	8	11	8	8	10	12	9	9	9	11	3	1	1	0	0	0	0	0	0	0	254
1904–5:																																	
50-bird pens	81	4	7	4	8	6	8	5	8	3	10	5	8	7	11	11	13	6	16	9	16	16	11	4	1	2	2	1	0	0	0	0	283
100-bird pens	13	5	0	1	1	2	8	2	2	4	3	3	3	5	5	4	3	5	3	6	6	6	1	1	0	0	0	0	0	0	0	0	92
150-bird pens	28	4	1	4	3	8	2	4	8	2	6	1	3	1	4	7	10	8	8	7	5	9	1	1	4	1	0	0	0	0	0	0	140
1905–6:																																	
50-bird pens	20	2	3	2	2	3	6	5	2	3	4	4	8	4	8	11	9	12	12	12	5	12	14	10	5	0	0	0	0	0	0	0	178
100-bird pens	13	11	5	4	3	4	6	3	3	5	4	7	6	6	4	7	8	17	8	13	8	11	12	6	5	3	0	0	0	0	0	0	182
150-bird pens	46	15	6	6	2	5	3	5	4	5	5	7	10	12	7	10	11	12	9	14	25	19	15	12	5	2	3	0	0	0	0	0	275
1906–7:																																	
50-bird pens	81	10	3	5	7	5	6	4	5	7	5	6	6	4	6	5	3	2	6	4	2	3	0	2	0	0	0	0	0	0	0	0	187
100-bird pens	96	12	9	5	9	3	4	2	5	3	5	3	1	1	4	3	3	0	4	6	1	1	2	2	1	0	0	0	0	0	0	0	185
150-bird pens	136	13	22	16	6	1	6	8	7	9	4	5	1	5	4	4	5	4	10	1	5	0	7	0	0	1	0	1	0	0	0	0	281
Total birds	654	107	71	65	59	48	64	49	55	60	70	52	62	72	72	85	89	91	94	97	94	108	90	49	29	13	5	3	0	0	0	0	2,407

TABLE IV.—*Frequency distribution of egg production for the month of February.*

Laying year.	Number of hens laying number of eggs designated in box head.																													Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
1899–1900	10	2	4	3	1	2	0	6	3	2	3	4	1	2	5	4	4	4	3	2	1	3	1	0	0	0	0	0	0	70
1900–1901	0	2	3	0	2	4	2	3	1	5	2	3	2	2	2	10	7	4	6	4	6	8	5	0	2	0	0	0	0	85
1901–2	3	0	1	3	1	1	2	1	1	3	3	1	6	1	1	5	4	2	2	5	1	0	1	0	0	0	0	0	0	48
1902–3	24	11	2	1	3	2	3	9	9	4	5	10	10	7	7	7	6	6	5	7	5	2	1	0	0	0	1	0	0	147
1903–4	28	4	2	5	5	9	7	14	7	4	9	6	9	10	17	19	16	13	16	11	22	7	7	5	1	1	0	0	0	254
1904–5:																														
50-bird pens	15	6	6	7	4	8	4	9	10	13	6	7	4	10	15	18	20	25	32	14	22	10	12	4	2	0	0	0	0	283
100-bird pens	6	3	1	3	2	5	4	4	3	3	4	4	7	2	11	3	3	4	8	6	5	1	0	0	0	0	0	0	0	92
150-bird pens	24	5	4	3	8	7	8	7	3	6	10	1	1	3	7	6	10	5	10	6	5	0	1	0	0	0	0	0	0	140
1905–6:																														
50-bird pens	1	5	5	2	2	3	4	4	5	1	8	7	7	7	8	7	22	15	16	20	11	14	1	3	0	0	0	0	0	178
100-bird pens	11	3	4	3	4	6	5	3	5	4	7	7	8	6	9	6	12	13	16	18	17	8	6	0	1	0	0	0	0	182
150-bird pens	20	13	10	12	7	9	8	8	8	6	6	4	13	11	15	14	25	21	22	17	11	12	3	0	0	0	0	0	0	275
1906–7:																														
50-bird pens	23	5	4	3	7	9	6	4	11	5	13	8	12	7	8	8	11	10	14	6	6	3	3	0	1	0	0	0	0	187
100-bird pens	71	10	8	10	12	6	5	9	5	5	6	5	3	1	6	4	8	2	2	4	1	1	0	1	0	0	0	0	0	185
150-bird pens	86	17	14	10	10	13	11	9	7	5	11	6	11	6	5	12	10	11	10	6	6	3	0	1	1	0	0	0	0	281
Total birds	322	86	68	65	68	84	69	90	78	66	93	73	94	75	116	123	158	135	162	126	119	72	41	14	8	1	1	0	0	2,407

TABLE V.—*Frequency distribution of egg production for the month of March.*

Laying year.	Number of hens laying number of eggs designated in box head.																																Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899–1900	2	1	1	1	2	0	3	3	2	4	3	5	4	5	3	4	7	3	3	3	4	2	4	0	1	0	0	0	0	0	0	0	70
1900–1901	0	1	0	0	1	2	2	2	5	2	0	2	3	3	1	3	5	5	5	6	9	5	6	6	4	2	3	2	0	0	0	0	85
1901–2	0	0	0	0	0	0	0	1	0	3	1	1	4	3	4	1	3	1	3	5	5	3	4	1	3	1	1	0	0	0	0	0	48
1902–3	5	1	0	0	2	4	0	5	3	2	6	10	10	8	8	5	13	11	14	10	2	8	6	4	7	3	0	0	0	0	0	0	147
1903–4	12	2	0	1	2	1	3	2	2	4	7	10	7	10	8	15	9	8	17	14	19	13	32	23	18	10	3	1	1	0	0	0	254
1904–5:																																	
50-bird pens	6	0	1	1	0	1	1	3	8	3	6	7	15	11	11	13	10	17	15	12	23	28	19	28	22	14	3	3	2	0	0	0	283
100-bird pens	2	1	1	2	4	3	3	2	6	10	4	8	5	3	4	7	4	4	2	3	6	2	2	2	1	0	1	0	0	0	0	0	92
150-bird pens	16	4	2	5	0	0	3	8	6	6	8	9	5	3	4	7	5	8	4	16	4	9	3	3	1	1	0	0	0	0	0	0	140
1905–6:																																	
50-bird pens	0	0	2	2	0	2	0	2	2	3	3	3	6	2	9	7	11	9	13	15	16	20	15	13	12	7	2	2	0	0	0	0	178
100-bird pens	2	4	0	2	3	0	2	0	3	1	6	6	3	7	3	9	15	10	14	12	16	16	19	14	7	5	3	0	0	0	0	0	182
150-bird pens	3	0	0	1	0	2	1	4	7	8	4	8	11	12	14	11	17	27	18	38	19	16	23	16	14	1	0	0	0	0	0	0	275
1906–7:																																	
50-bird pens	3	1	0	2	4	3	0	2	3	5	2	3	8	6	4	3	10	8	7	18	25	16	18	16	11	3	4	0	1	1	0	0	187
100-bird pens	7	3	2	1	3	6	2	4	7	10	5	8	9	10	16	7	11	15	12	15	8	2	9	7	3	3	0	0	0	0	0	0	185
150-bird pens	10	6	4	1	5	5	5	6	10	8	8	14	19	15	12	11	12	24	16	11	15	18	18	14	7	4	2	0	1	0	0	0	281
Total birds	68	24	13	19	26	29	25	44	64	69	63	94	109	98	101	103	132	150	143	178	171	158	178	147	111	54	22	8	5	1	0	0	2,407

TABLE VI.—*Frequency distribution of egg production for the month of April.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1899–1900	2	0	1	0	0	2	0	1	1	0	2	4	1	3	5	4	4	9	3	4	10	8	3	0	2	1	0	0	0	0	0	70
1900–1901	2	0	0	1	0	0	0	1	2	0	0	3	3	5	5	4	6	2	2	8	8	7	6	6	7	4	2	0	1	0	0	85
1901–2	0	0	1	0	0	0	1	0	0	0	2	3	1	0	1	2	1	2	5	5	4	5	4	5	3	2	1	0	0	0	0	48
1902–3	2	1	0	0	2	3	1	1	2	4	2	5	5	3	4	5	5	10	11	12	14	16	8	11	14	4	1	1	0	0	0	147
1903–4	10	3	2	1	1	2	3	3	2	5	14	13	15	14	21	23	26	18	13	12	8	16	10	12	5	2	0	0	0	0	0	254
1904–5:																																
50-bird pens	10	4	0	5	0	3	3	5	4	8	6	9	15	18	15	12	25	21	20	23	23	19	13	8	6	3	2	2	1	0	0	283
100-bird pens	2	1	2	2	0	1	1	2	0	4	1	0	5	6	4	5	6	10	7	8	6	8	2	4	3	0	2	0	0	0	0	92
150-bird pens	5	8	5	2	0	2	1	5	0	7	8	6	7	5	8	9	9	8	9	17	8	6	3	2	0	0	0	0	0	0	0	140
1905–6:																																
50-bird pens	1	0	1	0	4	1	0	0	3	2	7	4	5	9	15	16	14	15	15	16	12	14	16	5	2	1	0	0	0	0	0	178
100-bird pens	4	0	1	2	1	2	2	2	2	6	7	7	8	13	12	18	11	19	24	10	12	11	4	2	0	1	1	0	0	0	0	182
150-bird pens	1	1	1	5	2	4	6	5	5	5	7	5	21	15	24	28	18	29	17	20	21	19	9	4	2	0	1	0	0	0	0	275
1906–7:																																
50-bird pens	4	1	1	0	2	1	1	1	2	4	7	4	8	11	6	11	9	11	20	5	20	8	17	17	6	6	3	1	0	0	0	187
100-bird pens	4	2	0	1	1	1	2	4	0	6	7	3	8	8	11	11	14	10	17	17	14	9	13	9	8	3	1	0	1	0	0	185
0-bird pens	8	8	1	0	5	1	4	3	5	7	6	13	17	15	19	20	14	22	22	14	24	18	16	10	5	2	1	1	0	0	0	281
Total birds	55	29	16	19	18	23	25	33	28	58	76	79	119	125	150	168	162	186	185	171	184	164	124	95	63	29	15	5	3	0	0	2,407

TABLE VII.—*Frequency distribution of egg production for the month of May.*

Laying year.	Number of hens laying number of eggs designated in box head.																																Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899–1900	1	1	0	0	1	2	1	0	2	1	4	7	3	3	3	2	6	6	7	2	4	2	6	2	2	2	0	0	0	0	0	0	70
1900–1901	3	2	1	0	0	0	2	1	2	2	3	2	3	5	11	3	6	5	1	5	9	3	5	7	4	0	0	0	0	0	0	0	85
1901–2	1	0	2	0	0	1	0	1	0	0	2	3	0	3	1	3	5	3	4	5	2	2	4	2	1	3	0	0	0	0	0	0	48
1902–3	3	1	0	1	0	1	1	1	2	0	4	11	4	11	4	11	10	5	12	7	9	13	13	7	7	5	4	0	0	0	0	0	147
1903–4	14	5	6	2	7	3	7	5	9	17	16	24	24	15	24	18	13	13	6	10	4	6	2	1	2	0	1	0	0	0	0	0	254
1904–5:																																	
50-bird pens	19	9	1	3	2	2	7	4	7	11	14	17	10	23	23	12	12	16	11	16	19	17	5	6	5	4	3	2	1	1	0	1	283
100-bird pens	2	3	2	0	0	1	0	1	2	5	4	6	4	6	9	8	4	5	4	6	6	5	5	3	1	0	0	0	0	0	0	0	92
150-bird pens	17	4	2	3	2	2	4	5	7	2	7	7	2	12	7	8	3	8	8	12	4	3	4	1	2	2	1	1	0	0	0	0	140
1905–6:																																	
50-bird pens	2	3	4	4	7	9	3	11	13	4	12	17	14	13	11	13	7	8	7	3	5	3	2	2	0	1	0	0	0	0	0	0	178
100-bird pens	13	3	5	4	5	5	9	4	13	9	9	13	21	14	8	11	9	5	3	6	4	4	3	2	0	0	0	0	0	0	0	0	182
150-bird pens	14	11	5	4	9	11	10	11	19	16	17	21	20	20	12	20	13	14	10	5	4	3	1	1	2	1	1	0	0	0	0	0	275
1906–7:																																	
50-bird pens	6	4	2	1	4	0	3	3	7	5	12	9	12	13	13	6	9	11	15	6	9	9	13	8	2	5	0	0	0	0	0	0	187
100-bird pens	6	1	2	1	4	2	2	3	3	5	5	10	9	4	13	14	14	17	14	11	8	14	7	8	4	0	1	2	1	0	0	0	185
150-bird pens	17	3	1	0	1	8	6	8	14	14	17	23	23	16	17	21	10	14	9	14	15	9	8	10	1	2	0	0	0	0	0	0	281
Total birds	118	50	33	23	42	47	55	58	100	91	126	170	149	158	156	150	121	130	111	108	102	93	78	60	33	25	11	5	2	1	0	1	2,407

TABLE VIII.—*Frequency distribution of egg production for the month of June.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1899–1900	4	1	2	3	2	1	0	1	1	5	1	4	4	4	7	9	6	1	2	3	4	0	2	1	1	0	1	0	0	0	0	70
1900–1901	7	1	0	1	0	0	1	1	2	5	6	8	1	2	3	2	7	6	5	4	4	8	6	2	1	1	0	1	0	0	0	85
1901–2	0	0	0	0	0	0	0	0	1	2	1	1	0	6	3	5	5	0	5	3	7	4	1	2	1	1	0	0	0	0	0	48
1902–3	6	0	2	2	0	2	0	0	2	5	4	8	7	7	8	8	9	7	9	12	13	14	11	6	2	2	1	0	0	0	0	147
1903–4	22	4	3	3	2	2	3	7	5	10	20	22	25	23	18	16	11	9	7	9	9	7	5	5	3	4	0	0	0	0	0	254
1904–5:																																
50-bird pens	25	5	2	1	7	1	13	2	10	13	12	18	19	18	22	21	14	16	9	10	17	8	9	5	1	0	3	1	1	0	0	283
100-bird pens	4	1	2	3	1	2	0	5	3	3	3	7	7	3	7	4	7	3	7	0	7	4	4	2	1	0	1	0	1	0	0	92
150-bird pens	28	7	1	2	3	3	2	3	5	4	6	12	6	5	9	6	5	7	4	7	2	4	5	3	0	0	0	0	1	0	0	140
1905–6:																																
50-bird pens	2	1	1	0	3	1	4	4	14	6	10	12	20	15	11	7	18	12	9	6	7	8	5	1	0	1	0	0	0	0	0	178
100-bird pens	7	4	6	0	2	3	4	2	8	7	17	15	15	9	13	10	9	9	6	9	10	8	2	4	1	1	1	0	0	0	0	182
150-bird pens	18	5	2	4	4	6	7	11	15	14	20	16	26	17	24	15	17	18	5	11	9	4	2	4	0	0	1	0	0	0	0	275
1906–7:																																
50-bird pens	14	6	3	2	6	4	3	10	13	11	9	14	14	10	7	13	12	5	8	8	3	6	3	3	0	0	0	0	0	0	0	187
100-bird pens	11	2	3	3	3	1	4	3	5	6	7	11	15	17	14	16	12	11	7	4	11	9	1	5	3	1	0	0	0	0	0	185
150-bird pens	45	7	3	3	1	5	4	5	16	17	13	24	22	15	15	13	19	12	14	11	4	6	4	1	2	0	0	0	0	0	0	281
Total birds	193	44	30	27	34	31	45	54	100	108	129	172	181	151	161	145	151	116	97	97	107	90	60	44	16	11	8	2	3	0	0	2,407

TABLE IX.—*Frequency distribution of egg production for the month of July.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899-1900	7	0	1	0	0	1	0	2	4	4	5	5	7	4	7	7	4	3	1	2	2	2	1	0	0	1	0	0	0	0	0	0	70
1900-1901	8	0	2	0	0	0	2	3	4	3	4	3	0	9	6	4	9	7	6	4	5	0	1	0	4	1	0	0	0	0	0	0	85
1901-2	2	1	0	2	0	1	0	0	0	2	1	2	0	2	5	3	8	2	3	1	4	5	1	2	1	0	0	0	0	0	0	0	48
1902-3	2	0	2	3	1	3	4	2	1	5	6	2	5	9	14	12	9	10	8	8	7	7	8	12	0	2	4	1	0	0	0	0	147
1903-4	28	5	5	3	7	6	7	5	15	14	20	25	32	19	18	7	11	5	5	3	4	3	1	2	1	3	0	0	0	0	0	0	254
1904-5:																																	
50-bird pens	39	6	7	1	7	5	7	6	8	15	15	18	22	26	15	15	11	15	13	10	4	11	1	2	2	1	1	0	0	0	0	0	283
100-bird pens	16	1	2	0	0	1	2	1	3	2	7	5	5	5	5	7	4	5	5	5	3	1	3	2	2	0	0	0	0	0	0	0	92
150-bird pens	36	5	1	2	0	2	3	5	6	5	6	6	8	11	3	1	5	7	5	5	6	1	3	3	4	0	0	1	0	0	0	0	140
1905-6:																																	
50-bird pens	12	2	2	3	4	3	12	14	9	7	15	19	13	17	9	6	8	5	7	3	2	4	1	1	0	0	0	0	0	0	0	0	178
100-bird pens	20	6	6	4	4	7	8	9	12	8	11	13	10	13	9	6	11	4	2	5	4	7	2	0	0	0	0	1	0	0	0	0	182
150-bird pens	47	7	3	2	7	13	11	16	20	13	17	17	21	16	12	11	8	9	6	5	4	5	1	4	0	0	0	0	0	0	0	0	275
1906-7:																																	
50-bird pens	14	6	0	0	3	3	4	4	8	8	13	19	12	13	13	13	8	6	7	12	4	5	5	3	3	1	0	0	0	0	0	0	187
100-bird pens	17	3	2	2	6	6	3	6	4	18	15	11	14	10	11	8	8	8	4	7	4	7	4	1	2	2	1	0	1	0	0	0	185
150-bird pens	49	6	2	5	3	4	2	13	16	21	22	23	15	13	17	19	5	9	9	9	7	1	4	3	1	0	0	1	0	0	0	2	281
Total birds	297	48	35	27	42	55	65	86	110	125	157	168	164	167	144	119	109	95	81	79	60	59	36	35	20	11	6	4	1	0	0	2	2,407

TABLE X.—*Frequency distribution of egg production for the month of August.*

Laying year.	Number of hens laying number of eggs designated in box head.																																Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899–1900	9	1	0	1	1	1	0	3	3	4	2	7	6	7	2	2	4	6	0	1	3	5	0	2	0	0	0	0	0	0	0	0	70
1900–1901	11	2	2	2	1	1	0	4	5	6	2	0	4	4	2	9	5	7	4	5	2	3	0	2	1	0	1	0	0	0	0	0	85
1901–2	4	0	0	1	1	2	0	3	0	2	1	3	2	3	4	2	3	3	1	1	8	2	0	1	0	1	0	0	0	0	0	0	48
1902–3	13	9	0	6	1	1	3	2	5	5	6	8	7	9	6	8	7	7	9	7	6	7	5	4	4	0	2	0	0	0	0	0	147
1903–4	41	6	3	4	2	7	4	6	15	9	24	16	29	15	14	17	5	8	9	5	3	3	4	2	1	2	0	0	0	0	0	0	254
1904–5:																																	
50-bird pens	46	10	6	2	1	5	5	8	14	8	18	18	15	17	17	17	15	17	10	14	9	2	4	2	0	3	0	0	0	0	0	0	283
100-bird pens	15	4	1	2	0	1	1	1	4	2	6	8	8	7	3	4	7	3	2	4	1	4	1	1	2	0	0	0	0	0	0	0	92
150-bird pens	41	4	2	3	1	1	4	6	6	5	7	8	8	11	10	5	5	1	3	4	1	1	0	3	0	0	0	0	0	0	0	0	140
1905–6:																																	
50-bird pens	19	5	3	4	4	7	9	11	12	13	13	20	12	6	7	5	6	8	5	4	2	1	0	2	0	0	0	0	0	0	0	0	178
100-bird pens	29	18	5	7	8	11	16	10	4	7	13	13	7	7	8	3	5	5	3	0	2	0	0	1	0	0	0	0	0	0	0	0	182
150-bird pens	59	22	15	10	11	19	18	7	11	12	16	8	9	12	11	11	6	3	5	4	4	2	0	0	0	0	0	0	0	0	0	0	275
1906–7:																																	
50-bird pens	10	4	2	1	3	5	4	9	8	10	13	13	15	16	17	19	7	4	11	4	3	3	2	4	0	0	0	0	0	0	0	0	187
100-bird pens	14	8	3	4	7	2	3	4	8	5	10	20	10	12	10	13	11	5	8	7	4	3	4	7	1	1	0	0	1	0	0	0	185
150-bird pens	34	16	11	8	12	5	10	8	8	17	20	27	19	16	19	8	8	7	5	7	5	4	1	4	2	0	0	0	0	0	0	0	281
Total birds	345	109	53	55	53	68	77	82	103	105	151	169	151	142	130	123	94	84	75	67	53	40	21	35	11	7	3	0	1	0	0	0	2,407

TABLE XI.—*Frequency distribution of egg production for the month of September.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1899–1900	14	1	2	1	1	1	2	2	6	2	4	6	4	5	4	2	3	1	3	4	0	2	0	0	0	0	0	0	0	0	0	70
1900–1901	14	1	4	2	2	3	4	0	5	2	3	4	6	4	1	7	2	5	2	3	2	6	3	0	0	0	0	0	0	0	0	85
1901–2	13	1	2	1	0	1	1	4	1	1	0	1	3	1	2	2	3	1	2	3	2	1	1	1	0	0	0	0	0	0	0	48
1902–3	24	3	5	2	5	5	6	7	3	5	5	5	6	9	4	6	4	7	9	9	7	7	1	2	0	1	0	0	0	0	0	147
1903–4	43	7	5	3	2	15	7	8	10	7	18	17	20	12	10	17	5	11	6	10	6	8	3	1	3	0	0	0	0	0	0	254
1904–5:																																
50-bird pens	60	8	2	2	3	5	3	8	9	13	17	17	9	14	13	11	10	12	13	8	8	17	10	6	1	3	1	0	0	0	0	283
100-bird pens	22	1	0	0	2	2	4	4	1	5	5	1	3	4	2	7	5	4	3	1	6	1	3	3	2	0	1	0	0	0	0	92
150-bird pens	49	3	2	4	4	3	2	6	4	7	3	7	6	8	7	2	3	6	0	3	4	3	3	1	0	0	0	0	0	0	0	140
1905–6:																																
50-bird pens	23	6	7	4	8	5	8	7	11	10	5	5	11	7	11	8	8	11	7	7	5	1	1	1	0	0	1	0	0	0	0	178
100-bird pens	44	15	8	4	4	7	9	6	6	8	6	11	5	10	7	6	5	4	4	6	2	3	2	0	0	0	0	0	0	0	0	182
150-bird pens	76	22	9	11	8	13	10	12	13	11	9	7	8	15	6	2	11	10	5	8	4	2	2	1	0	0	0	0	0	0	0	275
1906–7:																																
50-bird pens	56	22	11	8	10	11	15	7	11	6	2	9	4	2	2	3	4	2	1	0	1	0	0	0	0	0	0	0	0	0	0	187
100-bird pens	26	16	11	11	4	9	5	10	13	8	10	8	8	4	8	3	4	9	5	3	2	3	5	0	0	0	0	0	0	0	0	185
150-bird pens	90	41	18	11	16	9	15	14	7	4	6	2	6	10	9	3	4	9	1	2	2	1	0	1	0	0	0	0	0	0	0	281
Total birds	554	147	86	64	69	89	91	95	100	89	93	100	99	105	86	79	71	92	61	67	51	55	34	17	6	4	3	0	0	0	0	2,407

TABLE XII.—*Frequency distribution of egg production for the month of October.*

Laying year.	Number of hens laying number of eggs designated in box head.																															Total hens.
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
1899–1900	26	2	4	1	1	3	5	1	4	3	2	0	1	3	4	4	4	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	70
1900–1901	33	7	5	1	6	2	1	4	3	3	2	1	1	2	1	2	0	1	5	1	3	1	0	0	0	0	0	0	0	0	0	0	85
1901–2	28	4	0	2	0	1	2	1	0	3	0	0	1	0	0	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	48
1902–3	67	6	7	6	6	4	3	2	4	4	3	5	3	4	2	0	0	4	2	7	3	1	1	1	2	0	0	0	0	0	0	0	147
1903–4	90	26	13	9	3	6	6	10	7	12	7	15	12	6	6	10	7	5	3	0	1	0	0	0	0	0	0	0	0	0	0	0	254
1904–5:																																	
50-bird pens	110	14	5	7	4	6	3	5	8	6	8	9	10	10	12	8	18	11	13	2	5	5	1	1	1	1	0	0	0	0	0	0	283
100-bird pens	45	3	2	0	4	1	2	2	4	5	2	2	2	0	3	2	1	2	6	3	0	1	0	0	0	0	0	0	0	0	0	0	92
150-bird pens	57	10	4	0	2	3	7	5	7	4	1	7	9	4	8	4	2	3	0	1	0	0	0	0	1	0	0	1	0	0	0	0	140
1905–6:																																	
50-bird pens	70	15	4	5	4	6	3	5	5	6	8	5	4	4	4	9	4	8	5	0	2	0	1	1	0	0	0	0	0	0	0	0	178
100-bird pens	97	8	5	2	4	5	2	4	1	4	3	6	6	6	6	8	9	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	182
150-bird pens	150	19	3	5	7	7	4	2	2	6	4	11	6	13	8	9	10	4	2	3	0	0	0	0	0	0	0	0	0	0	0	0	275
1906–7:																																	
50-bird pens																																	
100-bird pens																																	
150-bird pens																																	
Total birds	773	114	52	38	41	44	38	41	45	56	40	61	55	52	54	57	56	43	38	19	15	9	3	4	4	1	0	1	0	0	0	0	1,754

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U. S. DEPARTMENT OF AGRICULTURE,

BUREAU OF ANIMAL INDUSTRY.—BULLETIN 110, PART III.

A. D. MELVIN, CHIEF OF BUREAU.

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

III. VARIATION AND CORRELATION IN THE PHYSICAL CHARACTERS OF THE EGG.

BY

RAYMOND PEARL, PH. D.,
Biologist, Maine Agricultural Experiment Station,

AND

FRANK M. SURFACE, PH. D.,
Biologist, Maine Agricultural Experiment Station.

WASHINGTON:
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LETTER OF TRANSMITTAL.

U. S. Department of Agriculture,
Bureau of Animal Industry,
Washington, D. C., December 23, 1913.

Sir: I have the honor to transmit herewith, for publication as Part III of Bulletin 110 of this bureau, the third paper in the series under the general title "A Biometrical Study of Egg Production in the Domestic Fowl," this paper being entitled "Variation and Correlation in the Physical Characters of the Egg," by Drs. Raymond Pearl and Frank M. Surface, of the Maine Agricultural Experiment Station.

Respectfully,

A. D. Melvin,
Chief of Bureau.

Hon. D. F. Houston,
Secretary of Agriculture.

CONTENTS.

Page.
Introduction 171
Material and methods 174
Variation of the Barred Plymouth Rock egg in size and shape 177
Nature of the frequency distributions 183
The correlation of the Barred Plymouth Rock egg in respect to size and shape characters 190
Index correlations 198
Comparison of correlation in the egg and the skull 200
Regression and the determination of the probable value of a particular egg dimension from a knowledge of other dimensions 203
Linearity of regression 204
Simple linear regression 205
Parabolic regression 210
- Capacity-length regressions 210
- Capacity-breadth regression 213

Multiple regression 214
- Multiple linear regression 216
- Multiple parabolic regression 217

Tests of regression equations 223
Variation and correlation in the eggs of wild birds as compared with the domestic fowl 225
Variation in the size of eggs in the individual as compared with the race 228
Summary 235
Appendix III 238

ILLUSTRATIONS.

Page.

Fig. 1. Histogram and fitted curve for variation of eggs in length.............. 186

2. Histogram and fitted curve for variation of eggs in breadth............ 186

3. Histogram and fitted curve for variation of eggs in shape, as measured by length-breadth index.. 187

4. Histogram and fitted curve for variation of eggs in bulk.............. 188

5. Histogram and fitted curve for variation of eggs in weight............. 189

6. Regression of egg weight on egg length............................ 207

7. Regression of egg bulk on egg length............................. 207

8. Regression of egg weight on egg breadth........................... 208

9. Regression of egg bulk on egg breadth............................. 208

10. Regression of egg weight on egg bulk............................. 209

11. Regression of egg bulk on egg weight............................. 209

12. Regression of egg weight on egg length........................... 213

13. Regression of egg bulk on egg length............................. 214

14. Showing the regression of weight of eggs on breadth (upper group) and the regression of bulk (volume) on breadth (lower group)............ 215

A BIOMETRICAL STUDY OF EGG PRODUCTION IN THE DOMESTIC FOWL.

III. VARIATION AND CORRELATION IN THE PHYSICAL CHARACTERS OF THE EGG.[1]

INTRODUCTION.

In the previous parts [2] of this bulletin attention has been directed primarily toward the statistical or mass phenomena of egg production. To reach anything approaching an adequate understanding of those phenomena it is essential that the production of an egg be analyzed as a physiological process pertaining to the individual. Such physiological studies [3] have been carried on concurrently with this "Biometrical Study."

The formation and deposition of a bird's egg involve a large series of complex processes. In order to get a better understanding of the exact bearing of this paper on the general problem it may be useful to enumerate a few of the more evident of these processes as follows: (1) The development of the ovary into the laying condition and its effect upon the somatic characters of the bird. (2) The accompanying increase in size and functional activity of the oviduct. (3) The growth of the oocyte and the elaboration of the yolk. (4) The rupture of the ovarian follicle (ovulation). (5) The inclusion of the yolk by the funnel of the oviduct. (6) The movement of the yolk down the oviduct. (7) The secretion of the albumen. (8) The formation of the shell membranes. (9) The secretion of the calcareous shell. (10) The determination of the characteristic shape and color of the shell. (11) The several processes involved in the extrusion of the egg.

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[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 45.

[2] A Biometrical Study of Egg Production in the Domestic Fowl. I. Variation in Annual Egg Production. U. S. Department of Agriculture, Bureau of Animal Industry, Bulletin 110, pp. 1-80, 1909. II. Seasonal Distribution of Egg Production. *Ibid.*, pp. 81-170, 1911.

[3] The results of these studies to date have been published in the following series of papers under the general title "Studies on the Physiology of Reproduction in the Domestic Fowl:"

I. Regulation in the morphogenetic activity of the oviduct. Jour. Exp. Zool., vol. 6, pp. 339-359, 1909.

II. Data on the inheritance of fecundity obtained from the records of egg production of the daughters of "200-egg" hens. Maine Agricultural Experiment Station, Annual Report for 1909, pp. 49-84.

III. A case of incomplete hermaphroditism. Biological Bulletin, vol. 17, pp. 271-286, 1909.

IV. Data on certain factors influencing the fertility and hatching of eggs. Maine Agricultural Experiment Station, Annual Report for 1909, pp. 105-164.

V. Data regarding the physiology of the oviduct. Jour. Exp. Zool., vol. 12, pp. 99-132, 1912.

Very little definite information regarding these various processes is to be found in the existing literature. The problem is a complex one and needs to be attacked from various standpoints. The essential facts regarding the physical characters of the normal hen's egg and the variation and correlation of these characters are fundamental data necessary for the most successful prosecution of experimental work on the problems of egg production. To supply these data is the purpose of the present paper.

We shall in this part of the study deal with the "product" phase of egg production. Quality and character of product are as essential as quantity in egg production, when the matter is looked at from the economic standpoint. In recent years increasing attention has been paid to the study of factors influencing the character and quality of eggs. Foremost in economic importance are the investigations dealing with the effect of the environmental vicissitudes to which the egg is subjected in the process of getting from the hen to the breakfast egg cup. Here may be mentioned the recent studies of Lamon and Opperman.[1] The bacteriology and chemistry of the egg are also receiving attention. In such studies as those of Pennington[2] and Willard and Shaw[3] the factors dealt with as influencing the quality and character of eggs are innate, in that they are determined by the hen in greater or less degree. Finally, however, beyond all these are the essentially innate biological factors which influence the character of eggs. The size, shape, and color of the egg are not determined by the way in which it is handled in getting to market. They are basic biological characters, which are presumably inherited. Also such qualities as texture of shell, thickness of shell, color of yolk and albumen, etc., are characteristics which depend for their precise determination on the physiological economy of the hen. But these qualities are not without economic significance. There are definite, if not as yet altogether precisely defined, market standards with regard to each of these characteristics which have been enumerated. The producer wants to put on the market an egg which will meet these standards. It is important that there be as clear an understanding and complete a knowledge as can be obtained of the underlying biological factors involved. The present study is intended as a contribution in this direction.

This Part III deals with the shape and size of eggs laid by "purebred" domestic fowls. There is good reason to believe that the final shape of an egg is determined in the lower portion of the oviduct known as the uterus or shell gland. The evidence for this conclusion has been stated in a previous paper[1] by one of the writers and need not be repeated in detail here. It was there pointed out that there are two possibilities with regard to the mechanism by which the shape is determined. These are (a) the active contractions of the muscular walls of the uterus and (b) the passive elasticity of these walls, causing them to act as a mold for the plastic egg mass. From a careful study of the successive eggs of an individual hen it was concluded that the finer and individual form characteristics of eggs were determined by muscular contractions before and during the deposition of the calcareous shell. It is only in this way that one can account for the changes undergone by successive eggs of the same individual. Hence the shape of an egg measures in an indirect way the physiological activity of the muscles in one portion of the oviduct.

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[1] Lamon, H. M., and Opperman, C. L. The improvement of the farm egg. U. S. Dept. Agr., Bur. Anim. Ind., Bulletin 141, pp. 1–43, 1911.

[2] Pennington, M. E. A chemical and bacteriological study of fresh eggs. Jour. Biol. Chem., vol. 7, pp. 109–132, 1909.

[3] Willard, J. T., and Shaw, R. H. Analysis of eggs. Kansas Agr. Expt. Sta., Bulletin 159, pp. 143–177, 1909.

The volume and weight—i. e., the size—of an egg are determined both by the size of the yolk elaborated before it leaves the ovary and by the amount of albumen secreted by the oviduct. In studying the variation in the size and volume of eggs we are indirectly studying the variation of certain physiological processes. These processes, connected with the formation of the yolk and albumen, are among the most important factors in egg production. Whatever light may be thrown upon them can not but aid in an ultimate understanding of the physiology of egg formation.

The ordinary concept of an egg is of a structure of fairly definite shape and size. This is reflected in such common expressions as "egg-shaped" and "the size of an egg." Nevertheless every poultryman knows that the size of eggs varies greatly—most markedly between different breeds of poultry. But within the same breed the variation is often scarcely less striking. The limits of this variability may extend, on the one hand, to the very small eggs occasionally found with a small amount of albumen and no yolk, or a very small amount without definite yolk membrane. On the other hand, a hen may occasionally lay an enormous double or triple yolked egg. It is not the purpose of this paper to deal with either these abnormally small or abnormally large eggs.

There is, however, a variation in the physical characters of what may be called normal eggs, as distinguished from such abnormal specimens as have just been mentioned. A "normal" egg may be regarded as one in which a single yolk surrounded by a yolk membrane is inclosed within the usual layers of albumen, shell membrane, and calcareous shell. It is with such normal eggs that the present investigation is concerned.

Among the general topics dealt with in this part of the study the following are the more important. The specific problems which arise in connection with these topics will, in order to avoid repetition, only be stated and discussed in the sections of the paper wherein the data bearing upon them are presented.

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[1] Pearl, R. Regulation in the morphogenetic activity of the oviduct. Jour. Exp. Zool., vol. 6, pp. 339-359, 1909.

1. The mean size and variability of the various physical characters of eggs from different individuals.
2. The nature of the frequency distributions to which measurements of these characters lead.
3. The correlation between the several physical characters of the egg.
4. The regression of the several characters upon one another, and the characteristic equations deduced from the regression constants, together with the use of such characteristic equations in predicting the probable condition of one character of the egg from a knowledge of the actual condition of other characters.
5. Comparison of the eggs of the domestic fowl with those of wild species of birds.
6. The degree of variation (or uniformity) exhibited in the eggs laid by an individual bird, and the relation of the variation in the product of an individual to variation in the eggs of the race (flock or strain).

MATERIAL AND METHODS.

The material upon which this paper is based consists of several series of egg measurements. These were obtained from eggs laid at the poultry plant of the Maine Agricultural Experiment Station during the years 1908 and 1909. Except when a specific statement to the contrary is made, all discussions in this paper refer to the data from the eggs of Barred Plymouth Rock pullets. The strain of Barred Plymouth Rocks from which these eggs came has been bred at the Maine Station since 1898.[1] Previous to that time they had been bred for a number of years by the late Prof. G. M. Gowell. They are pure-bred birds in the fancier's sense. The birds have been fed and cared for under conditions as nearly uniform as could be obtained. Hence the material with which we have to do is as nearly homogeneous as it is practically possible to obtain. This paper is based upon the measurement of about 5,500 eggs. It is believed that there are sufficient data in the various series to establish with considerable accuracy the constants derived from them for normal eggs of Barred Plymouth Rock pullets.

In each of the series of data here considered the length and maximum breadth of each egg were measured. From these two measurements there were calculated in most cases (*a*) the length-breadth index and (*b*) the bulk or volume of the egg. In addition to these data the weight of the fresh egg was obtained in series A.

The length and breadth of the eggs were measured with the aid of a large micrometer caliper provided with a ratchet head. This caliper was held in a wooden frame so to avoid the expansion and contraction due to the varying conditions of temperature which would be induced by handling. The ratchet head insures that the same pressure is exerted on each egg by the caliper jaws when the reading is taken. The measurements were recorded to hundredths of a millimeter. In cases where an egg was not perfectly symmetrical the largest diameter was taken. It not infrequently happened in the measurement of the breadth that one diameter was found to be larger than the others. In such cases the egg was turned until this diameter was found and its magnitude recorded.

---

[1] For a discussion of the breeding of this flock see Part I of this study.

The length-breadth index was obtained by dividing one hundred times the breadth by the length. This constant gives a measure of the shape of the egg. A long and narrow egg will have a relatively low index, while a short and broad egg will have a high index. Its value is independent of the absolute size of the egg except in so far as the length is correlated with the breadth. This correlation is very small.

In determining the volume or bulk, as it will be called here, the egg was regarded as a perfect prolate spheroid, and the formula for obtaining the volume of such a body was employed. This formula is:

$$V = \frac{4}{3}\pi a^2 b$$

where V denotes volume, and a and b are the semi-axes, a being the shorter. As actually used on eggs this formula becomes:

$$V_{egg} = 4.1888\ (\tfrac{1}{2}\ \text{breadth})^2 \times (\tfrac{1}{2}\ \text{length}).$$

The bulk gives us a measure which should bear a definite relation to the weight if the specific gravity of an egg is constant. In calculating both the index and bulk, the length and breadth measurements were used to the second decimal place. All of the weights used in this paper were obtained on balances sensitive to tenths of a milligram and the weights recorded to the nearest milligram.

In series A a determination of the color of each egg was made. As is well known, Barred Plymouth Rocks lay tinted eggs. The colors are, however, so delicate that their measurement offers special difficulties. After studying the matter carefully the following procedure was decided upon: To form a series of egg shells which should represent as nearly as possible equidistant steps in a graded series leading from the lightest egg anywhere to be found, on one end, to the darkest eggs which occur, on the other; then, having obtained such a "normal" series, to compare with the series each egg whose color it was desired to record, and note (by number) the "normal" egg with which it agreed in color, or the two members of the series between which it fell in case it exactly agreed with no single one. In attempting to follow this procedure in practice the first difficulty encountered was that more than one color series was found to exist in Barred Plymouth Rock eggs. Tinted hens' eggs are usually called "brown" eggs. As a matter of fact many of them are not brown at all but a delicate shade of pink. After handling hundreds of eggs with this point in mind we found it necessary to form three normal color scales instead of one as originally contemplated. These three series were designated in our records respectively as the "brown," "brown-pink," and "pink" series of comparison egg shells. The brown series included all eggs which showed no trace whatever of a reddish or pink pigment. The pink series included only eggs which showed no trace of a brown color. Finally the "brown-pink" included eggs intermediate in color, showing evidence of both the brown and the red pigments. From 10 to 14 eggs were included in each of these three normal series and numbered from 1 (the lightest egg in the series) up to whatever the number of the final (darkest) egg in the series might be. Each individual egg in series A was then compared (always by the same person, R. P.) with these series, and recorded. Discussion of these color measurements is deferred for the present, pending the collection of further and more adequate data.

The following is a description of the series of measurements which furnish the chief data for this paper:

Series A.—This consists of the determination of the length, breadth, and weight, the calculated index and bulk, and the color of 450 eggs. These eggs were all laid on the same day (Feb. 12, 1908), and consequently no two eggs were laid by the same hen. Each egg measured is from a different hen. They give the fundamental data on *inter-individual* or racial variation in Barred Plymouth Rock eggs. All the eggs were from pullets of approximately the same age, having been hatched in April and May, 1907. The detailed measurements for this series are given in Appendix III, Table I.

Series B.—This series contains the data for *intra-individual* variations in eggs. It consists of the measurements of the length and breadth of 25 eggs from each of 100 birds. The measurements were made during April and May, 1908. These eggs were likewise all obtained from pullets approximately 1 year old. The eggs were taken from those pullets which were in the breeding pens and the eggs measured before being put into the incubators. No selection with regard to egg characters of the pullets in the breeding pens was practiced in choosing the birds whose eggs were to be measured in this series. There is no reason to doubt that these 100 hens represent a random sample of the flock in respect to egg characters.

The statistical data obtained in the manner described were subjected to biometrical analysis. For a description of the technical methods used in this field of work the reader is referred to the original papers by Pearson and his associates in the Philosophical Transactions of the Royal Society and in Biometrika.[1] A useful condensed account of many of the methods is given by Elderton.[2]

Great care has been exercised in performing the various calculations. As many as possible of the fundamental constants have been worked twice independently. It is hoped that the paper is as free of arithmetical errors as is possible in so extensive a piece of work.

VARIATION OF THE BARRED PLYMOUTH ROCK EGG IN SIZE AND SHAPE.

The fundamental data for this section of the paper are contained in series A. It will be recalled that this series consists of the measurement of 450 eggs from as many different pullets. During the year in which these measurements were made there were 850 pullets in the houses. Every egg laid on February 12, 1908, with the exception of a few cracked ones, was used in this series. Consequently these 450 eggs constitute a random sample of the flock. The frequency distributions for size and shape of eggs are found in the correlation tables on pages 191–194. In order to present the variation data in a more compact form Table 1 has been prepared.

TABLE 1.—*Frequency distributions of variation in the size and shape of eggs.*

Length.		Breadth.		Shape index.		Bulk.		Weight.	
Millimeters.	Frequency.	Millimeters.	Frequency.	Per cent.	Frequency.	Cubic centimeters.	Frequency.	Grams.	Frequency.
49.0–49.9	1	38.0–38.4	2	60.0–60.9	1	40.0–40.9	1	42.0–42.9	1
50.0–50.9	1	38.5–38.9	4	61.0–61.9	1	41.0–41.9	1	43.0–43.9	1
51.0–51.9	7	39.0–39.4	9	62.0–62.9	2	42.0–42.9	2	44.0–44.9	0
52.0–52.9	22	39.5–39.9	18	63.0–63.9	1	43.0–43.9	6	45.0–45.9	3
53.0–53.9	36	40.0–40.4	41	64.0–64.9	3	44.0–44.9	5	46.0–46.9	6
54.0–54.9	71	40.5–40.9	52	65.0–65.9	3	45.0–45.9	15	47.0–47.9	10
55.0–55.9	68	41.0–41.4	41	66.0–66.9	5	46.0–46.9	25	48.0–48.9	17
56.0–56.9	77	41.5–41.9	65	67.0–67.9	6	47.0–47.9	28	49.0–49.9	20
57.0–57.9	78	42.0–42.4	73	68.0–68.9	7	48.0–48.9	26	50.0–50.9	20
58.0–58.9	35	42.5–42.9	48	69.0–69.9	13	49.0–49.9	50	51.0–51.9	29
59.0–59.9	29	43.0–43.4	41	70.0–70.9	34	50.0–50.9	36	52.0–52.9	34
60.0–60.9	10	43.5–43.9	26	71.0–71.9	27	51.0–51.9	47	53.0–53.9	42
61.0–61.9	4	44.0–44.4	15	72.0–72.9	46	52.0–52.9	43	54.0–54.9	42
62.0–62.9	3	44.5–44.9	7	73.0–73.9	41	53.0–53.9	35	55.0–55.9	38
63.0–63.9	6	45.0–45.4	5	74.0–74.9	47	54.0–54.9	32	56.0–56.9	47
64.0–64.9	1	45.5–45.9	2	75.0–75.9	50	55.0–55.9	28	57.0–57.9	24
65.0–65.9	0	46.0–46.4	1	76.0–76.9	55	56.0–56.9	16	58.0–58.9	25
66.0–66.9	0			77.0–77.9	33	57.0–57.9	20	59.0–59.9	21
67.0–67.9	1			78.0–78.9	22	58.0–58.9	11	60.0–60.9	19
				79.0–79.9	27	59.0–59.9	5	61.0–61.9	18
				80.0–80.9	12	60.0–60.9	5	62.0–62.9	13
				81.0–81.9	5	61.0–61.9	3	63.0–63.9	1
				82.0–82.9	5	62.0–62.9	2	64.0–64.9	5
				83.0–83.9	2	63.0–63.9	5	65.0–65.9	3
				84.0–84.9	1	64.0–64.9	3	66.0–66.9	4
				85.0–85.9	1			67.0–67.9	2
								68.0–68.9	3
								69.0–69.9	2
Total....	450		450		450		450		450

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[1] For detailed references see Part I of this study, p. 30.

[2] Elderton, W. P. Frequency Curves and Correlation, London, 1907, pp. xiii +172

The units of grouping of the several measurements are as follows: Length, 1 mm.; breadth, 0.5 mm.; index, 1 per cent; bulk, 1 c. c.; weight, 1 gram. With slightly larger groupings the distributions would be somewhat smoother. For the analytical treatment (pp. 183–189) the grouping of the index, bulk, and weight has been doubled, but for present purposes, and for the determination of the correlation coefficients, the smaller grouping gives more accurate results. The chief constants of variation deduced from the above distributions are given in Table 2.

TABLE 2. *Constants of variation in size and shape of Barred Plymouth Rock eggs.*

Character.	Mean.	Standard deviation.	Coefficient of variation.
Length .... millimeters..	56.322±0.076	2.386±0.054	4.237±0.095
Breadth .... do....	41.917± .044	1.378± .031	3.287± .074
Index .... per cent..	74.522± .120	3.788± .085	5.053± .115
Bulk .... cubic centimeters..	51.880± .135	4.233± .095	8.160± .181
Weight .... grams..	55.264± .147	4.622± .103	8.364± .189

From the above tables the following points may be noted:

1. The means for the various characters may be considered to give close approximations to the true values for normal eggs of Barred Plymouth Rock pullets. It is of interest in this connection to compare the values of Table 2 with those given by Willard and Shaw in Bulletin 159 of the Kansas Agricultural Experiment Station. This bulletin, while dealing chiefly with the chemical analysis of eggs, gives, among other things, the length and breadth measurements of a series of eggs from different breeds of poultry. The mean length and breadth of Plymouth Rock eggs as given on page 171 of the bulletin referred to are as follows:

Mean length = 56.15 mm.

Mean breadth = 41.53 mm.

These results are based upon the measurement of the eggs from only 6 different hens, and no statement of the age of these birds is given. It will be seen that considering the small numbers involved these results correspond very closely to the values given in Table 2.

2. Since the means of all the characters tabled do not differ greatly among themselves in absolute numerical value, it follows that the absolute variation, as measured by the standard deviation, is very nearly proportional to the relative variability as measured by the coefficient of variation. Consequently in comparing the several characters in respect to variability it makes little difference which measure one uses. On general grounds it is better to measure variability in proportion to the size of the thing varying. Consequently the discussion which follows will be chiefly based on the coefficients of variation.

3. The variation of the egg in length and breadth, both absolutely and relatively, is quite small. Variabilities of only 3 to 8 per cent

indicate that the distributions are closely grouped about their means. This is evident from inspection of the data of Table 1.

4. The egg is relatively more variable in length than in breadth. From the values given in Table 2 it appears that the difference between the coefficient of variation for the length and that for the breadth of the egg is 0.957 ± 0.121.[1] The difference is thus approximately eight times its probable error and certainly significant. This result, that the egg is more variable in respect to its length than in respect to its breadth, is only what might be expected in view of the factors concerned in the shaping of the egg. If a plastic body generally oval in shape gets its final definite form in the lumen of a tube it is obvious that there will be a greater opportunity for variation in its longitudinal dimensions than in any of its lateral dimensions. The degree of diversity which can occur in respect to lateral dimensions, or breadth of the egg, is primarily conditioned by the degree of tonus of the muscular layers of the wall of the oviduct. It is reasonable to suppose that in normal, healthy birds this will be a relatively constant factor.

As the absolute difference here is small, a better idea can be gained of the actual magnitude of the amount by which the egg is more variable in length than in breadth by a relative or percentage comparison. Taking coefficients of variation as the basis of comparison, it appears that length is more variable than breadth by an amount which is 29.1 per cent of the variability of the breadth and 22.6 per cent of the variability of the length. In other words, it may be said that the egg is roughly a fourth more variable in length than it is in breadth.

5. The index is considerably more variable than either the length or breadth. Since the index is a mathematical function of the length and breadth it is possible to determine its mean and standard deviation from a knowledge of the means and coefficients of variation of the length and breadth. It was shown by Pearson[2] that for indices of the form $z=\frac{x_1}{x_2}$ the mean of z is $\frac{m_1}{m_2}(1+v^2_2-r_{1,2}v_1v_2)$, and that the standard deviation of z is $\frac{m_1}{m_2}\sqrt{v^2_1+v^2_2-2r_{1,2},v_1v_2}$, where m_1 and m_2 are the means of x_1 and x_2; v_1 and v_2 are the coefficients of variation *divided by 100*, and $r_{1,2}$ denotes the coefficient of correlation between x_1 and x_2. Substituting the proper values from Table 2 in these formulæ and evaluating, we have

Calculated mean index = 74.548
Calculated standard deviation of index = 3.833

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[1] This probable error is calculated by the ordinary formula for the probable error of a difference.

[2] Pearson, K. On a Form of Spurious Correlation which may arise when Indices are used in the Measurement of Organs. Proc. Roy. Soc., vol. 60, pp. 489–498, 1897.

These values each differ by less than 0.05 from the values determined directly from the observations in Table 2. This difference is well within the probable error of each constant. Considering the grouping of the data, these formulæ give excellent values, and furnish an additional check on the accuracy of the arithmetic involved in the calculation of the constants of Table 2.

6. The bulk and weight are much more variable than any of the other characters discussed. The bulk appears to be slightly the less variable of the two, both absolutely and relatively, although the difference between the coefficients of variation there is only about 0.2, and is probably not significant. The bulk and weight of the egg are almost twice as variable as the length and more than two and one-half times as variable as the breadth.

It is of interest to note that the variability of these egg characters is of about the same degree as that exhibited by similar characters of the human skull. Table 3 shows a few coefficients of variation for skull characters.

TABLE 3.—*Variation in certain skull measurements in man.*

Character.	Coefficient of variation.
Skull:	
Length (English)[1]	3.31
Length of base[1]	4.07
Greatest breadth[1]	3.75
Least breadth of forehead[1]	4.29
Height[1]	4.21
Cross circumference[1]	3.70
Ratio of height to greatest length[1]	3.80
Ratio of height to horizontal length[1]	4.61
Height to breadth index[1]	4.86
Breadth to height index[1]	4.83
Capacity[1]	8.28
Brain weight (Bavarian)[2]	8.12

[1] Macdonell, W. R. Biometrika, Vol. III, p. 222.

[2] Pearl, R. Biometrika, Vol. IV, p. 30. The Bavarian weights are given here because they represent about the average of several series from different nationalities.

From this table it will be noted that measurements of length, breadth, height, and circumference of the skull are subject to a variation of the same order of magnitude as egg length and breadth, namely, about 3 and 4 per cent. The skull indices exhibit a slightly smaller variation than those of the egg, averaging about 4.5 per cent. This is about 0.5 lower than the coefficient of variation for the egg index. The more important point is that in both cases the indices are more variable than the measurements upon which they are based. The difference between the coefficients of variation for index and length of eggs is less than the difference between the coefficients of length and breadth. The actual differences with the probable errors are:

$$\text{C. V. of index} - \text{C. V. of length} = 0.846 \pm 0.149$$

$$\text{C. V. of length} - \text{C. V. of breadth} = 0.857 \pm 0.121$$

When we consider measurements of weight and capacity, we find a still more marked increase in the variability in both eggs and skulls. In the case of skull capacity and brain weight the coefficient ranges about 8 per cent. This is practically the same value as that found for egg bulk and weight.

This close agreement between the egg of the domestic fowl and the human skull in respect to variation in size and shape is somewhat remarkable. At first thought it might be considered to be entirely fortuitous, but further consideration of the matter suggests the possibility that this is not the case; but that, on the contrary, there is a real and definite analogy between the two things, when considered from the standpoint of developmental mechanics. When we measure the shape, size, and capacities of either egg or skull, we are dealing with bodies having at least the following points in common:

1. Hollow structure, with hard, firm wall. In its development this structure passes through stages in which the wall is to some degree plastic and capable of being molded into different forms.

2. Both are normally completely filled at the time when their definite shape is being fixed with a soft, semifluid substance which is increasing in volume either by growth or by absorption of fluid. In other words, in both cases there is exerted on the outer walls some degree of pressure from within while this wall is in a plastic stage.

3. In both cases muscular pressure is brought to bear on the wall from the outside during the period when it is in a plastic condition and while its definitive shape is being fixed. In the one case (the skull) this pressure is exerted by muscles (temporal and masseter muscles chiefly) which have their attachment upon the outside of the wall itself. In the other case (the egg) the external pressure comes from the intrinsic muscle layers of the walls of the uterus or "shell gland."

The size and shape of an egg are determined by a complex of what may be spoken of as intrinsic and extrinsic factors. On the one hand in influencing size we have as a primary intrinsic factor the size of the yolk (determined by the deposition of yolk before the ovum is discharged from the ovary). On the other hand as extrinsic factors may be distinguished (*a*) the amount of albumen secreted, (*b*) the tonic condition of the muscular walls of the oviduct in determining the manner in which the albumen shall be deposited about the yolk, (*c*) the muscular activity of the walls of the uterus in molding the shell as it is secreted about the albumen and its inclosing shell membrane. So far as concerns the actual determination of the size and shape of the shell, the secretion of albumen should be regarded as an intrinsic factor. Observations[1] made in this laboratory show that after the shell membrane has been secreted around the albumen by the glands in the isthmus, the egg increases in weight by a considerable amount. This means that pressure is being exerted upon the formed membrane from within. The observations referred to show further that the increase in weight is due to the passage of fluids into the egg by osmosis. So then, when the shell is being formed and is receiving its definite shape we may think of something like the following processes occurring: On the one hand, the shell membrane is being kept tense and in places where it is least resistant is being bulged out by the pressure from within. On the other hand, the muscular walls of the uterus are pressing upon this membrane and forming shell, from without, and molding it into the definite shape which becomes fixed as the shell hardens.

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[1] Pearl, R., and Curtis, M. R. Studies on the Physiology of Reproduction in the Domestic Fowl. V. Data Regarding the Physiology of the Oviduct. Jour. Exp. Zool., vol. 12, pp. 99-132, 1912.

If we may suppose that a set of processes essentially similar in their mechanical action and results to those which have been described as determining the shape of the egg are operative in the determination of the definitive form of the skull we then have a definite basis for the observed close similarity in variation of these two structures. That these two factors of internal pressure (from the growing brain) and the external molding action of the temporal muscles are of primary importance in the developmental mechanics of the skull is a view which has been definitely advocated by certain anthropologists.

Thomson [1] had this to say in regard to the matter (*loc. cit.*, p. 682):

> For purposes of convenience, it may be well to group the influences at work in the determination of cranial form under two heads:
>
> (*a*) Those operative from within.
> (*b*) Those operative from without.
>
> The former includes such factors as volume of brain, growth of sutures, and ossification of bones, compensation as to balance of head, heredity and breeding. The latter embraces muscular influence dependent upon jaw development more or less intimately related to the size of the teeth and consequently associated with diet. In this way environment plays a part in the modification of man's cranial form in the same manner as it is productive of the variety of shapes met with in other types.

Thomson demonstrates by means of models how the primary factors of internal pressure and muscular action operate to influence the form of the skull.

To summarize this discussion, it may be said that in the first place it is a fact that in respect to the variability of their chief dimensions—length, breadth, length-breadth index, capacity, and weight of contents—the egg of the domestic fowl and the human skull show a very close similarity.[2]

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[1] Thomson, A. Cranial Form in Man, together with some Remarks on the Attitude of the Profession toward Anthropology. Medical Record, vol. 3, pp. 681-687, 1903.

[2] The human skull is taken for illustration in this discussion simply because it has been more thoroughly studied from a biometric standpoint than the skull of any other higher vertebrate. The work of Hatai (Amer. Jour. Anat., vol. VIII, p. 432) on the skull of the white rat indicates that the general features regarding variation and correlation in skull characters are probably similar in all of the mammals.

A consideration of the processes concerned in the production of the definitive size and shape of both of these structures suggests, though it does not prove, that this similarity with reference to variation constants is not fortuitous but rests upon a basis of general similarity in regard to mechanical factors operative in both cases during the development and fixation of the final form.

NATURE OF THE FREQUENCY DISTRIBUTIONS.

Having presented the general facts regarding variation in egg size and shape we may next turn to the analytical discussion of these frequency distributions. In the mathematical analysis the methods of Pearson have been used. A detailed analysis has been carried out and fitted curves obtained for the distributions of length, breadth, index, bulk, and weight of the 450 eggs of series A. Both for convenience in calculation and to secure somewhat smoother distributions, the data with regard to index, bulk, and weight were grouped in larger units than in Table 1. Table 4 gives the distributions as they were used for purposes of graduation.

TABLE 4.—*Frequency distributions used in the analytical discussion.*

[Data from Series A.]

Length.[1]		Breadth.[1]		Index.		Bulk.		Weight.	
Millimeters.	Frequency.	Millimeters.	Frequency.	Per cent.	Frequency.	Cubic centimeters.	Frequency.	Grams.	Frequency.
49.5	1	38.25	2	61.0	2	41.0	2	42.0	1
50.5	1	38.75	4	63.0	3	43.0	8	44.0	1
51.5	7	39.25	9	65.0	6	45.0	20	46.0	9
52.5	22	39.75	18	67.0	11	47.0	53	48.0	27
53.5	36	40.25	41	69.0	20	49.0	76	50.0	40
54.5	71	40.75	52	71.0	61	51.0	83	52.0	63
55.5	68	41.25	41	73.0	87	53.0	78	54.0	84
56.5	77	41.75	65	75.0	97	55.0	60	56.0	85
57.5	78	42.25	73	77.0	88	57.0	36	58.0	49
58.5	35	42.75	48	79.0	49	59.0	16	60.0	40
59.5	29	43.25	41	81.0	17	61.0	8	62.0	31
60.5	10	43.75	26	83.0	7	63.0	7	64.0	6
61.5	4	44.25	15	85.0	2	65.0	3	66.0	7
62.5	3	44.75	7	....	....	....	....	68.0	5
63.5	6	45.25	5	....	....	....	....	70.0	2
64.5	1	45.75	2	....	....	....	....	....	....
65.5	0	46.25	1	....	....	....	....	....	....
66.5	0	....	....	....	....	....	....	....	....
67.5	1	....	....	....	....	....	....	....	....

[1] The distributions for length and breadth are the same as in Table 1. They are repeated here for convenience.

The units of grouping for the various characters are as follows: Length, 1 mm.; breadth, 0.5 mm.; index, 2 per cent; bulk, 2 c. c.; weight, 2 grams. In Table 5 are set forth the analytical constants for the distributions of Table 4. The moments are given in terms of the units of grouping stated above. The values for the mean, mode, standard deviation, and y_0 are in terms of the units of measurements.

TABLE 5.—*Analytical constants of variation in egg size and shape.*

[Data from Series A.]

Constant.	Length.	Breadth.	Index.	Bulk.	Weight.
	Mm.	*Mm.*	*Per cent.*	*C. c.*	*Grams.*
Mean	56.3222	41.9167	[1] 74.5556	[1] 51.8400	[1] 55.2400
Standard deviation	2.3862	1.3777	[1] 3.7503	[1] 4.2438	[1] 4.5923
Coefficient of variation	4.2367	3.2868	[1] 5.0302	[1] 8.1964	[1] 8.3133
μ_2	+ 5.6940	+ 7.5922	+ 3.5162	+ 4.5025	+ 5.2723
μ_3	+ 8.3979	+ 2.7141	+ 2.6249	+ 3.6878	+ 4.2776
μ_4	+141.7823	+166.1906	+46.4494	+64.8049	+90.1964
β_1	0.3820	0.0168	0.1585	0.1490	0.1249
β_2	4.3732	2.8832	3.7561	3.1967	3.2448
$\sqrt{\beta_1}$	0.6181	0.1297	0.3981	0.3860	0.3533
β_2-3	+ 1.3732	− 0.116842	+ 0.7561	+ 0.1967	+0.2448
κ_1	+ 1.6002	− 0.2842	+ 1.0367	− 0.0536	+0.1151
κ_2	+ 0.1985	− 0.0446	+ 0.1199	− 2.1636	+0.8388
Skewness	+ 0.2155	....	− 0.1523	+ 0.1964	+0.1704
Modal divergence	+ 0.5142	....	− 0.5712	+ 0.8335	+0.7825
Mode	55.8080	....	75.1268	51.0065	54.4575
Type of curve	IV	II	IV	I	V
+ end of curve	+∞	51.5951	+∞	260.6816	....
− end of curve	−∞	32.2383	−∞	32.0609	2.8514
y_0	26.345	64.157	41.887	90.472	[2] ......

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[1] These values for the mean, standard deviation, and coefficient of variation for the index, bulk, and weight were calculated from the distributions in Table 4. Since the units of grouping in this case are twice as large as for the same characters in Table 1, the values differ slightly from those previously given. The differences, however, are insignificant in comparison with their probable errors as given in Table 2.

[2] In Type V curves y_0 very often has a very large value. In this case the logarithm of y_0=247.66959.

The first thing that is noted from this table is that there is no uniformity in regard to the type of curve demanded by the several characters under discussion. The criterion ($\kappa_1=2\beta_2-3\beta_1-6$) is negative in only two cases, viz, the breadth and bulk. This is the condition for either a Type I or a Type II curve. In regard to the breadth β_1 does not differ significantly from zero, and hence the curve is symmetrical. It is accordingly fitted with a Type II curve. In the bulk distribution, however, β_1 is significantly greater than zero and hence demands a curve of Type I.

In the other distributions κ_1 is positive. This suggests that the distributions belong to Pearson's Type IV curve, but in order to determine this it is necessary to know in addition the value of

$$\kappa_2=\frac{\beta_1(\beta_2+3)^2}{4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)}$$

To fulfill the condition of a Type IV curve the value of κ_2 must be >0 and <1. Strictly speaking this is true for all three of the remaining distributions and accordingly they could be fitted with Type IV. This has been done in the case of the length and index. In regard to the weight, however, the value of κ_2 ($=+0.84$) so nearly approaches unity that having regard to the probable errors of the β's it may be looked upon as not significantly <1. We shall thus very likely get a better fit by using the transition curve of Type V.

The equations of the theoretical curves are as follows:

Length:

$$y=\frac{26.2446}{\left\{1+\left(\frac{x}{6.8278}\right)\right\}^{6.6076}}\cdot e^{5.5811\tan^{-1}x/6.8278}$$

Breadth:

$$y = 64.157\left\{1 - \frac{x^2}{374.6880}\right\}^{23.1757}$$

Index:

$$y = \frac{41.887}{\left\{1 + \left(\frac{x}{6.5901}\right)^2\right\}^{8.5172}} \cdot e^{5.5194 \tan^{-1} x/6.5901}$$

Bulk:

$$y = 90.472\left(1 + \frac{x}{9.4728}\right)^{19.8392}\left(1 - \frac{x}{104.8375}\right)^{208.4977}$$

Weight:[1]

$$\text{Log } y = 247.6695959 + 133.1410 \log\left(\frac{1}{x}\right) + \overline{1492}.1386\frac{1}{x}$$

The histograms and their fitted curves are given in figures 1–5. It is evident that the graduations are as good as could be expected for the distributions.

From Tables 4 and 5 and figures 1–5 we note the following points:

1. The skewness is positive for all characters except the index. In each case the value of the constant is numerically small, but with the exception of the breadth it is significant. The probable error of the skewness on the assumption that the distribution follows the normal curve of errors is given by the formula

$$P.\ E._{sk} = 0.67449\sqrt{\frac{3}{2N}}$$

In these distributions N is 450 and we have

$$P.\ E._{sk} = \pm 0.0389,$$

for all distributions.

The skewness of the breadth distribution is less than twice the probable error and consequently the distribution does not deviate sensibly from the normal curve. Of the remaining distributions the skewness for the index is the smallest. This, however, is more than five times the probable error, and the distribution is certainly significantly asymmetrical. The length distribution shows the greatest skewness and the breadth the smallest.

2. The distance from the mean to the mode ($d = Sk.\sigma$) is again significant in all distributions except that for the breadth. For the normal distribution the probable error of $d = P.\ E._{sk} \times \sigma$. For this breadth distribution it is equal to ±0.150. The values of d for the bulk and weight distributions approach each other rather closely and are the largest of the five values.

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[1] For obvious reasons this curve is put in the logarithmic form. The equation to a curve of Type V is $y = y_0 x^{-p} e^{-\gamma/x}$.

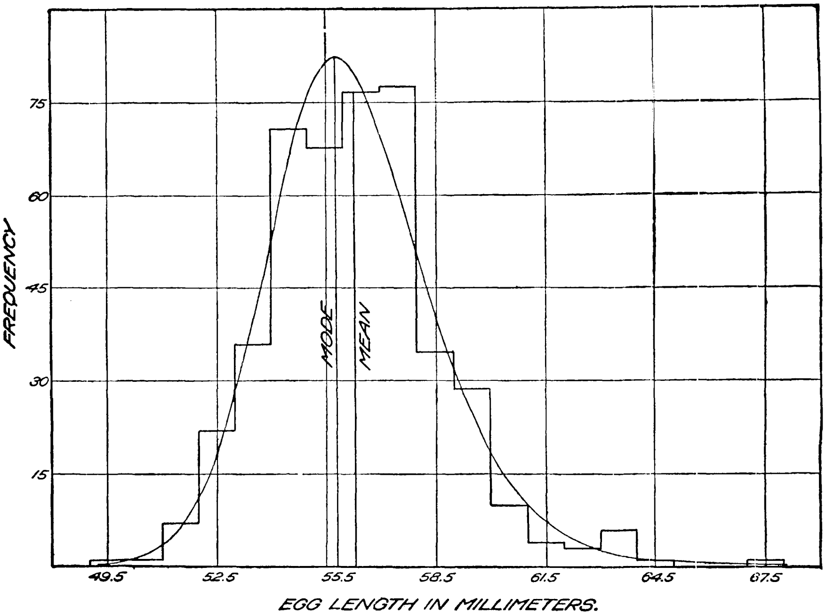

FIG. 1.—Histogram and fitted curve for variation of eggs in length.

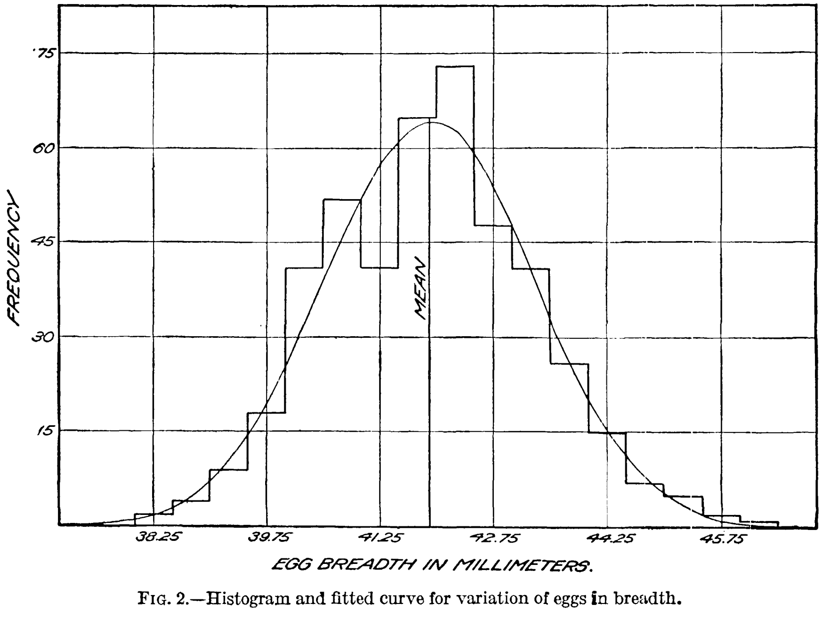

FIG. 2.—Histogram and fitted curve for variation of eggs in breadth.

3. The kurtosis ($\eta = \beta_2 - 3$) is positive or greater than zero in all the distributions except that for the breadth. Since the

$$P.\ E._{\beta_2} = 4\ P.\ E._{sk} = \pm 0.1556$$

the kurtosis of the breadth, bulk, and weight does not differ significantly from zero. These curves are thus mesokurtic. From the graphs (figs. 1–5) it is evident that the curves for bulk and weight are rather more peaked than the mesokurtic condition. On the other hand the kurtosis of the curves for length and index is significantly positive, thus indicating a leptokurtic or peaked curve. This is shown in figures 1 and 3.

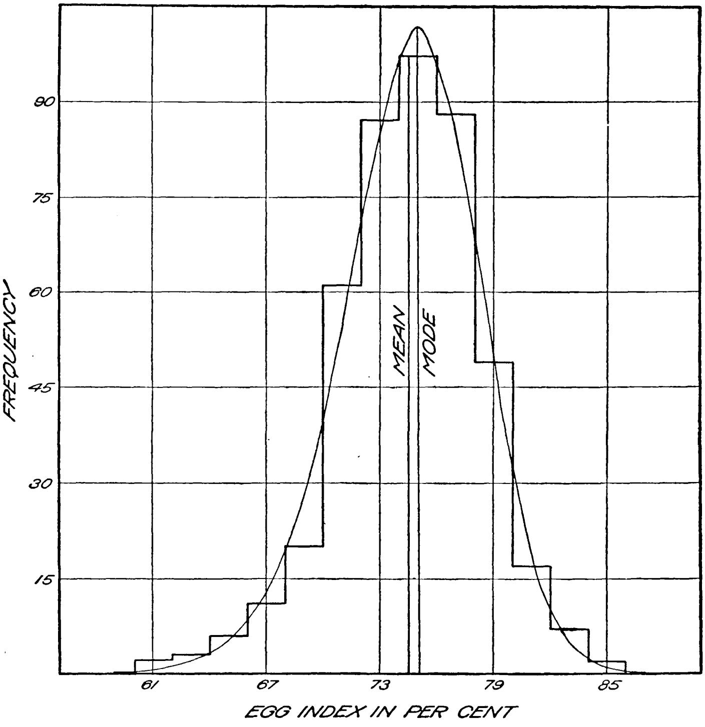

FIG. 3.—Histogram and fitted curve for variation of eggs in shape, as measured by length-breadth index.

4. In only three cases are the theoretical curves of limited range. In the curve for variation in breadth the estimation of both ends of the range is very good. This is seen if the values given for the range ends are compared with the observed measurements of extremely large and small eggs. In the direction of large size the cases of triple-yolked eggs are to be considered. The measurements given by Akers [1] for such a specimen lead to a breadth of about 52 mm. Small eggs with a diameter of 30–35 mm. are not uncommonly found. In

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[1] Akers, L. C. A Three-yolked Egg. Farm Poultry, vol. 16, p. 558, 1905.

the case of the curve for variation in bulk the lower range end is a very fair estimation. The upper range, however, is considerably overestimated. A volume of 260 c. c. would certainly mean an exceptionally large egg, and perhaps a physically impossible value. A triple-yolked egg studied in this laboratory had a volume of 79.1 c. c.[1] From the weight of Akers' specimen (4½ oz. = 127.6 gms.) we would estimate the bulk to be about 120 c. c., on the basis of correlation data discussed farther on in this bulletin. Since this value seems to approach the limit of observed eggs, it is hardly likely that an egg

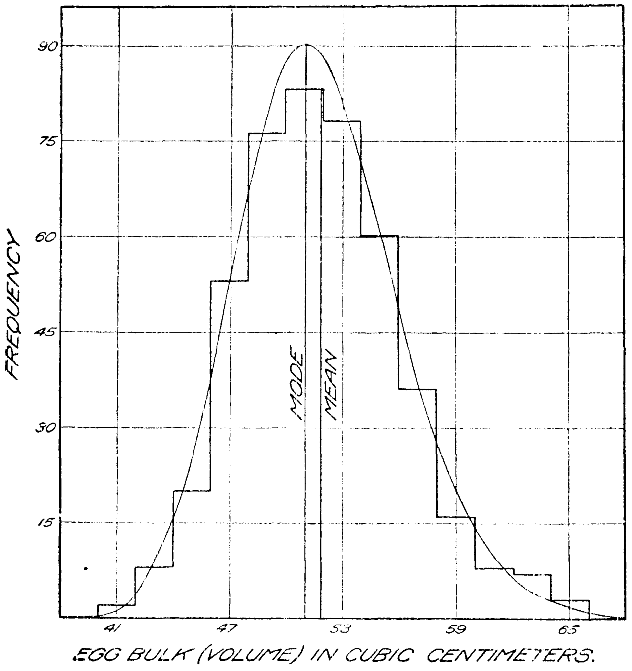

FIG. 4.—Histogram and fitted curve for variation of eggs in bulk.

with a volume of over 200 c. c. would be found among hen's eggs. In this connection it must be noted that the frequency above say 70 c. c. is very small and practically negligible. Thus the expected absolute frequency of eggs with a bulk of 80 c. c. is, according to the theoretical curve, 0.000118, or the expectation is that one would get 3 such eggs out of every 10,000,000 examined. The expectation of getting eggs with a bulk of 100 c. c. is about 5 in every 100,000,000,000,000. These two values taken as examples roughly correspond to the average bulks for double and triple-yolked eggs respectively. It is evident that the theoretical curve underestimates the frequency of such eggs.

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[1] Pearl, R. A Triple-yolked Egg. Zool. Anz. Bd. XXXV, pp. 417-432, 1910.

Valenciennes[1] stated that marketmen in Paris find about 3 double-yolked eggs in every 14,000,000 eggs handled. Market data on such a question, however, can not be a very reliable indication of the actual frequency with which double-yolked eggs occur, for most double-yolked eggs are never sent to market. It is more likely that the actual frequency of double-yolked eggs is nearer 1 to every 5,000 or 6,000 eggs laid. In any event it is evident from the preceding discussion that variation in the bulk of double-yolked eggs must follow some other law than that which describes the variation of normal eggs in respect to the characters.

The theoretical curve for variation in weight is of Type V, which is a curve limited in one direction only. In this case we have the lower range limit at 2.85 grams. This clearly underestimates the weight of the smallest of normal eggs, our lowest observed value being 42 grams. However, here again the expected frequency below 40 is practically negligible. The expectation at 40 grams is less than 0.5 of an egg per 1,000. The upper range is unlimited, but it is evident from the graph of the theoretical curve (fig. 5) that it makes considerable allowance for eggs heavier than any observed. Thus the curve passes 70 grams, the highest observed value, with a frequency of about 2.6 eggs per 1,000.

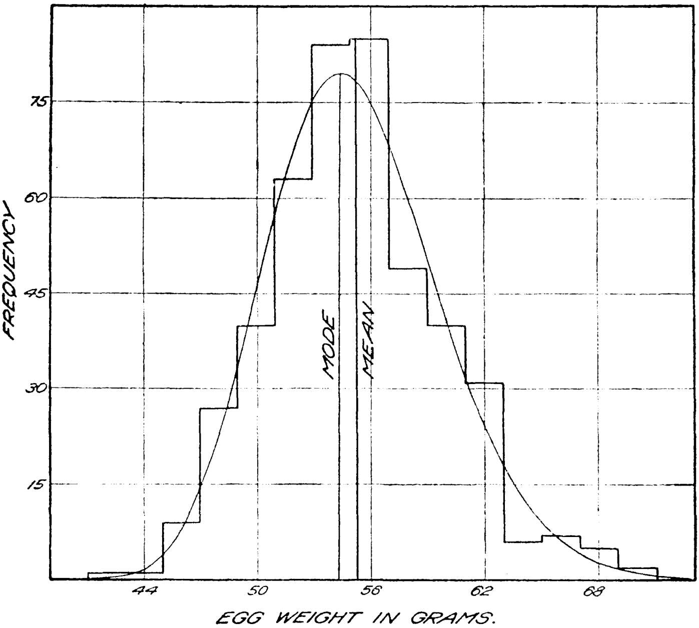

FIG. 5.—Histogram and fitted curve for variation of eggs in weight.

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[1] **Valenciennes.** Note sur les œufs à plusieurs jaunes contenus dans la même coque. C. R., Ac. Sci. Paris, T. 42, pp. 3–6, 1856.

THE CORRELATION OF THE BARRED PLYMOUTH ROCK EGG IN RESPECT TO SIZE AND SHAPE CHARACTERS.

Having considered the variation of eggs in the size and shape characters taken singly, we may next turn to the problem of correlation in the variation of these characters. The size of an egg is primarily determined by a set of physiological processes entirely different from those which fix its shape. On this account it is of considerable theoretical interest to know the degree of correlation which exists between the shape and the size of eggs. Do eggs of a certain shape always have a larger volume than eggs of another shape?

It is also of importance, both theoretically and practically, to know the relation between the bulk and weight and the two diameters of eggs. Is an exceptionally long egg likely to have an exceptionally large volume? Will one be more likely to get an exceptionally large, heavy egg if he selects long eggs than if he selects broad eggs?

From the standpoint of causal morphology (Entwicklungsmechanik) the comparison between the correlations found in the size and shape characters of eggs and those and similar characters in the vertebrate skull is of much interest. These considerations apply with reference to the length and breadth dimensions of the egg. If, as has been previously proposed,[1] the chief factor in determining the shape of an egg is the contraction of the uterine muscles, we could scarcely expect a high degree of correlation between length and breadth. If such should be found it would indicate a greater degree of coordination of muscular contractions in such an organ than has hitherto been supposed to exist.

The data used in this section are those contained in Series A. This consists of the measurements of 450 eggs. As in the previous section, five characters are dealt with here. These are length, breadth, index, bulk, and weight. Arranging these in all possible pairs, we get 10 combinations. It is thus necessary to obtain 10 coefficients of correlations. Two of these correlations (namely, index with length and with breadth) will be obtained by means of formulæ. For the other eight correlations, it is necessary to form the usual correlation tables. These are shown below in Tables 6 to 14, inclusive. The units of grouping used in these tables are the same as those employed in calculating the variation constants (p. 178).

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[1] Pearl, R. Jour. Exp. Zool., vol. 6, pp. 339–359, 1909.

Table 6.—*Showing the correlation of egg length and egg breadth.*

Egg length, in millimeters.	Egg breadth, in millimeters.																	Total.
	38.0–38.4	38.5–38.9	39.0–39.4	39.5–39.9	40.0–40.4	40.5–40.9	41.0–41.4	41.5–41.9	42.0–42.4	42.5–42.9	43.0–43.4	43.5–43.9	44.0–44.4	44.5–44.9	45.0–45.4	45.5–45.9	46.0–46.4	
49.0–49.9	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	1
50.0–50.9	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	1
51.0–51.9	....	1	....	....	1	1	....	....	1	1	2	....	....	....	....	....	....	7
52.0–52.9	....	....	....	1	2	5	3	2	4	1	3	1	....	....	....	....	....	22
53.0–53.9	....	1	2	1	3	4	6	2	8	4	2	2	1	....	....	....	....	36
54.0–54.9	1	....	1	3	9	11	4	15	12	3	8	4	....	....	....	....	....	71
55.0–55.9	....	....	1	3	6	3	7	11	13	9	3	4	4	2	2	....	....	68
56.0–56.9	1	....	1	5	9	8	6	9	8	9	11	4	3	2	1	....	....	77
57.0–57.9	....	....	1	2	6	8	6	17	14	11	2	6	4	....	....	....	1	78
58.0–58.9	....	....	2	1	....	3	6	2	4	3	7	3	1	....	1	2	....	35
59.0–59.9	....	....	....	....	3	2	3	2	8	4	3	1	1	1	1	....	....	29
60.0–60.9	....	1	....	1	1	1	....	3	1	....	....	....	....	2	....	....	....	10
61.0–61.9	....	....	....	....	....	3	....	....	....	....	....	1	....	....	....	....	....	4
62.0–62.9	....	1	....	....	....	1	....	1	....	....	....	....	....	....	....	....	....	3
63.0–63.9	....	....	....	1	....	1	....	....	....	3	....	....	1	....	....	....	....	6
64.0–64.9	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
65.0–65.9	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
66.0–66.9	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
67.0–67.9	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	1
Total.....	2	4	9	18	41	52	41	65	73	48	41	26	15	7	5	2	1	450

Table 7.—*Showing the correlation between egg length and egg weight.*

Egg length, in millimeters.	Egg weight, in grams.																												Total.
	42.0–42.9	43.0–43.9	44.0–44.9	45.0–45.9	46.0–46.9	47.0–47.9	48.0–48.9	49.0–49.9	50.0–50.9	51.0–51.9	52.0–52.9	53.0–53.9	54.0–54.9	55.0–55.9	56.0–56.9	57.0–57.9	58.0–58.9	59.0–59.9	60.0–60.9	61.0–61.9	62.0–62.9	63.0–63.9	64.0–64.9	65.0–65.9	66.0–66.9	67.0–67.9	68.0–68.9	69.0–69.9	
49.0–49.9	..	1	..	..	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	1
50.0–50.9	..	..	..	..	..	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	1
51.0–51.9	1	..	..	..	..	1	1	...	...	...	1	2	1	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	7
52.0–52.9	..	..	..	1	1	3	3	2	1	5	...	3	1	1	1	...	...	...	...	...	...	..	..	..	..	..	..	..	22
53.0–53.9	..	..	..	1	2	2	3	3	2	6	6	2	4	2	1	1	...	...	...	1	...	..	..	..	..	..	..	..	36
54.0–54.9	..	..	..	1	2	1	6	8	4	5	11	9	8	3	7	2	4	...	...	...	...	..	..	..	..	..	..	..	71
55.0–55.9	..	..	..	..	1	1	3	1	4	4	3	5	15	6	8	5	4	1	1	3	3	..	..	..	..	..	..	..	68
56.0–56.9	..	..	..	..	..	1	...	4	5	6	6	8	4	7	8	5	7	5	5	3	2	..	..	1	..	..	..	..	77
57.0–57.9	..	..	..	..	..	...	1	1	2	2	4	8	5	11	15	5	4	8	2	5	3	1	..	..	..	..	1	..	78
58.0–58.9	..	..	..	..	..	...	...	1	1	1	...	2	3	4	1	1	1	3	7	4	2	..	..	1	2	..	1	..	35
59.0–59.9	..	..	..	..	..	...	...	...	...	...	1	3	...	2	3	3	3	2	2	2	3	..	2	1	..	1	..	1	29
60.0–60.9	..	..	..	..	..	...	...	...	1	...	1	...	...	...	1	1	2	1	1	...	...	..	..	..	1	..	1	..	10
61.0–61.9	..	..	..	..	..	...	...	...	...	...	...	...	...	1	2	...	...	...	...	...	...	..	..	..	..	1	..	..	4
62.0–62.9	..	..	..	..	..	...	...	...	...	...	1	...	...	...	...	...	...	1	1	...	...	..	..	..	..	..	..	..	3
63.0–63.9	..	..	..	..	..	...	...	...	...	...	...	...	1	...	...	1	...	...	...	...	...	..	3	..	..	..	..	1	6
64.0–64.9	..	..	..	..	..	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	..	..	..	..	..	..	..	1
65.0–65.9	..	..	..	..	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	0
66.0–66.9	..	..	..	..	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	..	..	..	..	0
67.0–67.9	..	..	..	..	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	1	..	..	..	1
Total....	1	1	0	3	6	10	17	20	20	29	34	42	42	38	47	24	25	21	19	18	13	1	5	3	4	2	3	2	450

TABLE 8.—*Showing the correlation between egg length and egg bulk.*

Egg bulk, in cubic centimeters.	Egg length, in millimeters.																			Total.
	49.0-49.9	50.0-50.9	51.0-51.9	52.0-52.9	53.0-53.9	54.0-54.9	55.0-55.9	56.0-56.9	57.0-57.9	58.0-58.9	59.0-59.9	60.0-60.9	61.0-61.9	62.0-62.9	63.0-63.9	64.0-64.9	65.0-65.9	66.0-66.9	67.0-67.9	
40.0-40.9			1																	1
41.0-41.9						1														1
42.0-42.9	1				1															2
43.0-43.9			1	1	2	1		1												6
44.0-44.9		1		3	1															5
45.0-45.9			1	4	2	5	2	1												15
46.0-46.9				2	5	9	4	3	1	1										25
47.0-47.9				1	3	9	6	5	2	1		1								28
48.0-48.9			1	4	3	3	1	8	4	1				1						26
49.0-49.9			1	3	8	12	7	7	9	1	1	1								50
50.0-50.9			2		5	10	8	5	3	1	2									36
51.0-51.9				2	1	6	12	10	6	5	2	1			1	1				47
52.0-52.9				2	2	5	7	5	13	3	3	1	2							43
53.0-53.9					2	7	5	5	14	1			1							35
54.0-54.9						3	5	7	8	3	2	2		1	1					32
55.0-55.9					1		3	9	5	3	6	1								28
56.0-56.9							2	3	2	2	5	1		1						16
57.0-57.9							3	5	5	6	1									20
58.0-58.9							1	1	3	3	3									11
59.0-59.9							1	1	2		1									5
60.0-69.9							1	1		1					2					5
61.0-61.9											1		1		1					3
62.0-62.9										1									1	2
63.0-63.9										2	2	1								5
64.0-64.9									1			1			1					3
Total....	1	1	7	22	36	71	68	77	78	35	29	10	4	3	6	1	0	0	1	450

TABLE 9.—*Showing the correlation between egg breadth and egg weight.*

Egg breadth, in millimeters.	Egg weight, in grams.																												Total.
	42.0-42.9	43.0-43.9	44.0-44.9	45.0-45.9	46.0-46.9	47.0-47.9	48.0-48.9	49.0-49.9	50.0-50.9	51.0-51.9	52.0-52.9	53.0-53.9	54.0-54.9	55.0-55.9	56.0-56.9	57.0-57.9	58.0-58.9	59.0-59.9	60.0-60.9	61.0-61.9	62.0-62.9	63.0-63.9	64.0-64.9	65.0-65.9	66.0-66.9	67.0-67.9	68.0-68.9	69.0-69.9	
38.0-38.4				1		1																							2
38.5-38.9	1			1					1		1																		4
39.0-39.4					3	1		2	1	1				1															9
39.5-39.9				1	2	1	4	4	3	1	1		1																18
40.0-40.4		1			1	2	7	6	8	6	4	5			1														41
40.5-40.9						5	5	6	4	7	7	6	4	3	2	2		1											52
41.0-41.4							1	2	3	7	4	11	5	5	2	1													41
41.5-41.9										3	7	10	12	16	9	3	1	1	2						1				65
42.0-42.4										4	8	5	15	8	15	4	7	5	2										73
42.5-42.9											2	3	3	1	9	8	5	8	2	3	1		3						48
43.0-43.4												2	2	4	6	3	7	3	7	3	3		1						41
43.5-43.9															3	2	4	2	3	6	3	1	1			1			26
44.0-44.4																1	1	1	3	3	3			2				1	15
44.5-44.9																				3	1				1	1	1		7
45.0-45.4																					2			1	1			1	5
45.5-45.9																									1		1		2
46.0-46.4																											1		1
Total....	1	1	0	3	6	10	17	20	20	29	34	42	42	38	47	24	25	21	19	18	13	1	5	3	4	2	3	2	450

TABLE 10.—*Showing the correlation between egg breadth and egg bulk.*

	Egg bulk, in cubic centimeters.																									
	40.0–40.9	41.0–41.9	42.0–42.9	43.0–43.9	44.0–44.9	45.0–45.9	46.0–46.9	47.0–47.9	48.0–48.9	49.0–49.9	50.0–50.9	51.0–51.9	52.0–52.9	53.0–53.9	54.0–54.9	55.0–55.9	56.0–56.9	57.0–57.9	58.0–58.9	59.0–59.9	60.0–60.9	61.0–61.9	62.0–62.9	63.0–63.9	64.0–64.9	Total.
Egg breadth, in millimeters. 38.0–38.4		1		1																						2
38.5–38.9	1		1					1	1																	4
39.0–39.4				3		2	2	1				1														9
39.5–39.9				1	1	4	4	4	1	2		1														18
40.0–40.4			1	1	2	4	11	6	10	3	2	1														41
40.5–40.9					2	5	6	12	2	13	3	3	3	1	2											52
41.0–41.4							2	4	7	8	7	8	4	1												41
41.5–41.9									2	14	11	12	17	2	4	1	1						1			65
42.0–42.4									3	8	8	17	7	14	6	7	3									73
42.5–42.9										2	3	2	6	7	13	7	4	1			2	1				48
43.0–43.4											2	2	5	7	3	8	5	6	3							41
43.5–43.9													1	3	4	4	1	6	5	1		1				26
44.0–44.4																1	2	6	1	2	1	1			1	15
44.5–44.9																		1	2	1				2	1	7
45.0–45.4																				1	2		1	1		5
45.5–45.9																								2		2
46.0–46.4																									1	1
Total.....	1	1	2	6	5	15	25	28	26	50	36	47	43	35	32	28	16	20	11	5	5	3	2	5	3	450

TABLE 11.—*Showing the correlation between egg index and egg weight.*

	Egg weight, in grams.																												
	42.0–42.9	43.0–43.9	44.0–44.9	45.0–45.9	46.0–46.9	47.0–47.9	48.0–48.9	49.0–49.9	50.0–50.9	51.0–51.9	52.0–52.9	53.0–53.9	54.0–54.9	55.0–55.9	56.0–56.9	57.0–57.9	58.0–58.9	59.0–59.9	60.0–60.9	61.0–61.9	62.0–62.9	63.0–63.9	64.0–64.9	65.0–65.9	66.0–66.9	67.0–67.9	68.0–68.9	69.0–69.9	Total.
Egg index, in per cent. 60.0–60.9														1															1
61.0–61.9																									1				1
62.0–62.9											1		1																2
63.0–63.9						1																							1
64.0–64.9									1					1		1													3
65.0–65.9											1							1					1						3
66.0–66.9															3				1				1						5
67.0–67.9								1	1		1	2											1						6
68.0–68.9						1		1		1		1		1	1	1													7
69.0–69.9							2		2	1	1	1		2	1		1	1	1										13
70.0–70.9				1	1			4	2	4	2	4	5	2		3	3	1	1									1	34
71.0–71.9		1			1		1		3	2	1	4	1	3	3	1	1	2		1	1					1			27
72.0–72.9				1	2	2	2	1	3	1	6	5	3	6	5	3	1	1	2	1	1								46
73.0–73.9					1		1	5		4	1	4	2	4	6			2	3	2	2		2		1		1		41
74.0–74.9							4	1	4	3	2	3	3	5	7	2	3	4	3	2				1					47
75.0–75.9	1			1		1	1	4	1	1	3	4	5	5	6	3	4	3	2	2	1			1		1			50
76.0–76.9					1	1	1	2	2	2	6	3	11	2	3	4	5	3	1	3	3	1						1	55
77.0–77.9						2	1		1	3	1	3	4	1	4	3		2	4	1	1				2				33
78.0–78.9							3	1		3	2	1	2	2	2		1		1	2	1						1		22
79.0–79.9						2	1			2	4	2	3	1	3	1	5	1		1				1					27
80.0–80.9										2	1	1		1	1	1	1			2	1						1		12
81.0–81.9												1			1					1	2								5
82.0–82.9											1	1	1	1		1													5
83.0–83.9												1			1														2
84.0–84.9													1																1
85.0–85.9												1																	1
Total....	1	1	0	3	6	10	17	20	20	29	34	42	42	38	47	24	25	21	19	18	13	1	5	3	4	2	3	2	450

TABLE 12.—*Showing the correlation of egg index and egg bulk.*

Egg index, in per cent.	Egg bulk, in cubic centimeters.																									Total.
	40.0–40.9	41.0–41.9	42.0–42.9	43.0–43.9	44.0–44.9	45.0–45.9	46.0–46.9	47.0–47.9	48.0–48.9	49.0–49.9	50.0–50.9	51.0–51.9	52.0–52.9	53.0–53.9	54.0–54.9	55.0–55.9	56.0–56.9	57.0–57.9	58.0–58.9	59.0–59.9	60.0–60.9	61.0–61.9	62.0–62.9	63.0–63.9	64.0–64.9	
60.0–60.9	...	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	1
61.0–61.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	...	1
62.0–62.9	...	...	...	...	...	...	...	...	1	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	2
63.0–63.9	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
64.0–64.9	...	...	...	...	...	...	...	1	...	...	...	...	1	...	1	...	...	...	...	...	...	...	...	...	...	3
65.0–65.9	...	...	...	...	...	...	...	...	...	1	...	...	...	...	1	...	...	...	...	...	1	...	...	...	...	3
66.0–66.9	...	...	...	...	...	...	...	...	...	...	...	1	1	1	...	...	1	...	...	...	1	...	...	...	...	5
67.0–67.9	...	...	...	...	...	...	1	1	...	1	2	...	...	...	...	...	...	...	...	...	...	1	...	...	...	6
68.0–68.9	...	...	...	1	...	...	1	...	1	...	...	2	2	...	...	...	...	...	...	...	...	...	...	...	...	7
69.0–69.9	...	...	...	...	...	2	...	2	2	1	...	1	2	...	2	1	...	...	...	...	...	...	...	...	...	13
70.0–70.9	...	1	...	...	...	1	3	4	3	6	2	4	1	...	2	4	2	...	...	...	...	...	...	...	1	34
71.0–71.9	...	...	1	1	...	...	1	1	5	2	2	2	4	1	1	2	2	1	...	...	...	1	...	...	...	27
72.0–72.9	...	...	1	1	...	3	3	3	2	9	2	3	8	5	2	1	2	...	1	...	...	...	...	...	...	46
73.0–73.9	...	...	...	...	1	1	6	3	...	...	3	3	5	5	2	2	2	3	2	1	...	...	...	1	1	41
74.0–74.9	...	...	...	...	...	1	3	6	1	5	3	8	2	4	6	2	...	3	2	...	...	1	...	...	...	47
75.0–75.9	1	...	...	1	1	...	2	4	3	4	2	7	4	5	5	3	2	3	1	...	1	...	...	1	...	50
76.0–76.9	...	...	...	...	1	1	3	1	2	7	9	5	4	1	5	7	2	1	3	2	...	...	...	1	...	55
77.0–77.9	...	...	...	...	1	2	1	2	1	3	2	4	1	4	1	3	1	5	...	...	...	...	1	1	...	33
78.0–78.9	...	...	...	1	...	3	...	...	2	2	3	2	1	3	...	1	...	2	...	1	...	...	...	1	...	22
79.0–79.9	...	...	...	...	1	1	1	...	1	5	3	1	4	2	3	1	2	...	1	...	1	...	...	...	...	27
80.0–80.9	...	...	...	...	...	...	...	...	1	2	1	...	1	2	1	...	...	2	1	...	...	...	...	...	1	12
81.0–81.9	...	...	...	...	...	...	...	...	...	1	...	...	...	2	...	...	...	...	...	1	1	...	...	...	...	5
82.0–82.9	...	...	...	...	...	...	...	...	1	...	...	2	1	...	...	1	...	...	...	...	...	...	...	...	...	5
83.0–83.9	...	...	...	...	...	...	...	...	...	1	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	2
84.0–84.9	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
85.0–85.9	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
Total.....	1	1	2	6	5	15	25	28	26	50	36	47	43	35	32	28	16	20	11	5	5	3	2	5	3	450

TABLE 13.—*Showing the correlation between egg weight and egg bulk.*

Egg bulk, in cubic centimeters.	Egg weight, in grams.																												Total.
	42.0–42.9	43.0–43.9	44.0–44.9	45.0–45.9	46.0–46.9	47.0–47.9	48.0–48.9	49.0–49.9	50.0–50.9	51.0–51.9	52.0–52.9	53.0–53.9	54.0–54.9	55.0–55.9	56.0–56.9	57.0–57.9	58.0–58.9	59.0–59.9	60.0–60.9	61.0–61.9	62.0–62.9	63.0–63.9	64.0–64.9	65.0–65.9	66.0–66.9	67.0–67.9	68.0–68.9	69.0–69.9	
40.0–40.9	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
41.0–41.9	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
42.0–42.9	...	1	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2
43.0–43.9	...	...	...	1	2	2	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	6
44.0–44.9	...	...	...	...	2	3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	5
45.0–45.9	...	...	...	...	2	3	8	1	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	15
46.0–46.9	...	...	...	...	...	2	3	11	8	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	25
47.0–47.9	...	...	...	...	...	...	5	8	6	8	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	28
48.0–48.9	...	...	...	...	...	...	...	...	5	12	6	3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	26
49.0–49.9	...	...	...	...	...	...	...	...	1	7	19	16	6	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	50
50.0–50.9	...	...	...	...	...	...	...	...	...	...	6	16	13	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	36
51.0–51.9	...	...	...	...	...	...	...	...	...	...	2	6	17	17	5	...	...	...	...	...	...	...	...	...	...	...	...	...	47
52.0–52.9	...	...	...	...	...	...	...	...	...	...	...	1	6	17	18	1	...	...	...	...	...	...	...	...	...	...	...	...	43
53.0–53.9	...	...	...	...	...	...	...	...	...	...	...	...	...	2	19	10	3	...	...	1	...	...	...	...	...	...	...	...	35
54.0–54.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4	9	10	8	1	...	...	...	...	...	...	...	...	...	32
55.0–55.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	4	9	8	3	3	...	...	...	...	...	...	...	...	28
56.0–56.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	3	4	6	3	...	...	...	...	...	...	...	...	16
57.0–57.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	9	6	4	...	...	...	...	...	...	...	20
58.0–58.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	5	4	1	1	...	...	...	...	...	11
59.0–59.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4	...	1	...	...	...	...	...	5
60.0–60.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	2	2	...	...	...	...	5
61.0–61.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	1	...	1	...	...	3
62.0–62.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	...	...	...	2
63.0–63.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	1	1	1	5
64.0–64.9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	1	3
Total....	1	1	0	3	6	10	17	20	20	29	34	42	42	38	47	24	25	21	19	18	13	1	5	3	4	2	3	2	450

In Table 14 are exhibited the coefficients of correlation deduced from Tables 6 to 13, together with their probable errors. The correlation coefficients were determined according to the usual Bravais formula,

$$r=\frac{S(xy)}{N\sigma_1\sigma_2}$$

when x and y are the deviation from the means, and σ_1 and σ_2 are the standard deviations of the two variables.

TABLE 14.—*Coefficients of correlation. Egg size and shape. Series A.*

Characters.	Coefficient of correlation.	Characters.	Coefficient of correlation.
Length and breadth	+0.0837±.0316	Length and weight	+0.5797±.0211
Index and weight	+0.0851±.0316	Breadth and weight	+0.8357±.0096
Index and bulk	+0.1005±.0315	Breadth and bulk	+0.8602±.0083
Length and bulk	+0.5751±.0212	Weight and bulk	+0.9804±.0012

From Table 14 and the accompanying Tables 6–13, inclusive, the following points may be noted:

1. In every case the correlation is positive.

2. In Table 14 the coefficients of correlation are arranged in the order of their magnitude, beginning with the lowest and ending with the highest coefficient. Under this arrangement it is seen that the first three coefficients show very low values and are barely, if at all, significant. The next two show moderately high values, while the last three are exceptionally high.

3. In the case of the correlation between length and breadth, the coefficient is considerably less than three times its probable error. It is thus clear that there is very little if any correlation between variation in the length and the breadth of an egg. It seems most probable that the shape of an egg is determined by the molding influence of the longitudinal and circular muscles of the uterine wall. If the circular muscles contract more strongly than the longitudinal we shall get a relatively long and narrow egg. On the other hand a reduced tonus of the circular muscle layer, with relatively strong contractions of the longitudinal layer, will mean the production of a short, round egg. The absence of correlation between length and breadth of the egg indicates that these two sets of muscles are to a large degree independent in their action.

4. The next two coefficients, index and weight and index and bulk, may be considered together. These two coefficients do not differ greatly from each other. The difference is certainly not significant in comparison with its probable error (difference$=0.0154\pm0.0446$). The probable error is several times the difference. Further, neither

of these coefficients taken by itself can be considered as certainly significant. The coefficient for index and bulk is a little more than three times its probable error, while that for index and weight is slightly less than three times its probable error. However, from the fact that these two correlations in which index enters are both positive and of about the same value and these values approximately three times their probable errors, it can not safely be positively concluded that there is no correlation between the shape of an egg and its weight. The results, in fact, are not entirely conclusive. They appear to indicate that there is a slight correlation between the shape of an egg as measured by the length-breadth index and its bulk or weight. Thus a broad, short egg is on the average slightly more likely to have a larger proportionate capacity than a narrow, long egg.

5. Taking the next two correlations together (viz, length with bulk and length with weight), there is no question as to the significance of the coefficients. It is to be noted that these two coefficients, in which the length is correlated with the two measures of capacity, have approximately the same value. The difference between them is entirely insignificant.

6. The next two correlations are breadth with weight and breadth with bulk. These coefficients have exceptionally high values. Again it will be noted that the correlation of breadth with the two measures of capacity has about the same value in each case. The difference here, while greater than that mentioned in paragraph 5, is still not to be regarded as certainly significant. It is 0.0245 ± 0.0127. The results clearly show that inter-racially both egg length and egg breadth are highly correlated with the volume and weight. Of the two linear dimensions the breadth is distinctly more highly correlated with bulk characters than the length. That this difference is significant may be shown by taking the difference between the highest coefficient in which the length enters and the lowest one in which the breadth is concerned. This difference with its probable error is $0.8357 - 0.5797 = 0.2560 \pm 0.0232$. The difference is more than 10 times its probable error. As to the measure of capacity, it plainly makes little difference whether one uses the calculated volume in cubic centimeters or the actual weight in grams.

7. Turning finally to the correlation between weight and bulk, the very high coefficient of $+0.9804$ is noted. The significance of this is that the formula which has been used throughout for calculating the bulk gives excellent results. This is rather remarkable when it is remembered that in calculating the bulk the egg is regarded as a perfect prolate spheroid, which, of course, it never actually is. The correlation between this measure of capacity and the actually determined weight is almost perfect. In fact, the greater part of the difference between the determined coefficient of $+0.9804$ and that

for absolutely perfect correlation +1.0000 may be regarded as due to experimental errors either in the weighing or in calculating the bulk with only two decimal places in the observed length and breadth.

It is perfectly clear that the calculated bulk is a very excellent *relative* measure of the volume of an egg. It is another question as to how closely this measure approximates the *absolute* volume. It is clear that if the calculated bulk is a good absolute measure of the volume the specific gravity calculated from the mean weight and mean bulk ought to equal the actually observed mean specific gravity of eggs, since—

$$\text{Specific gravity} = \frac{\text{Weight in grams}}{\text{Volume in cubic centimeters}}$$

Substituting the means for weight and bulk given in Table 2, we have—

$$\text{Specific gravity} = \frac{55.264}{51.880} = 1.0652$$

The average specific gravity of fresh hen's eggs as determined by Wheeler at the New York Agricultural Experiment Station[1] was 1.090.

Since there is a considerable discrepancy here, it was thought advisable to make a series of determinations of the specific gravity of eggs from the same strain that furnished the material of Series A of this bulletin. This was done, with the results shown in Tables 15 and 16. Table 15 gives the frequency distribution for variation in the specific gravity of 61 Barred Plymouth Rock eggs. Each one of these eggs was laid by a different bird. The determinations were made within a few hours of the time of laying.

TABLE 15.—*Frequency distribution for the variation in the specific gravity of fresh eggs.*

[Data from Barred Plymouth Rocks.]

Specific gravity.	Frequency.	Specific gravity.	Frequency.
1.075-1.079	4	1.095-1.099	4
1.080-1.084	6	1.100-1.104	3
1.085-1.089	19		
1.090-1.094	25	Total	61

The chief variation constants for this distribution are shown in Table 16.

TABLE 16.—*Constants for variation of eggs in specific gravity.*

Constant.	Value.
Mean	1.0898±0.0005
Standard deviation	.0054± .0003
Coefficient of variation	.499

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[1] New York Agr. Exp. Sta., Seventh Ann. Rept., p. 67, 1889.

From these tables we note:

1. There is an extremely small degree of variation in respect to specific gravity among different eggs of an approximately equal freshness.
2. The mean value for the specific gravity of the egg obtained in this series of observations differs by only 0.0002 from that found by Wheeler.
3. The difference between the observed mean specific gravity of the egg and that determined from the calculated bulk and the weight is 0.0246. This difference is approximately 50 times the probable error of the observed mean.

It is plain that the calculated bulk on the average somewhat overestimates the absolute volume of the egg. Since both the mean weight and the mean specific gravity have been directly determined from the eggs, with small probable errors, it is possible to make a rather close determination of the average amount by which the bulk formula as here used overestimates the true volume of the egg. We have

$$\text{Mean volume} = \frac{55.264}{1.0898} = 50.710$$

The mean bulk determined from the frequency distribution is 51.880. It is then seen that on the average the calculated bulk overestimates the volume of the egg by 1.170 cubic centimeters. In other words, by assuming the egg to be a perfect prolate spheroid an average error of about 2.2 per cent is made in the calculation of the volume of the egg. This error is in excess. Evidently, then, the formula for bulk used in this paper not only gives an excellent relative measure of the volume of the egg, but also makes a fair absolute determination of this quantity, certainly sufficiently accurate for all practical purposes.

If still greater accuracy is desired it may be obtained by applying a correction factor, based on the calculated bulk. The determination of the volume of the egg would then take this form:

$$\text{Approximate volume of egg} = \{4.1888\ (\tfrac{1}{2}\ \text{breadth})^2 \times (\tfrac{1}{2}\ \text{length})\} - 0.022\{4.1888\ (\tfrac{1}{2}\ \text{breadth})^2 \times (\tfrac{1}{2}\ \text{length})\}$$

INDEX CORRELATIONS.

Before entering on the general discussion of the results of this section it is necessary to consider the two coefficients measuring the correlation of index with length and index with breadth, respectively. It was stated earlier that these index correlations would be calculated by formulæ. Pearson[1] has shown that if we have two indices $\frac{x_1}{x_3}$ and $\frac{x_2}{x_4}$ the gross correlation ρ between these indices will be

$$\rho = \frac{r_{12}v_1v_2 - r_{14}v_1v_4 - r_{23}v_2v_3 + r_{34}v_3v_4}{\sqrt{v_1^2+v_3^2-2r_{13}v_1v_3}\ \sqrt{v_2^2+v_4^2-2r_{24}v_2v_4}}$$

where $r_{12}\text{----}r_{34}$ represent the calculated coefficients of correlation and $v_1\text{----}v_4$ the coefficients of variation of the variables of $x_1\text{----}x_4$, respectively.

It has further been pointed out by Pearl[2] that the gross correlation ρ between the length-breadth index $\frac{x_1}{x_3}$ and the breadth x_1 is

$$\rho = \frac{v_3 - r_{13}v_3}{\sqrt{v_1^2+v_3^2-2r_{13}v_1v_3}} \text{ ------------------ (i)}$$

Between the index $\frac{x_1}{x_3}$ and the length x_3 the gross correlation is

$$\rho = \frac{r_{13}v_1 - v_3}{\sqrt{v_1^2+v_3^2-2r_{13}v_1v_3}} \text{ ------------------ (ii)}$$

In the above-mentioned memoir Pearson showed that a large portion of the gross correlation when an index is correlated either with another index or with one of the actual measurements which enter into the index is not due to organic relationship, but is a spurious correlation arising from arithmetical reasons. It has been shown that the measure of this spurious correlation ρ_0 in the case of index-breadth is

$$\rho_0 = \frac{v_3}{\sqrt{v_1^2+v_3^2}} \text{ -------------------- (iii)}$$

Similarly for index-length correlation

$$\rho_0 = \frac{-v_3}{\sqrt{v_1^2+v_3^2}} \text{ -------------------- (iv)}$$

In Table 17 are given first the gross correlations, ρ, between index and length and index and breadth as calculated from (i) and (ii). Next are given the corresponding spurious correlations calculated from (iii) and (iv). Finally are given the net correlations $\rho - \rho_0$. These represent the part of the gross correlations that may be considered to be due to true organic interrelationship. If the indices were arbitrarily made up from a random series of measurements of the lengths and breadths of different eggs the values in the last column, $\rho - \rho_0$, would be significantly zero. The meaning of the expression $\rho - \rho_0$ had been fully discussed by Pearl (*loc. cit.*) and need not be repeated here. Expressed in terms of regression, it indicates the amount of shift of the gross regression line beyond the spurious line. This shift is due to the organic correlation. The probable errors given in the spurious and net columns are calculated from the formula $\text{P.E.}_r = 0.67449 \frac{1-r^2}{\sqrt{n}}$. This assumes that r has been calculated from the usual formula $r = \frac{S(xy)}{N\sigma_1\sigma_2}$. The error involved in this assumption is probably not significant.

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[1] Pearson, K. On a form of spurious correlation which may arise when indices are used in the measurement of organs. Proc. Roy. Soc., vol. 60, pp. 489–498, 1897.

[2] Pearl, R. A Biometrical Study of Conjugation in Paramecium. Biometrika, vol. V, pp. 213–297, 1907.

TABLE 17.—*Index correlations.*

Characters.	Gross, ρ	Spurious, ρ_0	Net, $\rho-\rho_0$
Index-length	—0.7707	—0.7901±0.0119	+0.0194±0.0318
Index-breadth	+ .7552	+ .7901± .0119	— .0349± .0318

From an examination of the last column of this table it is clear that there is no significant correlation between the shape of an egg as measured by the length-breadth index and either its length or breadth. In neither case is the net correlation significantly greater than its probable error.

In discussing the correlation between the index and volume (p. 195), it was said that there also the values of r were not significant, taken independently. From the fact that both the index-bulk and index-weight correlations were positive and of values bordering on significance, it was concluded that there might be some slight correlation between shape and capacity variation of the egg. With the additional evidence afforded by Table 17 it becomes still more certain that if there is any real correlation between size and shape characters of the egg it must be extremely slight in amount or degree.

COMPARISON OF CORRELATION IN THE EGG AND THE SKULL.

Earlier in this bulletin it has been shown that the egg and the human skull show a marked similarity in respect to variation in corresponding dimensions. It was suggested that this correspondence between two such dissimilar things probably owes its origin to the fact that the developmental mechanical forces which come into play in determining the form of an egg are essentially of the same kind as those which give the skull its definitive form. If this view of the matter is correct it would be expected that a similar correspondence would be found in respect to the degree of correlation between the different dimensions in the two objects. As a matter of fact, such is the case, as is shown by the data given in Table 18.

In this table are collected from the literature correlation coefficients for skull dimensions in several human races and also in the albino rat. In the case of the human data only the constants for males are tabled, in order to save space and at the same time avoid the mental confusion which arises with a long series of constants, all showing about the same thing, but differing slightly among themselves. Nothing different in principle would be brought out if all the constants for females were given in the table. In obtaining the means the individual coefficients have been weighted in proportion to the number of cases on which they were based.

TABLE 18.—*Comparison of correlations in egg and skull characters.*

Characters and race.	Number of cases.	Coefficient of correlation.	
Skull length and breadth:			
French males[1]	860	0.089±0.024	
German males[2]	100	.286± .062	
English males[3]	131	.240± .055	
Naqada males[1]	139	.344± .050	
Weighted mean, human males	1,230	*0.150*	
Albino rat[4]	53	♂=0.665±0.052	
	51	♀=0.397± .080	
Egg length and breadth	450	*0.084± .032*	
Skull length and capacity:			
English males[3]	72	0.597±0.051	
Naqada males[1]	89	.501± .054	
German males[2]	100	.515± .050	
Weighted mean, human males	261	*0.533*	
Albino rat[4]	53	♂=0.761±0.039	
	51	♀=0.577± .062	
Egg length and weight	450	*0.580± .021*	
Egg length and bulk	450	*.575± .021*	
Skull breadth and capacity:			
English males[3]	72	0.631±0.048	
Naqada males[1]	89	.434± .058	
German males[2]	100	.672± .037	
Weighted mean, human males	261	*0.580*	
Albino rat[4]	53	♂=0.838±0.028	
	51	♀=0.632± .057	
Egg breadth and weight	450	*0.836± .010*	
Egg breadth and bulk	450	*.860± .008*	
Skull B/L index and length:		*Gross.*	*Spurious.*
English males[3]	131	−0.547±0.041	−0.658±0.033
French males[1]	860	− .652± .014	− .686± .013
Naqada males[1]	130	− .551± .041	− .694± .031
Weighted mean, human males	1,121	*−0.628*	*−0.684*
Egg B/L index and length	450	*− .771*	*− .790± .012*
Skull B/L index and length, *net* correlation (mean)		*+0.056*	
Egg B/L index and length, *net* correlation		*+0.019±0.032*	
Skull B/L index and breadth:			
English males[3]	131	0.682±0.032	0.753±0.026
French males[1]	860	.699± .012	.727± .011
Naqada males[1]	130	.594± .038	.720± .028
Weighted mean, human males	1,121	*0.685*	*0.729*
Egg B/L index and breadth	450	*.755*	*.790± .012*
Skull B/L index and breadth, net correlation (mean)		*−0.044*	
Egg B/L index and breadth, net correlation		*−0.035±0.032*	

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[1] Fawcett, C. D. Biometrika, Vol. I, p. 457.

[2] Lee, A. Phil. Trans, Vol. 196A, p. 231.

[3] Macdonell, W. R. Biometrika, Vol. III, p. 233.

[4] Hatai, S. Amer. Jour. Anat., Vol. VII, p. 432.

From this table it is at once apparent that the same general correspondence between the egg and the skull, which was found to hold for relative variability, also obtains with reference to the degree of correlation in variation. Taking the various points seriatim, we note:

1. That while the correlation between length and breadth in the human skull tends to be higher than that for the corresponding egg dimensions, yet taking the mean for all the coefficients it is not greatly so. The coefficient for skull length and skull breadth in the French series (the most extensive of all) is almost identical with that for the corresponding dimensions of the egg. In the case of the rat skull the correlations are distinctly higher than for either human skull or egg. But here two facts must be taken into account. In the first place the rat series are small and may be comparatively "bad" samples of the general population of rat skulls. The unusually large differences between the constants for the male and the female series suggests that such may be the case. In the second place, the relative influence of the two morphogenetic factors chiefly concerned here (tension of temporal muscles and internal pressure from developing brain) must be distinctly different in the case of the rat's skull, with its proportionately small brain and almost no vaulting, its comparatively heavy jaw with associated muscular development, and its entirely different facial development, from what obtains in the case of the human skull. In other words, there is not the same morphogenetic basis for the comparison of the rat's skull with the bird's egg that there is for comparing the human skull with this object.
2. That the agreement between human skull and egg in respect to the correlation between length and capacity is very close. This is true whether weight or volume be taken as the capacity measure in the case of the egg.
3. That this accord between egg and skull is not quite so close in the case of the breadth-capacity correlation as for the length-capacity correlation. Here there is closer agreement between the rat skull and the egg. The most significant point, however, is that all the cases (rat skull, human skull, etc., and egg) agree in showing a somewhat closer correlation between breadth and capacity than between length and capacity. This difference is least for the human skull and greatest for the egg, but it is in the same sense for all three forms.
4. That in respect to index correlations—shape with size—there is a remarkably close agreement between egg and skull. The net organic correlations are, within the limits of error of random sampling, identical.

Taking all the facts together it may fairly be said that in respect to relative variability and correlation of dimensions there is a close similarity between the hen's egg and the human skull in particular, and probably to a lesser extent the mammalian skull in general.

The most probable explanation of this similarity is to be found in the fact that the immediate physical and mechanical forces which have to do with the determination of the definitive form in the two cases are of essentially the same kind, and act in essentially the same way.

REGRESSION AND THE DETERMINATION OF THE PROBABLE VALUE OF A PARTICULAR EGG DIMENSION FROM A KNOWLEDGE OF OTHER DIMENSIONS.

One of the aims of science is to be able to predict the occurrence or attributes of one phenomenon from a knowledge of other related phenomena. It is clear that it would be of advantage in many ways if one could express with reasonable accuracy certain of the physical characteristics of eggs in terms of other characters. For example, it often happens that one is able to obtain directly the measurements of an egg but not its weight. This is the case with "blown" eggs in a museum. Further, in eggs which have been stored in such a way that they have lost water by evaporation it is impossible to obtain the correct weight directly, though their linear dimensions will not have changed. In such cases formulæ to determine the probable mean fresh weight of a series of such eggs would be valuable. Practical use might possibly be made of such formulæ in the investigation of cold-storage eggs. In the following pages especial attention will be given to the derivation of formulæ for estimating the weight and volume of eggs from the direct measurements of length and breadth.

For such characters as are capable of numerical expression the simplest method of predicting one character in terms of another is furnished by the principle of regression.

The degree of accuracy with which such a prediction can be made depends upon two things; namely, (a) the relation between the characters considered, i. e., the degree of correlation, and (b) the variability of the characters. It is well known that if from the ordinary correlation table the means of the arrays for either correlated character are plotted the points so obtained usually lie very nearly on a straight line. By determining the equation of the straight line best fitting these points it is clear that we obtain an expression of one of the characters in terms of the other. Occasionally, however, it happens that the plotted means do not lie in a straight line. In such cases the regression is said to be "nonlinear," and the correlation "skew." Some curve must then be found to graduate the means of the arrays.

Before entering upon the detailed discussion of the regression lines for egg characters it will be of advantage to consider briefly certain points of mathematical theory regarding tests for linearity of regression.

LINEARITY OF REGRESSION.

In his memoir on "Skew Correlation and Non-Linear Regression,"[1] Pearson has pointed out a method of measuring the approach to linearity of the regression curve. In addition to the coefficient of correlation, r, the two most significant constants are (a) the square root of the mean square deviation of the means of the arrays from the regression line Σ_M, and (b) the correlation ratio, η. The latter gives the mean reduction in the variability of an array as compared with that of the whole population. It is evident that

$$\eta = \frac{\sigma_{my}}{\sigma_y} \text{ - (i)}$$

where σ_{my} is the standard deviation of the weighted means of the y arrays about the mean of the population, and σ_y is the standard deviation of the variates about the same mean. In calculating σ_{my} the squared deviations of means are weighted with their frequencies. Pearson has further pointed out that

$$\Sigma_M{}^2 = \sigma_{my}{}^2 - r\sigma_y{}^2 \text{ - (ii)}$$

Hence from (i) and (ii)

$$\Sigma_M = \sigma_y \sqrt{\eta^2 - r^2}$$

If the regression is linear, Σ_M should equal zero and η should equal r within the limits of random sampling. Table 19 gives the values of Σ_m and η derived from the data in the correlation tables (pp. 192–193). The correlation coefficients for length with breadth and for the index correlations were so low that it was not deemed worth while to consider their regression equations.

Also from the inspection of Table 13 and particularly from an examination of figures 10 and 11 (p. 209) it is clear that the regressions of weight on bulk and of bulk on weight are essentially linear. Hence, we have not gone to the trouble of calculating the constants for the linearity of regression in these cases.

For convenience there is included in Table 19 the values of r and the corresponding differences with their probable errors. It has been shown by Pearson (*loc. cit.*) that a very close approximation to the probable error of η is given by the formula

$$P.E._\eta = 0.67449 \frac{1-\eta^2}{\sqrt{N}}$$

Blakeman[2] has given approximate formulæ for the probable errors of $\eta - r^2$ and $\eta - r$.

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[1] Pearson, K. Mathematical Contributions to the Theory of Evolution XIV. Draper's Company Research Memoirs, Biometric Series II, 1905.

[2] Blakeman, J. On Tests for Linearity of Regression in Frequency Distributions. Biometrika, Vol. IV, pp. 332-350, 1905.

His formulæ have been used in obtaining the probable errors of these constants.

TABLE 19.—*Analytical constants for linearity of regression.*

Correlated characters.	r	η[1]	$\eta - r$	Σ_M	$\eta^2 - r^2$
Bulk and length	0.5751±0.0213	0.6163±0.0197	0.0412±0.0116	0.2214σ	0.0490±0.0136
Weight and length	.5797± .0211	.6127± .0198	.0330± .0113	.1983σ	.0393± .0133
Weight and breadth	.8357± .0096	.8377± .0095	.0020± .0022	.0582σ	.0034± .0037
Bulk and breadth	.8602± .0083	.8620± .0082	.0018± .0021	.0563σ	.0032± .0036

[1] Calculated from the weighted squared deviations of the means from the mean of the population. Sheppard's corrections were *not* used.

From Table 19 we note that in the regression of bulk on length and weight on length the values of $\eta - r$ and $\eta^2 - r^2$ are more than three times their probable errors. In these cases η is significantly greater than r. Likewise Σ_M deviates sensibly from zero. Judged by this test we must conclude that the regressions of bulk and weight on length deviate sensibly from linearity.

Turning to the regression of bulk on breadth and weight on breadth we note that the values $\eta - r$ and $\eta^2 - r^2$ are just about equal to their probable errors. Hence, η does not differ significantly from r. From this we may conclude that the best fitting straight line will give as good results in the prediction of weight or bulk from breadth as any other curve.

The deviation of r from η and of Σ_M from zero necessarily implies departure from linearity. This may be due to either one or both of two causes. Either the points determined by the means do not lie in a straight line and can be better described by some other curve, or else the points deviate from a straight line but nevertheless can be better described by that than by any other curve. In order to decide in which class the present series lies we may have recourse to the diagrams showing the plotted means of the arrays.

SIMPLE LINEAR REGRESSION.

Before taking up these diagrams, however, it will be of advantage to have before us the equations to the respective regression lines, and these may then be plotted on the same figure. It has been shown by Yule[1] and others that the simplest forms of the characteristics or regression equations with two variables are

$$x = r\frac{\sigma_1}{\sigma_2}y \qquad \text{(i)}$$

and

$$y = r\frac{\sigma_2}{\sigma_1}x \qquad \text{(ii)}$$

where x and y represent deviations from their respective mean values; σ_1 and σ_2 are the standard deviations of X and Y respectively; and r is their coefficient of correlation as usual.

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[1] Yule, G. U. On the Theory of Correlation. Jour. Roy. Stat. Soc., Vol. LX, 1897.

Transferring these values to their means we have

$$(X - \text{mean } x) = r\frac{\sigma_1}{\sigma_2}(Y - \text{mean } y) \ldots\ldots\ldots \quad \text{(iii)}$$

$$(Y - \text{mean } y) = r\frac{\sigma_2}{\sigma_1}(X - \text{mean } x) \ldots \quad \text{(iv)}$$

The expressions $r\frac{\sigma_1}{\sigma_2}$ and $r\frac{\sigma_2}{\sigma_1}$ are spoken of as the regression coefficients and often designated as b_1 and b_2, respectively.

Further it is evident from the mathematical relations involved that the standard error, Σ, made in estimating X from equation (iii) is $\sigma_1\sqrt{1-r^2}$. Similarly $\sigma_2\sqrt{1-r^2}$ is the probable error of Y as estimated from equation (iv).

Now applying these equations to the data in hand we obtain the following regression equations together with their standard errors. Only those equations are given in which weight or bulk is estimated from other characters.

The abbreviations used are evident; *W.* = Weight; *L.* = Length; *Br.* = Breadth; *Blk.* = Bulk.

(1) $W. = 1.1229L. - 7.9792$; $\Sigma = 3.7661$.
(2) $W. = 2.8035Br. - 62.2483$; $\Sigma = 2.5383$.
(3) $W. = 1.0704Blk. - 0.2681$; $\Sigma = 0.9610$.
(4) $Blk. = 1.0203L. - 5.5879$; $\Sigma = 3.4631$.
(5) $Blk. = 2.6431Br. - 58.9081$; $\Sigma = 2.1591$.
(6) $Blk. = 0.8979W. + 2.2563$; $\Sigma = 0.8344$.

Since there is a very high correlation between weight and bulk the corresponding regression lines for weight and for bulk on length, as well as for weight and bulk on breadth, show a striking similarity. The regression lines for weight on bulk and bulk on weight are practically identical. For this reason we may discuss these regressions in pairs, taking first those in which the length forms are variable. These are exhibited in figures 6 and 7.

From these figures it is clear that the regressions of weight on length and of bulk on length follow some other law than that of the straight line. The deviations at either extremity are in both cases definitely directed and can not be accounted for as random deviations. This furnishes confirmatory evidence concerning the conclusions drawn from a study of the correlation ratio. We are undoubtedly dealing with nonlinear regression in these cases. The figures show that the fitted regression line represents the facts well in the middle of the range, but prediction as to the weight or bulk of very long or very short eggs made according to these equations would vary rather widely from the true values. However, in a random sample of eggs the majority will lie in that portion of the curve which is well graduated by the regression line. Thus in the above figures 6 and 7 it is evident that the points at which the line of the plotted means begins to deviate sensibly from the fitted line may be taken as corresponding approximately to egg lengths of 51 and 61 mm. Referring to the frequency distribution for length (Table 1), it is seen that 433 eggs out of the total 450 eggs measured have lengths between 51 mm. and 61 mm. In other words, over 96 per cent of the eggs in this sample fall within that portion of the curve which is satisfactorily fitted by the regression straight line. It may further be noted that these points of 51 mm. and 61 mm. lie on either side of the mean length at distances nearly equal to -2σ and $+2\sigma$, respectively. It is thus clear that for any random sample of eggs having a mean length and a standard deviation in length approximately the same as those of the present distribution one will get for over 96 per cent of the eggs fairly good predictions of weight and bulk from equations (1) and (4). If the regression line will estimate the weight or volume of 96 per cent of the eggs as closely as can be done by any equation which takes account of the length only, we may expect to get fairly good results when the equation is applied to a random sample of eggs.

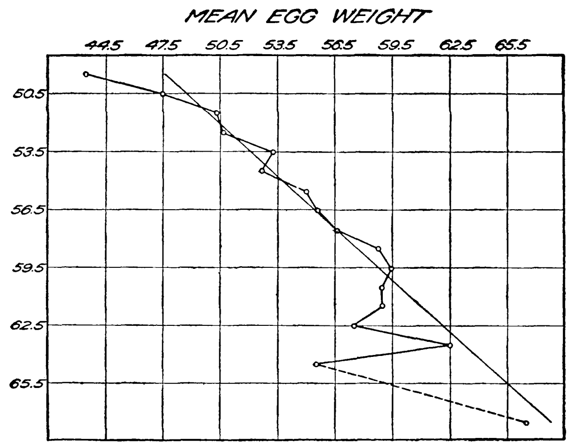

FIG. 6.—Regression of egg weight on egg length.

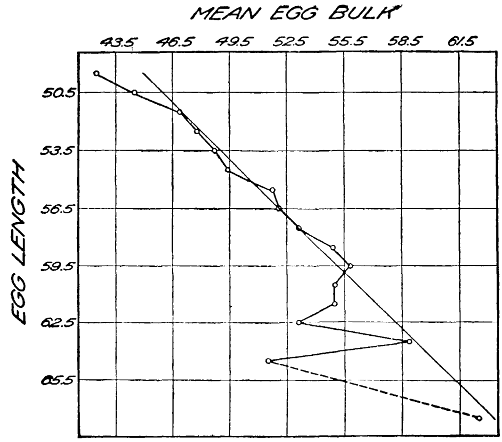

FIG. 7 —Regression of egg bulk on egg length.

Such a test for this and other equations will be made in a later section of this bulletin (p. 223).

A still further test as to the estimation of such equations is given by the standard error $\Sigma=\sigma\sqrt{1-r^2}$. The value for each case is given with the equations. In the regression of weight on length

$\Sigma = 3.7661$ and for bulk on length $\Sigma = 3.4631$. In other words, the standard error of the weight as predicted by equation (1) is about 3.8 grams, or about 7 per cent of the mean weight.

We may next consider the regressions in which weight and bulk are associated with breadth. The equations to the fitted lines are given as (2) and (5) on page 206. The graphs of these lines, together with the plotted means, are given in figures 8 and 9.

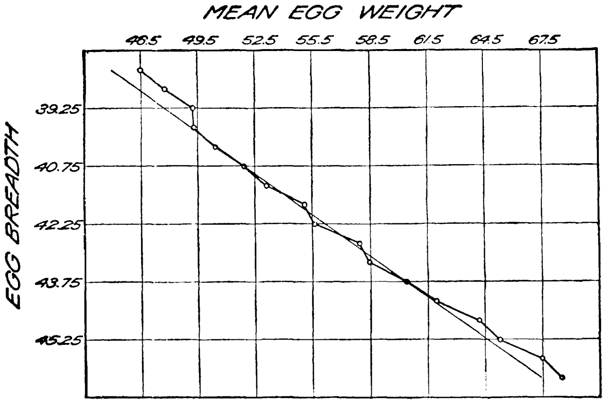

FIG. 8.—Regression of egg weight on egg breadth.

It is at once evident that here the regression lines fit the data much better than in figures 6 and 7. However, there is here also an evident deviation from linearity at the ends of the curves. Looking at the line of plotted means it appears clearly probable that one could fit them better with a parabola than with a straight line. In these cases the results of inspection apparently do not confirm the conclusion drawn from a study of the correlation ratio (p. 205). It was there concluded that, since the value of $\eta - r$ was no greater than its probable error, these regressions could be considered as essentially linear. The reason for this discrepancy is that both η and r are calculated from the ordinates weighted with their frequency. For this reason the regression lines fit the middle portions of the curves very well indeed. Further, the deviations at the ends are not great in magnitude, and the total frequency in these regions is very small.

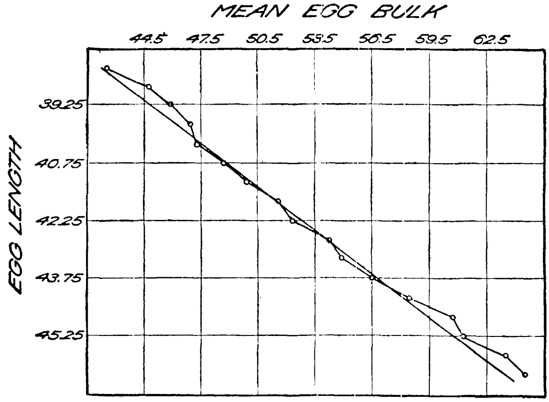

FIG. 9.—Regression of egg bulk on egg breadth.

If, as in the case of capacity with length, the points where the curves begin to deviate sensibly from the fitted line are examined, it is found that these correspond, taking the two curves together, to breadths of about 39.50 mm. and 45 mm. Referring to the frequency distribution for breadth (Table 1), it is seen that 427 out of the 450 eggs are included between these limits. In other words, the regression line fits with considerable accuracy about 95 per cent of the eggs. While the deviations at either end are not very great, nevertheless the data from the present sample of eggs can be somewhat more closely described by a parabola than by a straight line, as will be shown later.

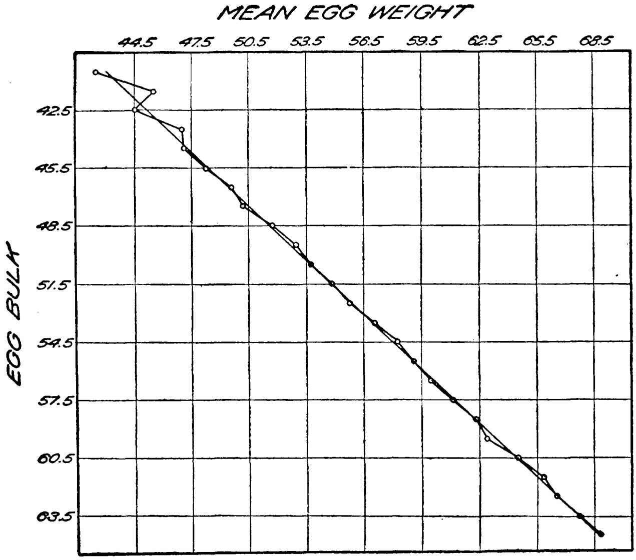

FIG. 10.—Regression of egg weight on egg bulk.

The standard error made in estimating the capacity of an egg from its breadth is in the case of the weight 2.5383 grams and in the case of the bulk 2.1519 grams. Thus the average estimation of capacity is more than 1 gram closer when made from the breadth rather than from the length as the second variable.

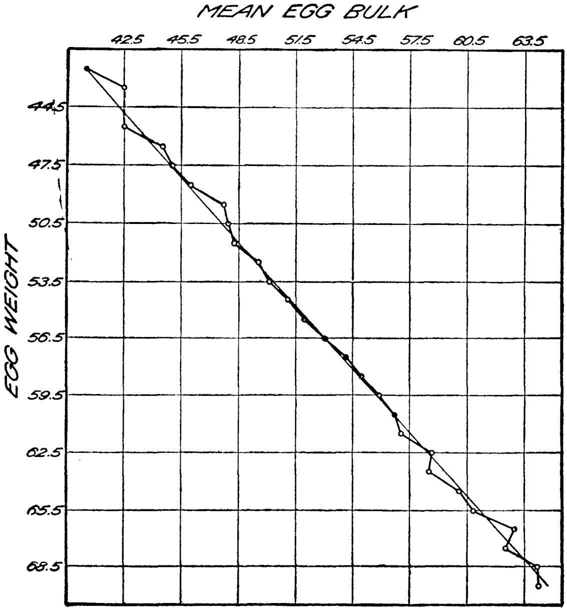

FIG. 11.—Regression of egg bulk on egg weight.

The regressions of weight on bulk and of bulk on weight may next be considered. The equations to these regression lines are given as equations (3) and (6) on page 206. Figures 10 and 11 show the graphs of these equations along with the plotted means.

The diagrams afford sufficient proof of the strict linearity of these regressions, without further demonstration. It would clearly be impossible to get any curve which would estimate weight from bulk or bulk from weight better than the present straight lines. Further, since the correlation coefficient between these two characters has such a high value, it is to be expected that the absolute estimation would be very close in each case. Judging from the above diagrams, this would certainly seem to be the case. However, it must be remembered that the regression line might fit the plotted means very closely, and yet if the standard deviations of the characters are large the standard error of the estimation would be correspondingly large. Here, however, this is not the case. The standard error of equation (3) is 0.9610, and for equation (6) is 0.8344. Taking the case of the weight on bulk (equation 3) we note that the error of 0.96 gram is only about 1.7 per cent of the mean egg weight. In the case of bulk on weight the estimation is made with a standard error of less than 1.5 per cent of the mean bulk.

PARABOLIC REGRESSION.

CAPACITY-LENGTH REGRESSIONS.

It has been shown above (pp. 205) that the regressions of egg weight and bulk on length are nonlinear. In such cases the problem of obtaining the best fitting curve for the graduation of the observed regression line is greatly complicated. In his previously mentioned work on "Skew Correlation and Non-Linear Regression" Pearson has developed the theory of higher types of regression. He classifies these types as linear, parabolic, cubic, and quartic, corresponding respectively to the straight line, second, third, and fourth order parabolas. As a first approach to fitting these regression curves we have used Pearson's parabolic type.

The general form for this type of equation is

$$y_{xp} = a_o + a_1 x_p + a_2 x_p^2$$

where y_{xp} is the mean of an x-array of y's.

Evaluating the a's in terms of the correlation and variation constants and subsequently simplifying the equation, Pearson arrives at the following expression for parabolic regression:

$$\frac{y_{xp}}{\sigma_y} = r\frac{x_p}{\sigma_x} \pm \sqrt{\frac{\eta^2 - r^2}{\phi_2}}\left\{\left(\frac{x_p}{\sigma_x}\right)^2 - \sqrt{\beta_1}\left(\frac{x_p}{\sigma_x}\right) - 1\right\}$$

where y_{xp} is the deviation from the mean of the y-arrays corresponding to the given deviation from the x-arrays, viz, x_p; η and r are the correlation ratio and correlation coefficient respectively; and σ_x and σ_y the standard deviations. $\phi_2 = \beta_2 - \beta_1 - 1$.

The values of β_1 and β_2 and $\sqrt{\beta_1}$ are given in Table 5. Except in the case of very nearly linear regression there will be no doubt as to the sign of $\sqrt{\eta^2 - r^2}$.

Inspection will tell whether the parabola is concave or convex to the x axis.

Applying this formula to the capacity-length regressions, we get for weight on length,

$$y_{xp} = 1.2601x_p - 0.0931x_p^2 + 0.5299$$

and for bulk on length,

$$y_{x\dot{p}} = 1.1607x_p - 0.0952x_p^2 + 0.5420$$

It was found upon trial of these curves that the fit was not good. The graduation toward the upper ends of the curves was particularly bad. Through the middle and over a portion of the lower end the curves represented the facts fairly well. However, at the lower end of the curves the fit was again bad. It is unlikely that either the weight or bulk of eggs over 64 mm. in length decreases at anything like the rate shown by these theoretical curves. It is to be remembered that the curves under discussion are second order parabolas fitted to the weighted means of the arrays. As has been pointed out in the case of the simple regression lines (p. 209), the frequencies at the two ends are very small. The curves attempt to fit the heavily weighted ordinates at the middle of the curve and let the ends take care of themselves.

It is of course possible that Pearson's cubic or quartic regression curves might graduate these data more satisfactorily. However, because of the very considerable probable errors connected with the use of the higher moments, it did not seem worth while to calculate these equations. Instead the attempt has been made to graduate the data with parabolas using the unweighted means.

Such parabolas can be fitted either by the method of least squares commonly used by physicists, or by Pearson's [1] method of moments. Where a series of parabolas of different orders are to be fitted to the same data the method of moments offers many advantages. This latter method has been used in the present instance.

The general equation to these parabolas is

$$y = e_0 + e_1x + e_2x^2 + \text{------} e_{n-1}x^{n-1}$$

where e_0, e_1, e_2, etc., are constants to be determined from the data.

Parabolas of the first, second, third, fourth, fifth, and sixth orders have been fitted to both the regression of weight on length and of bulk on length. This makes 12 parabolas in all.

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[1] Pearson, K. On the Systematic Fitting of Curves to Observation and Measurements. Biometrika, Vol. I, p. 265, and Vol. II, p. 1, 1902 and 1903.

The equations to the parabolas graduating the regression of weight on length are as follows:

Straight line—

$$y = 56.1367\left(1 + 0.1328\frac{x}{l}\right)$$

Second order parabola—

$$y = 56.1367\left\{1.0218 + 0.1328\frac{x}{l} - 0.0654\left(\frac{x}{l}\right)^2\right\}$$

Third order parabola—

$$y = 56.1367\left\{1.0218 + 0.0724\frac{x}{l} - 0.0654\left(\frac{x}{l}\right)^2 + 0.1007\left(\frac{x}{l}\right)^3\right\}$$

Fourth order parabola—

$$y = 56.1367\left\{1.0256 + 0.0724\left(\frac{x}{l}\right) - 0.1037\left(\frac{x}{l}\right)^2 + 0.1007\left(\frac{x}{l}\right)^3 - 0.0447\left(\frac{x}{l}\right)^4\right\}$$

Fifth order parabola—

$$y = 56.1367\left\{1.0256 + 0.1274\frac{x}{l} - 0.1037\left(\frac{x}{l}\right)^2 - 0.1562\left(\frac{x}{l}\right)^3 + 0.0447\left(\frac{x}{l}\right)^4 + 0.2312\left(\frac{x}{l}\right)^5\right\}$$

Sixth order parabola—

$$y = 56.1367\left\{1.0870 + 0.1274\frac{x}{l} - 1.3925\left(\frac{x}{l}\right)^2 - 0.1562\left(\frac{x}{l}\right)^3 + 3.9112\left(\frac{x}{l}\right)^4 + 0.2312\left(\frac{x}{l}\right)^5 - 2.8354\left(\frac{x}{l}\right)^6\right\}$$

In these equations l is equal to one-half the range, or 9 in the present instance, while x is the distance in units from the origin of the curve, which is at the middle of the range.

The equations of the parabolas fitted to the regression of bulk on length are as follows:

Straight line—

$$y = 52.5044\left(1 + 0.1288\frac{x}{l}\right)$$

Second order parabola—

$$y = 52.5044\left\{1.0203 + 0.1288\left(\frac{x}{l}\right) - 0.0610\left(\frac{x}{l}\right)^2\right\}$$

Third order parabola—

$$y = 52.5044\left\{1.0203 + 0.0718\left(\frac{x}{l}\right) - 0.0610\left(\frac{x}{l}\right)^2 + 0.0926\left(\frac{x}{l}\right)^3\right\}$$

Fourth order parabola—

$$y = 52.5044\left\{1.0287 + 0.0718\left(\frac{x}{l}\right) - 0.1450\left(\frac{x}{l}\right)^2 + 0.0926\left(\frac{x}{l}\right)^3 + 0.0980\left(\frac{x}{l}\right)^4\right\}$$

Fifth order parabola—

$$y=52.5044\left\{1.0287+0.1169\frac{x}{l}-0.1450\left(\frac{x}{l}\right)^2-0.1114\left(\frac{x}{l}\right)^3+0.0980\left(\frac{x}{l}\right)^4+0.1836\left(\frac{x}{l}\right)^5\right\}$$

Sixth order parabola—

$$y=52.5044\left\{1.0904+0.1169\frac{x}{l}+1.2305\left(\frac{x}{l}\right)^2-0.1114\left(\frac{x}{l}\right)^3+13.5352\left(\frac{x}{l}\right)^4+0.1836\left(\frac{x}{l}\right)^5+11.7305\left(\frac{x}{l}\right)^6\right\}$$

Upon plotting out these curves against the observations it was found that results which must be considered on the whole as excellent were obtained with certain of the higher order parabolas. Others gave bad results owing to having too many points of inflection. In these cases while the curve came close to the individual observations the result could not be considered a real graduation. On the whole the third, fourth, and fifth order parabolas gave the best results. Accordingly these have been chosen for graphical representation in figures 12 and 13.

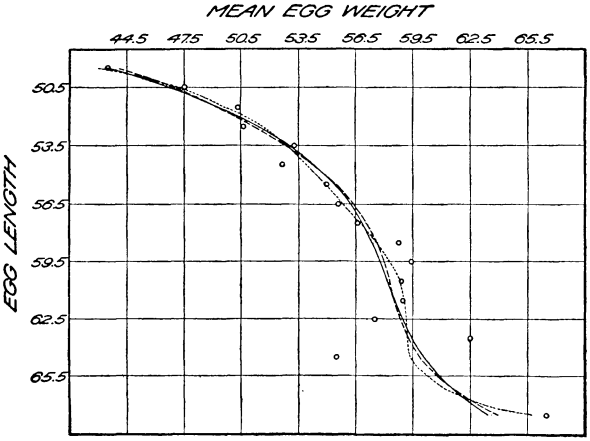

FIG. 12.—Regression of egg weight on egg length. The zigzag line gives the observations; ——— third order parabola; — — — fourth order parabola; ...... fifth order parabola.

CAPACITY-BREADTH REGRESSION.

As has been pointed out in a previous section (*cf.* p. 208), the regressions of weight on breadth and of bulk on breadth are fitted fairly well by the ordinary regression line. According to the test for linearity these regressions do not deviate sensibly from the straight line. However, inspection of the actual observations shows that toward the ends there is a marked tendency for the regression line to turn downward. It must be remembered here again that both in the test for linearity and in the actual fitting of the ordinary regression line weighted lines are used. Since in these cases the weight is mostly toward the middle of the curve, the points toward the end have little influence. Owing to the constancy of these deviations in the same direction, however, it seemed possible that they possess some real significance, even though the test for linearity used failed to attribute any to them.

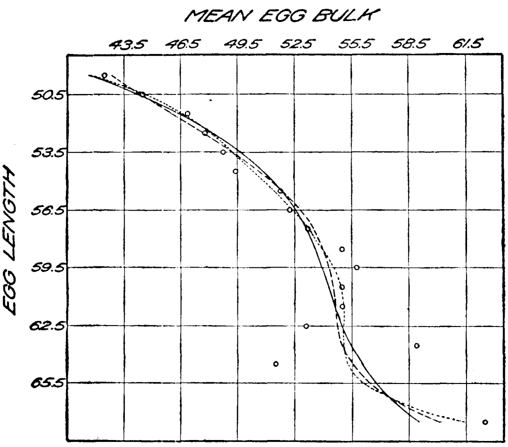

FIG. 13.—Regression of egg bulk on egg length. Significance of lines as in figure 12.

For these reasons it was decided to attempt to fit the unweighted ordinates with parabolas. In these calculations the method of least squares was used. Both the straight line and the second order parabola have been determined. It is to be remembered that these are fitted to the unweighted ordinates.

The equations to these lines are as follows:
Regression of weight on breadth:
Straight line—

$$Wt. = 2.793\ Br. - 61.345$$

Second order parabola—

$$Wt. = 0.111\ (Br.)^2 - 6.620\ Br. + 136.855$$

Regression of bulk on breadth:
Straight line—

$$Blk. = 2.692\ Br. - 60.558$$

Second order parabola—

$$Blk. = 0.043\ (Br.)^2 - 2.347\ Br. + 31.598$$

The observations and fitted lines are shown in figure.

It is evident from the diagrams that the graduations here are excellent, particularly those given by the parabolas.

MULTIPLE REGRESSION.

In the preceding section we have discussed the methods of estimating the probable magnitude of one character of an egg from a knowledge of the actual magnitude of another character. More satisfactory results can sometimes be obtained by using two or more characters from which to reach an estimation of the probable value of another. In taking up this phase of the subject we shall deal first with ordinary multiple regression, which assumes that all the simple regressions involved are linear. After that will be discussed parabolic multiple regression equations as applied to certain characters of the egg. No regressions involving more than three characters are discussed.

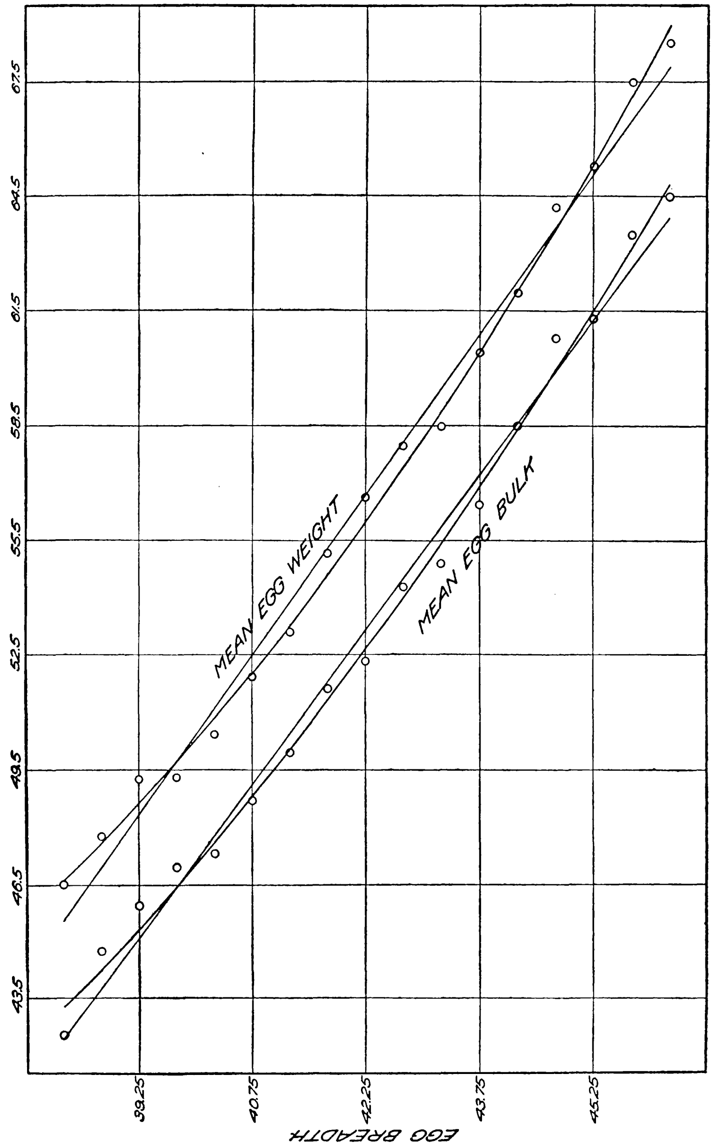

FIG. 14.—Showing the regression of weight of eggs on breadth (upper group) and the regression of bulk (volume) on breadth (lower group).

MULTIPLE LINEAR REGRESSION.

It has been shown by Yule[1] and others that the regression of x_1 on x_2 and x_3 may be represented by

$$x_1 = b_{12}x_2 + b_{13}x_3$$

where the coefficients b_{12} and b_{13} have the following values:

$$b_{12} = \frac{r_{12} - r_{13}r_{23}}{1 - r_{23}^2} \cdot \frac{\sigma_1}{\sigma_2},$$

$$b_{13} = \frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2} \cdot \frac{\sigma_1}{\sigma_3}$$

In these equations r_{12}, r_{13}, and r_{23} are the coefficients of correlation for the characters x_1, x_2, and x_3, and ς_1, ς_2 and ς_3 the standard deviations.

Further, the standard error of such a regression equation is given by

$$\Sigma_{12} = \sigma_1 \sqrt{1 - R_1^2}$$

where

$$R_1^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}$$

These equations have been used to determine the regression of weight and bulk on certain other characters. The following five regression equations have been deemed of sufficient importance to work out in detail. In a later section these equations, along with others, are tested as to their efficiency in estimating the several characters.

$$\begin{aligned}
&Wt. = 0.994Lt. + 2.660Br. - 112.217; \quad \Sigma = 0.924\\
&Wt. = 0.046Lt. + 1.056Blk. - 2.081; \quad \Sigma = 0.907\\
&Wt. = 1.098Blk. - 0.098Br. + 2.416; \quad \Sigma = 0.909\\
&Blk. = 0.018Lt. + 0.893\,Wt. + 1.534; \quad \Sigma = 0.652\\
&Blk. = 0.416Br. + 0.794\,Wt. - 9.463; \quad \Sigma = 0.773
\end{aligned}$$

In these equations *Wt.* denotes weight in grams; *Blk.* = bulk in cubic centimeters; *Lt.* = length in millimeters; and *Br.* = breadth in millimeters.

Considering the probable errors of these equations, as shown by Σ, it is clear that there is little choice between the three equations estimating weight in respect to the accuracy of the prediction. The equations in which the bulk is used as one factor have a slightly smaller error. In any case the error is about nine-tenths of a gram. This is roughly an error of 2 per cent in estimating the weight of an egg from dimensional characters.

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[1] Loc. cit.

In the case of the bulk the error is smaller when the length and weight are used as the basis of the prediction than when the breadth and weight are taken. In the former case the error is about 0.65 of a cubic centimeter, and in the latter about 0.77 of a cubic centimeter.

MULTIPLE PARABOLIC REGRESSION.

In the preceding section multiple regression equations have been determined on the assumption that the regression was linear for each character. We know, however, from previous determinations (*cf.* p. 205) that at least the regressions which involve the length of the egg as one factor are certainly not linear but parabolic. The same is probably true, though to a less marked degree, of the regressions involving the breadth of the egg. It is thus clear that the actual regression surface, in the case of multiple regression, is not a plane but rather a curved surface, either convex or concave, as the case may be. If an equation could be determined which would take account of this curvature of the regression surface, the prediction, especially near the bounding edges of the surface, should be much more satisfactory. The general theory of multiple nonlinear regression has not been developed mathematically. Hence it was necessary in dealing with the matter to start *de novo.*

It was decided first to test a paraboloid surface, generated by moving one parabola in such a way that its vertex was always on another parabola whose axis was parallel to the axis of the first, the vertical planes of the two parabolas being at right angles to each other.

The general equation for such a paraboloid surface is

$$Z=a+\frac{x}{b}+\frac{y}{c}+\frac{x^2}{d}+\frac{y^2}{e}$$

where a, b,.....e are constants to be determined from the data. The signs of the constants determine whether the surface is concave or convex.

On account of the considerable amount of calculation involved in the determination of the constants of such an equation, it was decided to try first of all an abbreviated form by dropping out the x and y terms, thus:

$$Z=a+\frac{x^2}{b}+\frac{y^2}{c}$$

It was found, however, that this equation did not give a satisfactory graduation, and the complete form was subsequently calculated.

Also on account of the labor involved in the calculations only one regression surface has been fitted. This is for the regression of weight on length and breadth. This regression is by far the most important one for practical purposes. If a reasonably close graduation of the data could be determined, such an equation would permit one to ascertain the approximate weight of blown eggs (e. g., museum specimens) and also the approximate fresh weight of eggs which had been stored for various lengths of time.

The data used for this determination, as for the other regression equations, were those of series A. Since the observations were somewhat scattered near the ends of the range, it was necessary to group the data in somewhat larger classes than those used for the other equations. As may be seen from a study of Tables 20 and 21, the length unit chosen was $1\frac{1}{2}$ millimeters. With the indicated units, distributions of the weights of the eggs falling in each length-breadth compartment were then made. The mean weight of the eggs falling in each compartment was taken as the observed weight for eggs of the indicated length and breadth. These observed weights are given in the tables in heavy-faced type, while the theoretical weights determined from the equations are given in light-faced type.

It may be observed here that the regression surface could be represented by erecting perpendicular ordinates from the center of each compartment, with the height of each ordinate proportional to the weight of the eggs in the class. A surface connecting the tops of these ordinates is the observed regression surface showing the relation of egg weight to length and breadth.

As stated above, the first attempt to graduate this paraboloid surface was made with the abbreviated general equation:

$$Z = a + \frac{x^2}{b} + \frac{y^2}{c}$$

For convenience $\frac{1}{b}$ and $\frac{1}{c}$ may be represented by q and r, and the equation becomes

$$Z = a + qx^2 + ry^2$$

where z = weight, x = length, and y = breadth. The constants of this equation were evaluated by the method of least squares.

The equation arrived at by this means was as follows:

$$Wt. = 0.00833\ (Lt.)^2 + 0.01987\ (Br.)^2 - 6.1270$$

From this equation the theoretical weights of eggs having the mean length and breadth of each class in Table 20 were calculated. These results are shown in the table in light-faced type.

TABLE 20.—*Showing the observed (bold-faced figures) and calculated (light-faced figures) mean egg weights associated with particular length and breadth dimensions.*

		Egg breadth, in millimeters.											
		38.00–38.74	38.75–39.49	39.50–40.24	40.25–40.99	41.00–41.74	41.75–42.49	42.50–43.24	43.25–43.99	44.00–44.74	44.75–45.49	45.50–46.24	46.25–46.99
Egg length, in millimeters.	49.0–50.4	43.75	44.91	48.09	**43.66** 47.29	48.51	49.75	51.02	52.31	53.62	54.95	56.31	57.69
	50.5–51.9	**42.98** 45.02	46.17	47.37	**47.71** 48.55	49.77	**53.14** 51.01	**53.62** 52.28	**54.26** 53.57	54.88	56.21	57.58	58.96
	52.0–53.4	46.32	**47.94** 47.47	**46.84** 48.65	**48.61** 49.85	**50.44** 51.07	**52.29** 52.31	**53.55** 53.58	**56.92** 54.87	56.18	57.51	58.87	60.25
	53.5–54.9	**45.65** 47.65	**46.01** 48.81	**49.01** 49.98	**49.76** 51.18	**52.25** 52.41	**53.80** 53.65	**55.43** 54.92	**57.52** 56.71	**57.39** 57.52	58.85	60.21	61.59
	55.0–56.4	**47.13** 49.03	**48.98** 50.18	**49.56** 51.36	**51.44** 52.56	**53.25** 53.78	**55.32** 55.02	**57.19** 56.29	**59.48** 57.58	**61.04** 58.89	**62.67** 60.23	61.58	62.96
	56.5–57.9	50.44	**49.73** 51.59	**50.49** 52.77	**52.79** 53.97	**54.67** 55.19	**56.36** 56.44	**58.56** 57.70	**60.42** 58.99	**61.45** 60.30	**65.72** 61.64	62.99	**68.48** 64.37
	58.0–59.4	51.89	**50.08** 53.04	**52.61** 54.22	**54.90** 55.42	**55.50** 56.64	**58.43** 57.89	**61.03** 59.15	**62.15** 60.44	**65.80** 61.75	**68.03** 63.09	**67.54** 64.44	65.82
	59.5–60.9	53.38	**50.45** 54.53	**52.64** 55.71	**55.71** 56.91	**57.90** 58.13	**59.41** 59.37	**62.20** 60.64	61.93	**65.07** 63.24	**67.82** 64.57	65.93	67.31
	61.0–62.4	54.90	**56.06** 56.06	57.23	**57.06** 58.43	**60.52** 59.65	60.90	62.17	**67.53** 63.45	64.77	66.10	67.46	68.83
	62.5–63.9	56.46	57.62	**54.59** 58.79	**57.36** 59.99	61.22	62.46	**64.62** 63.73	**69.47** 65.02	66.33	67.66	69.02	70.40
	64.0–65.4	58.06	**55.94** 59.22	60.39	61.59	62.82	64.06	65.33	66.62	67.93	69.26	70.62	71.99
	65.5–66.9	59.70	60.85	62.03	63.23	64.45	65.70	66.96	68.25	69.56	70.90	72.25	73.63
	67.0–68.4	61.37	62.53	63.71	64.91	66.13	**66.54** 67.37	68.64	69.93	71.24	72.57	73.93	75.31

From this table we note the following points:

1. If the observed and calculated weights for classes falling near the center of the table are compared, they clearly agree very closely. In a few cases the difference reaches as much as 1 gram, but these differences balance each other. The general effect, then, is to smooth out the irregularities of the observations, and for the center of the table the graduation must be regarded as excellent.

2. However, a good fit for the center of the table was obtained from the multiple linear equations. It is at the ends and sides of the table that the real test comes. We may compare first the columns of the table. In the first column on the left side there are three observations. In each of these cases the observed weight is less than the calculated by approximately 2 grams. In the second column on this side there are eight observations, and here, with the exception of two cases, the observed weights are markedly less than the calculated. The same is true to a somewhat less degree of the third column.

3. Passing to the right side of the table, it will be seen that here the observed weights are significantly larger than the calculated. Thus, in the third column from the right there are four observations in which the observed weight exceeds the calculated by from 2.4 grams to nearly 5 grams.

4. At the top and bottom of the table it is also seen that there is considerable discrepancy between the observed and calculated weights.

5. There is, then, a marked tendency for the calculated weights to overestimate the observed weight in the case of narrow eggs and to underestimate it in the case of broad eggs. There is also a slight tendency to underestimate the weights of both very long and very short eggs. The observations at the top and bottom of the table, however, are too few and based on too small a number of eggs to make this point entirely certain.

The constants to this equation were calculated twice independently, and there do not appear to be any significant arithmetical errors to explain the discrepancies.

In view of the unsatisfactory graduation obtained with this equation it was decided to employ more constants. Accordingly, the complete general equation for this type of surface was used, viz:

$$Z = a + \frac{x}{b} + \frac{y}{c} + \frac{x^2}{d} + \frac{y^2}{e}$$

This was also evaluated by the method of least squares. The solution of this equation involved a considerable amount of arithmetical work.

The final equation, stated in terms of the egg characters, was:

$$Wt. = 0.0198Lt. - 0.0426Br. + 0.0172(Lt.)^2 + 0.0322(Br.)^2 - 56.6665.$$

The results obtained with this equation, while better than in any of the previous cases, were still not entirely satisfactory. Since considerable time had already been spent in calculating these equations, it was decided to try to modify the present formula empirically rather than to make a new assumption and calculate another equation. This method of procedure has a precedent in much of the curve fitting that is regularly done, e. g., in the usual way of dealing with certain logarithmic curves.

The method followed was to take the equation given above as a basis and then to modify its constants by small amounts so as to produce the desired effect, which was tested by interpolation. After a number of attempts, the following equation was reached:

$$Wt. = 1.0513Lt. - 0.0426Br. - 0.00059(Lt.)^2 + 0.0322(Br.)^2 - 56.6665.$$

The theoretical weights for the various classes were calculated from this equation, with the results shown in Table 21.

TABLE 21.—*Showing the observed (bold-faced figures) and calculated (light-faced figures) mean egg weights associated with particular length and breadth dimensions.*

	Breadth, in millimeters.											
Length, in millimeters.	38.00–38.74	38.75–39.49	39.50–40.24	40.25–40.99	41.00–41.74	41.75–42.49	42.50–43.24	43.25–43.99	44.00–44.74	44.75–45.49	45.50–46.24	46.25–46.99
49.0–50.4	39.88	41.72	43.59	**43.66** 45.50	47.45	49.43	51.35	53.50	55.59	57.72	59.88	62.08
50.5–51.9	**42.98** 41.37	43.21	45.08	**47.71** 46.99	48.93	**53.14** 50.92	**53.62** 52.93	**54.26** 54.99	57.08	59.20	61.37	63.57
52.0–53.4	42.85	**47.94** 44.69	**46.84** 46.57	**48.61** 48.47	**50.44** 50.42	**52.29** 52.40	**53.55** 54.42	**56.92** 56.47	58.56	60.69	62.85	65.05
53.5–54.9	**45.65** 44.34	**46.01** 46.17	**49.01** 48.05	**49.76** 49.96	**52.25** 51.90	**53.80** 53.88	**55.43** 55.90	**57.52** 57.95	**57.39** 60.04	62.17	64.33	66.53
55.0–56.4	**47.13** 45.82	**48.98** 47.65	**49.56** 49.53	**51.44** 51.47	**53.25** 53.38	**55.32** 55.36	**57.19** 57.38	**59.48** 59.44	**61.04** 61.52	**62.67** 63.65	65.81	68.01
56.5–57.9	47.29	**49.73** 49.13	**50.49** 51.00	**52.79** 52.91	**54.67** 54.83	**56.36** 56.84	**58.56** 58.86	**60.42** 60.91	**61.45** 63.00	**65.72** 65.13	67.29	**68.48** 69.49
58.0–59.4	48.76	**50.08** 50.60	**52.61** 52.48	**54.90** 54.38	**55.50** 56.33	**58.43** 58.31	**61.03** 60.33	**62.15** 62.38	**65.80** 64.47	**68.03** 66.60	**67.54** 68.76	70.96
59.5–60.9	50.24	**50.45** 52.07	**52.64** 53.95	**55.71** 55.85	**57.90** 57.80	**59.41** 59.78	**62.20** 61.80	63.85	**65.07** 65.94	**67.82** 68.07	70.23	72.43
61.0–62.4	51.84	**56.06** 53.68	55.56	**57.06** 57.46	**60.52** 59.41	61.39	63.41	**67.53** 65.46	67.55	69.68	71.84	74.04
62.5–63.9	53.17	55.01	**54.59** 56.88	**57.36** 58.79	60.73	62.72	**64.62** 64.73	**69.47** 66.79	68.88	71.00	73.17	75.37
64.0–65.4	54.63	**55.94** 56.47	58.34	60.25	62.20	64.18	66.20	68.25	70.34	72.47	74.63	76.83
65.5–66.9	56.09	57.93	59.80	61.71	63.66	65.64	67.66	69.71	71.80	73.93	76.09	78.29
67.0–68.4	57.55	59.40	61.26	63.17	65.11	**66.54** 67.10	69.11	71.17	73.26	75.38	77.55	79.75

A study of the observed and calculated weights in this table shows that the graduation is much better than in the other cases.

While there are still some discrepancies, it will be seen that these are evenly distributed over the table. The plus and minus deviations practically offset each other. As a matter of fact the sum of the deviations in which the calculated weight exceeds the observed gives a total of 28.086 grams, while for cases in which the observed weight exceeds the calculated the sum is 27.368 grams. Thus the total deviation in both directions for the 70 observed classes is 55.454 grams, giving a mean deviation of 0.792 gram.

It is further seen that in relatively few instances do the calculated and observed weights differ by more than about 1 gram. Greater deviations are nearly always in classes where only a single egg has furnished the "observed" record. Thus the largest single deviation for any class is found in the second column and the third row. Here the observed weight exceeds the calculated by 3.249 grams. This observation is based upon the weight of a single egg. Comparing its observed weight with those which are near it in the other classes, it is clear that it is larger than it should be.

In five other cases the differences between the observed and calculated weights reach 2 grams. In four of these cases the observed data are from single eggs, while in the other case the observation is based on two eggs, in one of which the observed weight is considerably too high, as shown by other eggs in near-by columns.

Since the preceding discussion of regression has been rather extended, it seems desirable to collect together here in one place the various regression formulæ. This will then be followed by a discussion of tests of some of these various formulæ on random samples of eggs.

Table of regression equations.

(1) $Wt. = 1.1229\,Lt. - 7.9792$. (See p. 207.)
(2) $Wt. = 2.8035\,Br. - 62.2483$. (See p. 208.)
(3) $Wt. = 1.0704\,Blk. - 0.2681$. (See p. 209.)
(4) $Blk. = 1.0203\,Lt. - 5.5879$. (See p. 207.)
(5) $Blk. = 2.6431\,Br. - 58.9081$. (See p. 208.)
(6) $Blk. = 0.8979\,Wt. + 2.2563$. (See p. 209.)
(7) $Wt. = 0.9944\,Lt. + 2.6595\,Br. - 112.2168$. (See p. 216.)
(8) $Wt. = 0.0459\,Lt. + 1.0555\,Blk. - 2.0809$. (See p. 216.)
(9) $Wt. = 1.0978\,Blk. - 0.0980\,Br. + 2.4157$. (See p. 216.)
(10) $Blk. = 0.0182\,Lt. + 0.8925\,Wt. + 1.5335$. (See p. 216.)
(11) $Blk. = 0.4163\,Br. + 0.7942\,Wt. - 9.4630$. (See p. 216.)

WEIGHT ON LENGTH.

(12) $y_{xp} = 1.2601x_p - 0.093x_p^2 + 0.5299$. (See p. 211.)

BULK ON LENGTH.

(13) $y_{xp} = 1.1607x_p - 0.0952x_p^2 + 0.5420$. (See p. 211.)

WEIGHT ON LENGTH—PARABOLAS (METHOD OF MOMENTS).

(14) $y = 56.1367\left\{1 + 0.1328\left(\frac{x}{l}\right)\right\}$. (See p. 212.)

(15) $y = 56.1367\left\{1.0218 + 0.1328\left(\frac{x}{l}\right) - 0.0654\left(\frac{x}{l}\right)^2\right\}$. (See p. 212.)

(16) $y = 56.1367\left\{1.0218 + 0.0724\left(\frac{x}{l}\right) - 0.0654\left(\frac{x}{l}\right)^2 + 0.1007\left(\frac{x}{l}\right)^3\right\}$. (See p. 212.)

(17) $y = 56.1367\left\{1.0256 + 0.0724\left(\frac{x}{l}\right) - 0.1037\left(\frac{x}{l}\right)^2 + 0.1007\left(\frac{x}{l}\right)^3 + 0.0447\left(\frac{x}{l}\right)^4\right\}$. (See p. 212.)

(18) $y = 56.1367\left\{1.0256 + 0.1274\left(\frac{x}{l}\right) - 0.1037\left(\frac{x}{l}\right)^2 - 0.1562\left(\frac{x}{l}\right)^3 + 0.0447\left(\frac{x}{l}\right)^4 + 0.2312\left(\frac{x}{l}\right)^5\right\}$. (See p. 212.)

(19) $y = 56.1367\left\{1.0870 + 0.1274\left(\frac{x}{l}\right) - 1.3925\left(\frac{x}{l}\right)^2 - 0.1562\left(\frac{x}{l}\right)^3 + 3.9112\left(\frac{x}{l}\right)^4 + 0.2312\left(\frac{x}{l}\right)^5 - 2.8354\left(\frac{x}{l}\right)^6\right\}$. (See p. 212.)

BULK ON LENGTH—PARABOLAS (METHOD OF MOMENTS).

(20) $y = 52.5044\left\{1 + 0.1288\left(\frac{x}{l}\right)\right\}$. (See p. 212.)

(21) $y = 52.5044\left\{1.0203 + 0.1288\left(\frac{x}{l}\right) - 0.0610\left(\frac{x}{l}\right)^2\right\}$. (See p. 212.)

(22) $y = 52.5044\left\{1.0203 + 0.0718\left(\frac{x}{l}\right) - 0.0610\left(\frac{x}{l}\right)^2 + 0.0926\left(\frac{x}{l}\right)^3\right\}$. (See p. 212.)

(23) $y=52.5044\left\{1.0287+0.0718\left(\frac{x}{l}\right)-0.1450\left(\frac{x}{l}\right)^2+0.0926\left(\frac{x}{l}\right)^3+0.0980\left(\frac{x}{l}\right)^4\right\}.$ (See p. 212.)

(24) $y=52.5044\left\{1.0287+0.1169\left(\frac{x}{l}\right)-0.1450\left(\frac{x}{l}\right)^2-0.1114\left(\frac{x}{l}\right)^3+0.0980\left(\frac{x}{l}\right)^4+0.1836\left(\frac{x}{l}\right)^5\right\}.$ (See p. 213.)

(25) $y=52.5044\left\{1.0904+0.1169\left(\frac{x}{l}\right)+1.2305\left(\frac{x}{l}\right)^2-0.1114\left(\frac{x}{l}\right)^3+13.5352\left(\frac{x}{l}\right)^4+0.1836\left(\frac{x}{l}\right)^5+11.7305\left(\frac{x}{l}\right)^6\right\}.$ (See p. 213.)

WEIGHT ON BREADTH—PARABOLAS (METHOD OF LEAST SQUARES).

(26) $Wt.=2.793\ Br.-61.345.$ (See p. 214.)

(27) $Wt.=0.111\ (Br.)^2-6.620\ Br.+136.855.$ (See p. 214.)

BULK ON BREADTH.

(28) $Blk.=2.692\ Br.-60.558.$ (See p. 214.)

(29) $Blk.=0.043\ (Br.)^2-2.347\ Br.+31.598.$ (See p. 214.)

WEIGHT ON LENGTH AND BREADTH.

(30) $Wt.=0.0083\ (Lt.)^2+0.0199\ (Br.)^2-6.1270.$ (See p. 218.)

(31) $Wt.=0.0198\ Lt.-0.0426\ Br.+0.0172\ (Lt.)^2+0.0322\ (Br.)^2-56.6665.$

(32) $Wt.=1.0513\ Lt.-0.0426\ Br.-0.00059\ (Lt.)^2+0.0322\ (Br.)^2-56.6665.$

TESTS OF REGRESSION EQUATIONS.

A test of certain of the regression equations was made on a random sample of 20 eggs from Series A. Starting with egg No. 1, every fifteenth egg which had been used in the original data was selected until 20 were obtained. The theoretical weight of each of these 20 eggs was calculated by selected equations and the difference between this and the actual weight of the egg was found in each case. The mean deviation regardless of sign was taken as the measure of accuracy of the estimation.

Only the equations for estimating the weight have been used in this test, and in the case of the parabolas only those which have been shown to give the best fits.

In Table 22 the actual weight of the eggs used is given in the second column. In each succeeding column there is given the difference between the observed and calculated weights. Appropriate signs indicate the direction of the deviation in each case. Thus when the calculated weight is larger than the observed, the sign is + and when smaller the sign is −. The numbers at the tops of the columns refer to the equation numbers as given on pages 222–223. The total and mean deviations are given at the bottoms of the columns. The column of the observed weights shows whether the egg was exceptionally heavy or light as compared with the mean weight of eggs of this series as given in Table 2.

TABLE 22.—*Showing the results given by different regression equations on a random sample of eggs.*

Egg No.	Observed weight.	Difference between observed weight and the weight calculated by equation No. —												
		1	2	3	7	8	9	12	14	15	26	27	30	32
	Gms.	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*
1..........	56.88	− 0.50	−0.96	+1.72	+1.77	+1.81	+1.78	+ 1.71	+ 0.96	+ 0.74	−0.50	−1.15	+1.39	+1.83
2..........	52.96	− 1.19	+2.68	− .51	− .49	− .62	− .60	− 1.98	− 1.25	− .40	+3.09	+2.44	− .40	− .37
3..........	57.13	+ .76	−4.29	− .34	−1.84	− .25	−2.63	+ 1.10	+ .36	+ .30	−3.82	−4.31	−1.11	−1.74
4..........	51.07	+ 3.09	+ .22	− .46	− .55	− .44	− .44	+ 3.40	+ 3.22	+ 4.17	+ .70	+ .39	+ .90	− .37
5..........	53.15	+ 2.19	+1.98	+1.14	+1.18	+1.12	+1.12	+ 1.55	+ 1.32	+ 2.24	+2.45	+1.81	+1.21	+1.28
6..........	56.74	− 1.40	− .26	−1.20	−1.14	−1.24	−1.24	− 2.04	− 2.27	− 1.35	+ .20	− .45	−1.57	−1.03
7..........	59.88	− 2.31	−2.08	− .17	− .16	− .14	− .15	− 1.89	− 2.41	− 2.46	−1.63	−2.24	−1.20	− .08
8..........	69.47	− 6.64	−7.64	− .50	−1.27	− .39	− .39	− 9.41	− 9.32	−10.31	−7.20	−7.40	−3.55	−1.11
9..........	61.70	− 3.23	−4.29	−1.55	−1.56	−1.49	−1.51	− 3.06	− 3.80	− 4.05	−3.84	−4.47	−2.46	−1.49
10..........	65.80	− 7.74	−3.49	− .99	−1.37	−1.01	− .99	− 7.44	− 8.19	− 8.30	−3.05	−3.17	−3.90	−1.12
11..........	69.51	−11.05	−4.96	−1.81	−2.51	−1.85	−1.82	−10.95	−11.55	−11.83	−4.53	−4.19	−5.74	−2.12
12..........	59.83	− .19	−5.42	−1.74	−1.55	− .60	−1.64	− .49	− 1.24	− 1.82	−4.96	−5.56	−1.44	−1.54
13..........	50.11	+ 4.15	+ .90	− .56	− .74	− .57	− .56	+ 2.99	+ 3.14	+ 4.15	+1.38	+1.11	+ .89	− .54
14..........	52.81	− .79	+2.88	+ .96	− .01	− .14	− .12	− 1.43	− .81	− 2.09	+3.35	+2.70	+ .01	+ .11
15..........	52.31	+ 3.85	− .18	+ .72	+ .78	+ .78	+ .77	+ 4.43	4.79	+ 4.32	+ .30	− .12	+1.82	+ .92
16..........	49.63	+ 2.80	+2.00	− .15	− .32	− .18	− .17	+ 2.57	4.70	+ 3.80	+2.48	+2.12	+1.14	− .14
17..........	54.84	+ 1.71	−1.05	+ .05	+ .17	+ .11	+ .09	+ 2.28	+ 2.37	+ 2.00	− .58	−1.15	+ .58	+ .26
18..........	54.96	− .31	+ .94	+ .24	+ .31	+ .21	+ .21	+ 1.92	− .19	+ .69	+1.40	−1.91	+ .08	+ .41
19..........	54.08	− .29	+1.05	− .28	− .24	− .32	− .32	− .11	− .16	+ .83	+1.52	+ .88	− .17	− .14
20..........	43.66	+ 4.25	+7.58	+1.85	+1.27	+1.69	+1.74	− .15	+ 2.97	+ 1.00	+8.06	+7.75	+3.40	+1.48
Totals......	......	58.44	54.85	16.94	19.23	14.96	18.29	60.90	65.02	66.85	55.04	55.32	32.96	18.08
Means......	......	2.92	2.74	.85	.96	.75	.91	3.05	3.25	3.34	2.75	2.77	1.65	.90

From this table it is seen that five of the equations tested give an average error of less than 1 gram in the prediction of the weight of these 20 eggs. This means an error of about 2 per cent. It would appear that this is probably about as close as it is possible to approximate to the weight of an egg by means of regression equations based on linear dimensions.

Comparing the different equations it is seen that the best result on the 20 eggs taken for the test is given by equation No. 8, which is the multiple linear regression equation for weight on length and bulk. The computed bulk, of course, involves the breadth of the egg. It has been shown earlier in this bulletin that the bulk calculated by the formula here used gives a close approximation to the true volume. So virtually equation 8 is for the regression of weight on the length and volume of the egg.

It is of interest to note that in spite of the fact of nonlinearity of the weight regressions in general, nevertheless a linear equation when tested on a random sample of eggs gives as good a result as the best of the nonlinear equations.

If the weight of the egg is to be estimated on the basis of but a single linear dimension, breadth is apparently slightly the better dimension to choose, and equation 2 the best equation. Here again we see the regression line giving a better result than either the best fitting straight line to the unweighted means or the second order parabola fitted by least squares, even though the latter fits the observed regression line better than the regression equation 2.

The underlying purpose with which this investigation of regression was undertaken was to test concretely whether the true theory of regression which takes account of the variabilities of the related characters would give distinctly better results than the simple graduation, by least squares or moments, of the observed, unweighted regression means. On theoretical grounds there could be no doubt as to the outcome. It is a source of some satisfaction, however, to have a concrete and practical demonstration that even though we may fit with a theoretical curve or surface the observed regression points or surfaces much closer than they are fitted by the true regression equations, nevertheless, when the resulting prediction formulæ are tested on random samples from the same population, no substantial gain in accuracy of prediction is obtained. On the contrary, in not a few cases the results will be distinctly worse than with a true regression equation, even though the latter involves fewer characters.

VARIATION AND CORRELATION IN THE EGGS OF WILD BIRDS AS COMPARED WITH THE DOMESTIC FOWL.

Under conditions of domestication it is obvious that there can be no natural selection of the fowl on the basis of physical characters of the egg. Furthermore, in the stock which furnished the data of Series A there has been no artificial selection in respect to size, shape, or color of eggs, at least for a considerable period of years. It is evident, then, that whatever degree of variation is exhibited in this stock in respect to these characters is independent of any influence of selection. On the other hand, it has been generally assumed by followers of the neo-Darwinian school of evolutionary philosophy that the eggs of wild birds have been and are the objects of natural selective processes, and that their characteristics in respect of size, shape, and color are in some degree determined by natural selection. This conclusion has been definitely stated by Latter [1] for the cuckoo's egg.

In view of these considerations it is of interest to compare the eggs of the domestic fowl with those of wild birds in respect of variation and correlation. Considerable material is available for such comparison.

First may be considered variation and correlation in the egg of *Corvus frugilegus* L. The data here analyzed for this form are those published some years ago by Tur.[2] In his discussion of the data Tur considered only simple variation and calculated no correlations. The data include 200 eggs. One hundred and eighty-eight of these were collected from 73 nests. The remainder were "extra" eggs, for which the history was not known. In forming the correlation tables from these data we made use of units of grouping somewhat different from those used by Tur in his calculations of the frequency constants. This accounts for the slight discrepancies between his values and those here tabled.

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[1] Latter, O. H. The Egg of Cuculus canorus. An Enquiry into the Dimensions of the Cuckoo's Egg and the Relation of the Variation to the Size of the Eggs of the Foster Parent, with Notes on Coloration, etc. *Biometrika*, Vol. I, pp. 164–176, 1902.

[2] Tur, Jan. Wahania indywidualne jaj gawrona (*Corvus frugilegus* L.), zebranych w Wilanowie prod Warszawa w kwietniu r. 1906. Pamietnika Fiziograficznego, T. XIX, pp. (of reprint) 1-15, 1906.

In Table 23 are given the variation constants for the eggs of the rook.

Table 23.—*Constants for variation in the egg of Corvus frugilegus L.* (*Tur's data.*)

Character.	Mean.	Standard deviation.	Coefficient of variation.	Coefficient of variation for corresponding characters of fowls' eggs (from Table 2).	Difference between rook and fowl coefficients.
Length ....mm..	40.025±0.103	2.156±0.073	5.387±0.182	4.237±0.095	+1.150±0.205
Breadth................do....	28.183± .040	.843± .028	2.992± .101	3.287± .074	− .295± .125
Weight..................gm..	16.335± .071	1.497± .050	9.166± .312	8.364± .189	+ .802± .365

The coefficients of correlation for these characters of the rook's egg are as follows:

Correlation of length and breadth, $r = 0.257 \pm 0.045$.

Correlation of length and weight, $r = 0.672 \pm 0.026$.

Correlation of breadth and weight, $r = 0.844 \pm 0.013$.

Of these constants only the coefficients of variation and correlation are directly comparable between the fowl and the rook. By reference to Table 2 it will be seen that:

1. The relative variation and the degree of correlation are of the same general order of magnitude in the case of the egg of the fowl and that of the rook. There is no evidence whatsoever of that considerable increase in variability in the fowl's egg, which on *a priori* grounds might be supposed to result from the cessation of selection, both natural and artificial. with reference to the characters under discussion.

2. The egg of the rook is *more* variable in length than that of the fowl, to a degree which is statistically significant. In the case of weight the rook's egg is also more variable than the fowl's. The difference, however, is only a little more than twice the probable error and can not safely be regarded as statistically significant.

3. The egg of the rook is somewhat less variable in breadth than the fowl's egg, but the difference is probably not significant.

In considering these differences in variation it must be remembered that the rook's eggs are in clutches, and do not represent the product of 200 different individuals. On this account they might be expected to be somewhat less variable than the hens' eggs. In spite of this factor and in spite of any putative influence of natural selection, the eggs of the rook exhibit *greater* relative variability than hens' eggs in respect to two out of three characters measured.

A similar comparison of a wild bird's eggs with those of the domestic fowl may be made for the English sparrow (*Passer domesticus*). Pearson[1] has published the variation constants for tolerably long series of the eggs of this bird, including both his own and Bumpus' series of measurements on English birds, and the American series of Bumpus. This comparison is made in Table 24. Another character the breadth/length index, may be compared in this case. For this character the standard deviations are directly comparable.

TABLE 24.—*Showing the relative variability of eggs of (a) Passer domesticus and (b) the domestic fowl.*

Character.	English series (Pearson).	English series (Bumpus).	American series (Bumpus).	Fowl.
Length (coefficient of variation)	5.47	4.91	4.92	4.24
Breadth (coefficient of variation)	3.38	(?)	(?)	3.29
Index (standard deviation)	3.76	3.22	3.68	3.79

From this table it is evident that the sparrow's and the hen's egg are substantially equally variable in dimensional characters. In respect to length and breadth the egg of the sparrow is somewhat more variable than that of the fowl, not less variable, as it would be expected to be if natural selection were an important factor in determining the degree of variation exhibited by birds' eggs. None of the differences in Table 24 can be regarded as statistically significant. In other words, the egg of the wild bird is neither more nor less variable than that of the domesticated fowl in this case.

Table 25 gives the coefficients of variation for egg length and egg breadth in a number of different species of wild birds, as determined by Latter,[2] for comparison with the fowl.

TABLE 25.—*Showing the relative variability of eggs of (a) various species of wild birds, and (b) the domestic fowl.*

Species.	Coefficient of variation for egg length.	Coefficient of variation for egg breadth.	Species.	Coefficient of variation for egg length.	Coefficient of variation for egg breadth.
Cuckoo	4.72	3.93	House sparrow	5.47	3.38
Blackbird	4.61	3.62	Hedge sparrow	4.02	2.81
Song thrush	3.64	2.50	Robin	4.24	3.09
Starling	3.68	1.94	Linnet	3.49	2.69
Wagtail[3]	6.96	2.52			
Yellowhammer	3.17	2.53	Mean (unweighted)	4.48	2.98
Tree pipit	3.49	2.97	Domestic fowl	4.29	3.79
Meadow pipit	6.37	3.84			

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[1] Pearson, K. Variation of the Egg of the Sparrow (*Passer domesticus*). Biometrika, Vol. I, pp. 256–257, 1902.

[2] Latter, O. H. Loc. cit., p. 176.

[3] A mixed series.

From this table we see that again the wild birds' eggs are more variable on the average in length than are those of the domestic fowl. The difference here is probably not significant. It indicates merely that the domestic fowl is certainly not, as might have been supposed to be the case on *a priori* grounds, more variable than wild birds in respect to egg length.

In breadth the eggs of the fowl show a higher coefficient than the average for the wild birds. In view of the other facts set forth in this section, however, probably but little significance can be attached to this result. It may be related to the greater laying activity of the domesticated bird, as a result of which the muscular tonus of the wall of the oviduct is not so precisely maintained at a given point as in the case of wild species laying but a few eggs. This suggestion is purely hypothetical, however. It is not yet clear by any means that the eggs of wild birds are in general actually less variable in breadth than those of the domestic fowl.

Considering all the facts set forth in this section together, they seem to us to make it highly probable that the essential factors concerned in the determination of the degree of variation in size and shape exhibited by birds' eggs are purely physiological and in no direct or immediate way (if at all) related to the action of natural selection. The physiological mechanism concerned in the determination of the shape and size of a bird's eggs involves chiefly the secretory and muscular systems of the oviduct. This mechanism is essentially similar in wild and domestic birds. It seems reasonable to suppose that it would lead to the same sort of result in respect to variation of product in the one case as in the other.

It certainly does not appear from the available evidence that it is essential to the survival of a wild bird that its eggs shall be less variable than those of the domestic fowl, in which egg variation obviously had no survival value whatsoever.

VARIATION IN THE SIZE OF EGGS IN THE INDIVIDUAL AS COMPARED WITH THE RACE.

So far we have considered only intraracial variation in the size of eggs. Clearly egg production offers an excellent opportunity to carry the analysis of variation further, through the discussion of intra-individual variation. The same bird produces a considerable number of eggs in the course of a year. How much variation is shown by the eggs of the same individual bird as compared with that exhibited by the flock or race as a whole? Poultrymen are generally impressed with the similarity of the eggs of the same bird, which is indeed striking in some cases. On the other hand, as will presently appear, there are other individuals whose eggs vary among themselves quite as much or even more than do the eggs of the flock or the race. Individuality of a bird in respect to nonuniformity (i. e., variability) of its product is just as definite and real a phenomenon as is the more commonly known individuality in respect to average size, shape, or color of the product.

It should be said before entering upon the discussion of the data that intraindividual variation in eggs has been made the subject of a very thorough investigation by Dr. Maynie R. Curtis, working in this laboratory. Her researches have extended over a period of years. The results will be published shortly. The discussion of the data in the present bulletin is only preliminary to the more thorough treatment which she will give the matter.

The material for the present treatment of intraindividual variation in eggs (Series B, p. 176) was derived from 100 Barred Plymouth Rock hens, all approximately 1 year old at the time the data were taken. These birds were a part of the same stock which furnished the data for Series A. The intraindividual and the racial data are therefore strictly comparable in the present case. Twenty-five consecutive eggs were measured from each of the 100 birds, double-yolked or otherwise abnormal eggs being rejected. The eggs were taken during the spring laying cycle (cf. Part II of this study, p. 99); that is, in the months of March, April, and May. Dr. Curtis has shown in her unpublished work that the eggs of this season of the year constitute an almost absolutely perfect random sample of the total eggs produced during the year. This, of course, demonstrates the validity of the present data for the use to which they are put, namely, the study of intraindividual variation. The length and breadth of each of the 2,500 eggs used in this study were measured to hundredths of a millimeter. Then for each of the 100 birds the mean, standard deviation, and coefficient of variation for length and breadth were calculated from the sample of 25 eggs. In making these calculations the data were not grouped.

In Table 26 the constants for each bird are given.

TABLE 26.—*Constants for intraindividual variation in length and breadth of the eggs of 100 Barred Plymouth Rock hens.*

Case No.	Bird No.	Length.			Breadth.			Case No.	Bird No.	Length.			Breadth.		
		Mean.	Standard deviation.	Coefficient of variation.	Mean.	Standard deviation.	Coefficient of variation.			Mean.	Standard deviation.	Coefficient of variation.	Mean.	Standard deviation.	Coefficient of variation.
1	1	53.03	1.28	2.40	42.64	0.81	1.89	53	408	57.60	1.06	1.83	42.94	0.50	1.17
2	7	56.14	1.62	2.88	41.09	.51	1.25	54	409	59.40	1.73	2.92	45.33	.66	1.45
3	8	55.54	1.38	2.49	42.27	.72	1.69	55	410	55.04	1.48	2.70	41.25	1.00	2.42
4	10	54.45	1.76	3.24	40.61	1.16	2.86	56	414	58.68	1.48	2.52	43.28	1.09	2.52
5	12	55.38	1.39	2.51	42.11	.91	2.16	57	415	56.36	1.83	3.25	41.66	.89	2.15
6	18	57.23	2.17	3.79	41.07	1.00	2.44	58	419	64.89	1.54	2.80	41.57	.60	1.44
7	19	56.03	2.31	4.13	40.04	1.18	2.94	59	422	56.86	1.57	2.76	41.78	.59	1.41
8	20	55.77	1.38	2.48	42.73	1.06	2.47	60	424	54.72	1.36	2.48	43.15	1.28	2.98
9	21	55.31	1.84	3.33	41.08	.61	2.46	61	428	53.73	1.44	2.68	42.57	.71	1.66
10	23	52.10	1.12	2.14	38.36	.75	1.95	62	434	56.33	1.40	2.48	42.50	.47	1.10
11	27	59.78	1.87	3.13	42.05	1.12	2.66	63	438	56.20	1.40	2.48	42.24	.83	1.96
12	39	57.82	1.50	2.59	42.90	1.01	2.35	64	441	58.49	1.98	3.39	44.96	.93	2.08
13	42	54.71	1.23	2.25	42.71	.48	1.12	65	442	54.92	1.18	2.14	41.44	.62	1.51
14	52	54.78	1.73	3.15	41.49	1.21	2.92	66	443	54.33	1.07	1.96	41.23	.60	1.45
15	58	58.59	1.86	3.18	42.63	.77	1.81	67	444	55.09	1.03	1.87	42.27	.74	1.76
16	61	57.73	2.45	4.24	41.39	.76	1.84	68	447	54.08	1.42	2.63	41.90	.91	2.17
17	66	57.89	2.04	3.53	42.65	.69	1.63	69	450	55.60	1.61	2.89	42.90	.87	2.04
18	73	53.97	1.07	1.98	44.64	.90	2.01	70	C559	56.82	1.69	2.98	40.82	.65	1.59
19	87	55.19	1.29	2.34	41.61	.47	1.14	71	705	56.28	1.65	2.94	41.80	1.25	2.99
20	90	58.28	1.66	2.85	42.57	.66	1.54	72	707	54.72	1.47	2.68	38.87	.92	2.37
21	99	56.27	2.09	3.71	41.65	.79	1.91	73	709	55.91	2.68	4.79	40.79	.78	1.92
22	112	59.09	1.63	2.76	40.36	.64	1.58	74	712	53.32	1.41	2.65	40.69	.82	2.01
23	113	55.04	1.24	2.25	42.69	.61	1.42	75	717	57.10	1.82	3.19	41.70	.67	1.60
24	121	54.78	1.72	3.15	40.34	.75	1.86	76	725	56.36	1.86	3.30	40.49	.98	2.42
25	122	54.16	1.45	2.67	41.09	.73	1.77	77	726	58.67	2.42	4.13	41.87	.90	2.16
26	129	54.04	1.51	2.79	41.94	.98	2.34	78	731	56.42	1.56	2.76	38.99	1.32	3.39
27	152	55.55	1.26	2.27	42.22	.74	1.74	79	732	57.34	1.42	2.48	41.50	.70	1.68
28	160	58.80	1.94	3.29	42.03	.75	1.78	80	734	57.40	2.00	3.49	39.62	1.08	2.72
29	C161	59.63	1.58	2.64	41.60	.42	1.00	81	743	59.03	2.84	4.81	42.32	.89	2.11
30	168	56.10	1.15	2.05	41.29	.70	1.71	82	744	56.53	1.61	2.84	42.90	.83	1.94
31	172	54.35	1.96	3.61	42.29	.98	2.31	83	745	57.71	2.07	3.59	41.54	1.09	2.62
32	185	57.17	.94	1.64	43.41	.61	1.41	84	749	58.10	1.22	2.10	42.08	.40	.94
33	224	56.23	1.62	2.87	40.91	.71	1.74	85	752	52.57	1.31	2.49	39.95	.67	1.68
34	233	55.00	1.35	2.45	41.77	1.19	2.85	86	757	55.52	1.52	2.74	44.46	.51	1.14
35	243	55.90	1.60	2.87	43.14	.61	1.42	87	759	58.04	1.27	2.19	44.13	.51	1.15
36	352	55.60	1.27	2.29	40.73	.72	1.78	88	762	55.01	1.11	2.02	42.67	.64	1.51
37	357	54.14	2.14	3.95	42.39	1.33	3.13	89	763	58.22	1.54	2.65	42.18	.42	1.01
38	359	56.24	1.93	3.43	43.71	.51	1.17	90	764	55.38	1.52	2.75	42.86	.58	1.36
39	367	53.70	1.33	2.48	39.34	.84	2.14	91	765	55.10	1.53	2.78	40.95	.62	1.52
40	374	58.85	1.82	3.09	44.22	.95	2.15	92	766	52.41	1.45	2.76	41.91	.82	1.95
41	375	56.81	1.45	2.55	43.29	.57	1.31	93	768	55.54	1.68	3.02	39.54	.81	2.06
42	377	55.63	2.00	3.59	41.08	.76	1.84	94	770	55.01	1.55	2.82	41.36	.72	1.73
43	381	55.97	1.47	2.63	41.68	.88	2.11	95	771	57.28	2.21	3.91	42.99	.68	1.59
44	382	53.42	1.42	2.67	41.03	.95	2.32	96	774	52.95	1.40	2.64	41.70	1.71	4.11
45	389	56.34	1.98	3.51	41.38	1.03	2.50	97	777	55.37	1.90	3.43	41.38	.94	3.77
46	393	55.55	2.04	3.67	40.17	1.19	2.95	98	781	55.65	.87	1.56	39.18	.65	1.65
47	395	55.22	1.75	3.16	42.01	.97	2.32	99	803	57.52	1.82	3.17	42.41	1.13	2.66
48	397	57.06	1.35	2.36	43.48	.37	.86	100	1780	53.90	.81	1.56	42.77	.64	1.49
49	400	55.65	2.00	3.59	39.82	.44	1.11								
50	401	54.05	2.18	4.03	43.24	.96	2.23	General means		56.09	1.62	2.89	41.82	.81	1.97
51	402	56.40	1.94	3.44	40.61	.62	1.53								
52	406	57.43	2.25	3.92	40.87	1.27	3.10								

From this table we note that:

1. There is a great diversity among these 100 birds in respect not only to the mean size of the eggs produced but also in respect to the uniformity of the product. Some of the birds, as, for example, Nos. 185, 444, 784, and 1780, lay eggs which are all very closely alike in size (i. e., show little variation). On the other hand, other birds lay eggs which are extremely variable. Examples of this are Nos. 61 (in length), 19, 709 (in length), 774 (in breadth). The relative variability of the egg is plainly a definite character of the individual.

2. There is, *on the average*, a considerable reduction in variability as we pass from the race to the individual. The mean length and mean breadth deduced from the individual means agree almost exactly, as they would be expected to, with the racial means from Series A. For the variabilities the case is different. Since the means are substantially identical, we may compare standard deviations directly. We then have for length—

Racial S. D. (2.39) − Mean individual S. D. (1.62) = 0.77

and for breadth—

Racial S. D. (1.38) − Mean individual S. D. (0.81) = 0.57

3. Put in another way, the mean individual standard deviation in length is 68 per cent of the racial standard deviation, or there is a reduction in variability in length of 32 per cent in passing from race to individual. The mean individual standard deviation in breadth is 59 per cent of the racial standard deviation. In this character the reduction in variability is 41 per cent. It is of interest to note that the reduction is greater for breadth than for length. Dr. Curtis also finds this to be the case in her work.

4. There are 4 hens whose eggs show a greater standard deviation in length than does a random sample of eggs of the race. That is, the eggs of these birds are absolutely more variable in length than the eggs of the flock of Barred Rock birds from which they originated. Among the 100 individuals none laid eggs more variable (absolutely) than those of the race in respect to breadth.

5. Looking at the matter from the standpoint of relative variability in proportion to mean size, as measured by the coefficient of variation, we find two birds (Nos. 709 and 743) whose eggs are relatively more variable than those of the race in respect to length. Three birds (Nos. 731, 774, and 777) lay eggs which are relatively more variable in breadth than those of the race.

6. While in the race, and on the average for individuals, the egg is more variable in length than in breadth, there are individual birds in whose eggs this rule is reversed. Out of the 100 birds, two (Nos. 20 and 414) lay an egg which is equally variable in length and breadth. Seven birds out of the 100 produce eggs which are more variable in breadth than in length.

In 22 cases of repeated like parts in the vegetable kingdom, including various characters of leaves of trees, etc., Pearson[1] found an average reduction of variability in passing from race to individual of about 13 per cent. This is obviously very much smaller than what we find for eggs. The results indicate that the eggs of an individual hen are, on the average, from three to four times more uniform in size than are the leaves from an individual tree. This difference is interesting because of the physiological differences of egg formation and leaf formation (for example). In the one case the product, as measured, is essentially the result of secretory activity, whereas in the other case the product is a result of cell division and differentiation.

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[1] Pearson, K., and others. Mathematical Contributions to the Theory of Evolution.—IX. On the Principle of Homotyposis and in Relation to Heredity, to the Variability of the Individual, and to that of the Race. Phil. Trans. Roy. Soc., Vol. 197A, pp. 285–379, 1901.

The data of Table 26 offer material for the study of a number of problems in regard to the formation of the egg. We hope later to return to the discussion of some of these. At present only one further matter will be considered, namely, the interindividual correlations. Is there a correlation in respect to absolute and relative variability between length and breadth of the egg? In other words, does the hen which lays an egg with a high absolute variability (standard deviation) in length also tend to show a high absolute variability in breadth? Is relative variability (coefficient of variation) in length correlated with relative variability in breadth?

To answer these questions correlation tables have been prepared from the data of Table 26. Table 27 shows the interindividual correlation in respect to mean length and mean breadth. Table 28 gives the correlation between the standard deviation for length and that for breadth, or, in other words, the interindividual correlation in respect to absolute variability between length and breadth. Table 29 gives the interindividual correlation in respect to relative variability; that is, the correlation between the coefficients of variation for length and breadth.

TABLE 27.—*Showing the correlation between individual mean egg length and individual mean egg breadth.*

	Egg breadth means.																
Egg length means.	37.5–37.9	38.0–38.4	38.5–38.9	39.0–39.4	39.5–39.9	40.0–40.4	40.5–40.9	41.0–41.4	41.5–41.9	42.0–42.4	42.5–42.9	43.0–43.4	43.5–43.9	44.0–44.4	44.5–44.9	45.0–45.4	Total.
52.0–52.4	.....	1	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	2
52.5–52.9	.....	.....	.....	.....	1	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	2
53.0–53.4	.....	.....	.....	.....	.....	.....	1	1	.....	.....	1	.....	.....	.....	.....	.....	3
53.5–53.9	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	2	.....	.....	.....	1	.....	4
54.0–54.4	.....	.....	.....	.....	.....	.....	1	2	2	2	.....	1	.....	.....	.....	.....	8
54.5–54.9	.....	.....	1	.....	.....	1	.....	2	1	.....	1	1	.....	.....	.....	.....	7
55.0–55.4	.....	.....	.....	.....	.....	.....	1	4	2	3	3	.....	.....	.....	.....	.....	13
55.5–55.9	.....	.....	.....	1	2	1	2	1	1	3	1	1	.....	1	.....	.....	14
56.0–56.4	.....	.....	1	.....	.....	2	2	3	3	1	1	.....	1	.....	.....	.....	14
56.5–56.9	.....	.....	.....	.....	.....	.....	1	.....	1	.....	1	1	.....	.....	.....	.....	4
57.0–57.4	.....	.....	.....	.....	1	.....	1	1	2	.....	1	2	.....	.....	.....	.....	8
57.5–57.9	.....	.....	.....	.....	.....	.....	.....	1	1	1	3	.....	.....	.....	.....	.....	6
58.0–58.4	.....	.....	.....	.....	.....	.....	.....	.....	.....	2	1	.....	.....	1	1	.....	5
58.5–58.9	.....	.....	.....	.....	.....	.....	.....	.....	1	1	1	1	.....	1	.....	.....	5
59.0–59.4	.....	.....	.....	.....	.....	1	.....	.....	.....	1	.....	.....	.....	.....	.....	1	3
59.5–59.9	.....	.....	.....	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	.....	.....	2
Total....	0	1	2	2	4	5	9	15	17	15	16	7	1	3	2	1	100

Table 28.—*Showing the correlation between individual standard deviations for egg length and egg breadth.*

Egg length standard deviations.	Egg breadth standard deviations.								Total.
	0.30–0.49	0.50–0.69	0.70–0.89	0.90–1.09	1.10–1.29	1.30–1.49	1.50–1.69	1.70–1.89	
0.70–0.89		2							2
.90–1.09		3	2						5
1.10–1.29	3	4	5						12
1.30–1.49	2	3	8	7	2			1	23
1.50–1.69	2	9	6	2	1	1			21
1.70–1.89		3	3	3	4				13
1.90–2.09	1	3	3	6	1				14
2.10–2.29		1		2	1	1			5
2.30–2.49			1	1	1				3
2.50–2.69			1						1
2.70–2.89			1						1
Total....	8	28	30	21	10	2	0	1	100

Table 29.—*Showing the correlation between individual egg length coefficients of variation and individual egg breadth coefficients of variation.*

Egg length coefficients of variation.	Egg breadth coefficients of variation.																		Total.
	0.70–0.89	0.90–1.09	1.10–1.29	1.30–1.49	1.50–1.69	1.70–1.89	1.90–2.09	2.10–2.29	2.30–2.49	2.50–2.69	2.70–2.89	2.90–3.09	3.10–3.29	3.30–3.49	3.50–3.69	3.70–3.89	3.90–4.09	4.10–4.29	
1.50–1.69				2	1														3
1.70–1.89			1			1													2
1.90–2.09				1	1	1	1												4
2.10–2.29		1	2	1	1	2	1												8
2.30–2.49	1	2	1		3	1	1	1			1	1							12
2.50–2.69		2		1	1	1	1	3	4	1								1	15
2.70–2.89			2	4	3	2	3		1					1					16
2.90–3.09				1	1		1	1				1							5
3.10–3.29			1		1	3		1	2	1	1	1							11
3.30–3.49			1		1		1		1		1					1			6
3.50–3.69			1		1	1			2	1		1							7
3.70–3.89							1		1										2
3.90–4.09					1			1				1	1						4
4.10–4.29						1		1				1							3
4.30–4.49																			0
4.50–4.69																			0
4.70–4.89							2												2
Total...	1	5	9	10	15	13	12	8	11	3	3	6	1	1	0	1	0	1	100

The constants deduced from these tables are given in Table 30. The slight differences in the means in this table from those at the foot of Table 26 are due simply to the grouping of the data.

Table 30.—*Constants of interindividual variation and correlation in length and breadth of eggs.*

Intraindividual constant.	Mean.	Standard deviation.
Mean length	[1] 56.02±.1.14	1.68±.08
Mean breadth	41.82±.09	1.30±.06
Standard deviation for length	1.62±.03	.38±.02
Standard deviation for breadth	.82±.02	.25±.01
Coefficient of variation for length	2.88±.04	.66±.03
Coefficient of variation for breadth	1.95±.04	.64±.03

Correlation between length and breadth, intraindividual means	$r=+.300\pm.061$
Correlation between intraindividual standard deviations for length and breadth	$r=+.301\pm.061$
Correlation between intraindividual coefficients of variation for length and breadth	$r=+.327\pm.060$

All probable errors in this table are calculated on the basis of $n=100$.

From this table it will be noted that:

1. The length constants are in all cases more variable than the breadth constants. This means biologically (*a*) that if we determine the mean length and breadth of a series of eggs from each of a number of birds we shall find more variation among the mean lengths than among the mean breadths, just as in the eggs of one bird or of a flock we usually find a greater variation in length of egg than in breadth; (*b*) that if in each of our series of eggs we calculate the standard deviation and coefficient of variation for the length and for the breadth of the eggs of each individual bird, we shall find that the series of standard deviations and coefficients of variation vary more among themselves in the case of length of egg than in the case of breadth of egg. In other words, not only is the hen's egg generally more variable in length than in breadth, but in passing from individual to individual there is a greater variation in respect to degree of variability—or uniformity, as one pleases—in length than in breadth.

2. The interindividual correlation in mean length and mean breadth is higher than the general intraracial coefficient for the same two characters (Table 14). This is to be expected. It means that in general a hen which characteristically lays an egg longer than the average also lays an egg which tends to be wider than the average; that is, a relatively large egg altogether.

3. The same sort of correlation holds in regard to the variability of the product. A hen which characteristically lays an egg which is unusually variable in length also exhibits an unusual degree of variation in the breadth of her product.

The results of this section carry an important practical suggestion. Those having to do with the development and organization of the marketing side of the poultry industry are all the time laying more and more stress on the importance of quality to the producer. The farmer and poultryman is urged to grade his eggs carefully before marketing them. So far as size and shape go, eggs are graded before anything else for uniformity. It is commonly assumed that lack of uniformity in eggs is primarily the result of having many different birds in the flock each of which lays an egg more or less different from every other bird's eggs. The results set forth above show that this is not the whole story. A study of Table 26 will very quickly convince one that there is a considerable portion of the flock composed of individuals which are constitutionally incapable of laying a uniform egg. Variability of product is as definite a characteristic of such hens as their color. On the other hand, there are hens whose eggs are remarkably and characteristically uniform. It is plain that these facts offer a hopeful suggestion to the breeder who is desirous of producing the highest quality of market eggs at the lowest cost. His aim will be to breed a pure strain of birds characterized not only by the average excellence but also by the uniformity of their product. It should finally be said that while the facts set forth above deal only with size characters of eggs, unpublished results obtained in this laboratory show that the same considerations apply to other characters of eggs.

SUMMARY.

The object of the present part of this bulletin is to present quantitative data, together with a biometrical analysis of the same, regarding the normal variation and correlation of the egg of the domestic fowl in respect primarily to size and shape. Such data are of value in connection with all physiological and genetic studies in regard to the production of eggs. They also have a significant relation to the economic problem of the improvement of eggs in respect to quality and uniformity by breeding. The material used in this study comprised a considerable number of eggs of Barred Plymouth Rock pullets belonging to the Maine Station flock. The material was strictly homogeneous, and particularly well adapted in all respects for the study of the normal variation of the hen's egg. It will be understood that the results and conclusions of this investigation, while stated in general form, are not presumed to have a wider generality than the material on which they are based. It is probable that many of the results found for the egg of the Barred Plymouth Rock will also obtain for the eggs of other breeds.

Among the more important results obtained in this study the following may be mentioned:

1. The egg is, to a significant degree, relatively more variable in length than in breadth. The egg is considerably more variable in shape, as measured by the length-breadth index, than it is in either of the linear dimensions length or breadth. The weight of the egg and its volume (determined from length and breadth on the assumption that the egg is a perfect prolate spheroid) are much more variable than any of the other characters. (Pp. 117–180.)

2. There is a close agreement between the egg of the domestic fowl and the human skull in respect to the degree of variation exhibited in the corresponding size and shape characters of the two structures. A consideration of the processes concerned in the production of the definitive size and shape of the egg and the skull suggests, though it does not prove, that this similarity with reference to variation constants is not fortuitous, but rests upon a basis of general similarity in regard to mechanical factors operative in both cases during the development and fixation of the final form. (Pp. 180–183.)

3. With the exception of breadth, all other dimensional characters studied show significantly skew (asymmetrical) variation. The different egg dimensions give frequency distributions following different

ones of Pearson's curve types. Variation of the egg in length and in shape (length-breadth index) follows a curve of Type IV; variation in breadth Type II, and variation in volume Type I. (Pp. 183–189.)

4. All of the egg dimensions studied are positively correlated with each other in varying degree. Intraracially egg length and egg breadth are correlated only slightly. Furthermore, neither the weight nor the volume of the egg is more than very slightly, if significantly at all, correlated with its shape. As would be expected, both length and breadth are significantly correlated with the weight and the volume of the egg. The bulk measures (volume and weight) are more highly correlated with breadth than with length. There is a very high correlation between volume and weight. This implies what was found in fact to be the case, that the specific gravity of the egg exhibits only a very small degree of variation. There is no significant net correlation between the shape of an egg and its absolute size. (Pp. 190–200.)

5. The skull and the egg show a correspondence in respect to the degree to which their chief dimensions are correlated. The correlation results parallel those already set forth in paragraph 2 above for simple variation. (Pp. 200–203.)

6. A detailed study of the regression of the weight of the egg on length and breadth results in certain equations from which the former quantity can be estimated from a knowledge of the linear dimensions of the egg with a considerable degree of accuracy. (Pp. 203–225.)

7. A comparison of the egg of the domestic fowl with those of a number of species of wild birds in respect to relative variability leads to the result that in general the eggs of wild birds are neither less nor more variable than those of the domestic bird. It is concluded that probably the essential factors concerned in the determination of the degree of variation in size and shape exhibited by birds' eggs are purely physiological and in no direct or immediate way (if at all) related to the action of natural selection. It does not appear from the evidence that it is essential to the survival of a wild bird that its egg shall be less variable than those of the domestic fowl, in which egg variation obviously has no survival value whatsoever. (Pp. 225–228.)

8. A study of intraindividual variation indicates that the relative variability of eggs is a definite characteristic of the individual, some birds characteristically laying eggs which are extremely variable and other birds characteristically producing eggs which are very uniform. There is on the average a considerable reduction in variability in passing from the race to the individual. This amounts in the case of length to 32 per cent and in the case of breadth to 41 per cent. Some individuals may show a greater variability both

absolute and relative than a random sample of the eggs of the race from which the individual comes. The rule that length is more variable than breadth is reversed in the eggs of some individuals. Seven per cent of the birds studied produce eggs which are more variable in breadth than in length. In general a hen which characteristically lays an egg longer than the average also lays an egg which tends to be wider than the average. The same sort of correlation holds in regard to the variability of the product. A hen which characteristically lays an egg which is unusually variable in length also exhibits an unusual degree of variation in the breadth of her product. (Pp. 228–234.)

APPENDIX III.

In Table I of this appendix are given the original measurements for the eggs of Series A used in this investigation. These measurements are published here in the hope that they may be useful to other students of the physiology of egg production, and in general as a set of basic records of the normal variation of the egg in a homogeneous flock of domestic fowls.

TABLE I.—*Giving the original measurements of the eggs comprising Series A of this bulletin (cf. p. 176).*

Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.	Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.
	mm.	*mm.*	*gms.*	*Per ct.*	*c. c.*		*mm.*	*mm.*	*gms.*	*Per ct.*	*c. c.*
1	59.10	42.15	56.88	71.32	55.00	54	58.39	41.18	54.35	70.53	51.84
2	56.64	42.64	57.35	75.28	53.92	55	50.96	40.74	47.18	79.95	44.29
3	53.40	41.46	51.24	77.27	48.06	56	57.19	42.17	56.60	73.74	53.22
4	57.06	42.25	56.51	74.05	53.35	57	53.57	38.94	45.39	72.69	42.52
5	56.94	42.77	58.61	75.12	54.56	58	55.08	41.91	54.16	76.09	50.68
6	53.54	41.29	51.52	77.11	47.81	59	54.58	40.54	50.12	74.28	46.97
7	56.46	43.18	58.57	76.48	55.11	60	58.07	40.76	54.19	70.19	50.52
8	54.55	40.16	49.81	73.27	46.07	61	55.95	42.97	58.51	76.80	54.13
9	57.51	40.65	54.07	70.67	49.79	62	51.51	42.81	53.62	83.12	49.41
10	65.64	41.19	52.02	74.03	49.40	63	55.50	41.87	53.15	75.44	50.97
11	55.25	42.28	54.92	76.52	51.72	64	52.32	40.93	48.91	78.23	45.87
12	53.07	42.08	52.19	79.29	49.21	65	54.14	42.96	54.63	79.35	52.32
13	56.38	39.06	51.26	69.28	45.04	66	54.60	43.82	58.71	80.26	54.89
14	55.75	42.76	56.65	76.70	53.38	67	54.36	42.40	55.58	78.00	51.17
15	51.26	38.66	42.98	75.42	40.11	68	59.79	40.71	55.20	68.09	51.90
16	57.71	42.10	56.33	72.95	53.57	69	51.08	43.43	53.88	85.02	50.47
18	53.21	42.03	52.96	79.99	49.25	70	55.68	45.18	62.72	81.14	59.51
19	57.27	40.35	53.40	70.46	48.85	71	56.46	43.97	62.80	77.88	57.13
20	54.74	40.55	51.60	74.08	47.15	72	54.82	41.60	53.00	75.88	49.61
21	57.76	42.49	59.60	73.56	54.63	73	55.98	43.00	57.24	76.81	54.20
22	58.27	41.61	56.87	71.41	52.81	74	54.90	43.25	58.25	78.78	53.79
23	56.94	41.90	55.42	73.59	52.34	75	58.75	42.37	59.24	72.12	55.26
24	52.82	39.74	45.42	75.24	43.68	76	55.02	39.94	50.28	72.60	49.96
25	56.80	39.92	49.73	70.28	47.39	77	52.20	42.49	53.14	81.40	49.37
26	57.73	40.54	54.82	70.22	49.69	78	55.50	42.35	56.74	76.31	52.14
27	57.07	41.81	55.72	73.26	52.22	79	59.44	41.13	55.56	69.20	52.69
28	57.75	41.31	55.56	71.53	51.64	80	54.44	40.27	49.29	73.97	46.25
29	56.73	40.44	51.74	71.29	48.59	81	54.24	41.62	52.49	76.93	49.19
30	56.54	40.87	52.84	72.29	49.47	82	53.85	42.32	54.89	78.59	50.51
31	55.71	41.23	51.97	74.01	49.62	83	58.59	41.12	54.79	70.18	51.86
32	57.02	44.01	61.41	77.18	57.80	84	52.88	41.64	51.93	78.74	48.01
33	58.66	41.05	57.13	71.00	53.31	85	53.10	42.05	51.15	79.19	49.18
34	51.46	43.36	54.63	84.26	50.66	86	56.70	42.83	59.25	75.54	54.48
35	57.80	41.93	56.79	72.54	53.23	87	54.56	40.15	49.34	73.59	46.07
36	62.13	38.76	52.06	62.39	48.88	88	56.56	41.90	55.09	94.08	51.99
37	58.36	41.44	55.53	71.02	52.48	89	57.25	39.63	48.61	69.22	47.11
38	57.60	44.25	62.89	76.82	59.08	90	57.45	42.06	56.71	73.21	53.22
39	57.79	44.10	61.12	76.31	58.84	91	57.63	41.73	55.77	72.41	52.58
40	57.33	42.93	59.42	77.88	55.36	92	54.67	41.52	53.35	75.95	49.36
41	67.92	41.88	66.54	61.66	62.37	93	58.37	42.82	59.88	73.36	56.03
42	54.80	40.46	51.25	73.84	46.97	94	61.05	43.87	67.53	71.86	61.56
43	56.89	42.63	56.89	74.93	51.15	95	56.73	43.17	61.01	76.10	55.39
44	59.47	42.03	58.44	70.67	55.74	96	54.41	38.14	45.65	70.10	41.43
45	58.28	43.00	60.60	73.78	56.42	97	55.48	40.02	50.26	72.13	46.53
46	60.17	40.95	57.50	68.06	52.85	98	54.37	41.11	52.43	75.61	48.14
47	57.98	41.74	55.69	71.99	52.89	99	61.10	40.60	55.87	64.45	52.73
48	55.34	40.50	51.07	73.18	47.53	100	58.56	43.17	62.29	73.72	57.17
49	57.83	42.99	59.41	74.34	55.95	101	55.77	43.76	61.38	78.47	55.93
50	57.12	41.57	55.74	72.78	51.66	102	56.39	40.77	54.09	72.30	49.09
51	54.42	41.28	52.08	75.85	48.56	103	63.50	42.58	64.92	65.59	60.28
52	55.00	44.50	61.22	80.90	57.03	104	57.74	41.03	54.70	71.06	50.92
53	58.60	43.56	61.69	74.33	58.22	105	56.31	43.53	59.72	77.30	55.85

TABLE I.—*Giving the original measurements of the eggs comprising Series A of this bulletin (cf. p. 176)*—Continued.

Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.	Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.
	mm.	*mm.*	*gms.*	*Per ct.*	*c. c.*		*mm.*	*mm.*	*gms.*	*Per ct.*	*c. c.*
106	56.66	40.45	53.10	71.39	48.57	183	54.45	40.40	50.11	74.20	46.54
107	63.33	40.53	57.36	64.00	54.50	184	56.41	42.46	56.51	75.27	53.24
108	63.06	44.26	69.47	70.19	64.68	185	60.61	41.87	60.26	69.08	55.65
109	54.48	42.01	54.71	77.11	50.36	186	54.86	41.56	52.82	75.76	49.61
110	56.56	41.90	55.35	74.08	51.99	187	62.49	41.51	60.52	66.43	56.39
111	57.03	42.67	58.29	74.82	54.40	188	56.07	41.36	53.83	73.76	50.23
112	57.52	40.30	53.43	70.06	48.91	189	58.44	42.35	60.24	72.47	54.90
113	57.33	42.47	58.74	74.08	54.18	190	53.27	42.51	54.06	79.80	50.44
114	56.58	41.88	56.62	74.02	51.96	191	56.46	43.59	60.15	77.20	56.14
115	56.53	41.06	53.30	72.63	49.91	192	56.59	40.96	52.89	72.38	49.70
116	56.34	39.74	50.53	70.53	46.59	193	55.88	42.16	56.45	75.35	52.00
117	58.89	42.17	59.44	71.60	54.85	194	55.62	40.34	52.27	72.53	47.39
118	56.19	41.88	56.51	74.53	51.59	195	56.30	40.31	50.64	71.59	47.87
119	54.99	41.92	54.72	76.23	50.59	196	57.29	42.09	55.72	73.47	53.12
120	55.19	41.97	55.14	76.05	50.92	197	54.97	42.66	56.06	77.61	52.39
121	55.65	44.02	59.76	79.10	56.42	198	53.43	42.07	52.81	78.74	49.55
122	55.11	42.09	56.00	76.37	51.10	199	53.36	42.49	54.26	79.63	50.42
123	59.18	42.68	61.70	72.12	56.44	200	56.35	40.47	51.47	71.82	48.36
124	59.35	43.81	64.65	73.82	59.63	201	57.71	40.12	50.40	69.52	48.65
125	53.03	42.41	53.00	79.98	49.93	202	52.62	42.56	53.63	80.88	49.91
126	57.07	40.93	53.80	71.71	50.04	203	58.41	43.88	62.62	75.12	58.88
127	54.65	40.43	49.60	73.98	46.80	204	54.95	40.28	49.35	73.30	46.69
128	57.65	43.96	63.31	76.26	58.34	205	54.63	42.83	56.11	78.40	52.51
129	59.14	41.01	56.03	69.34	52.10	206	56.55	42.29	56.88	74.74	52.94
130	53.79	41.74	54.00	77.60	49.06	207	56.52	41.57	54.12	73.55	51.16
131	54.03	43.03	56.00	79.64	52.42	208	57.00	42.24	56.83	74.00	53.25
132	55.32	40.37	49.72	72.98	47.18	209	61.06	40.53	56.09	66.38	52.24
133	57.35	41.85	56.52	72.98	52.61	210	55.38	40.60	51.80	73.31	47.80
134	57.31	42.57	57.44	74.28	54.41	211	57.84	41.62	56.26	71.96	52.46
135	53.03	40.37	48.68	76.13	45.28	212	53.10	42.12	52.99	79.32	49.33
136	55.50	41.58	54.14	74.92	50.24	213	57.12	40.80	52.31	71.43	49.79
137	55.45	44.48	62.37	80.22	57.45	214	53.42	41.38	51.55	77.46	47.89
138	58.81	44.43	65.80	75.54	60.80	215	57.18	41.92	56.39	73.31	52.61
139	57.33	46.34	68.48	80.84	64.47	216	56.82	42.70	58.18	75.15	54.24
140	54.46	39.79	48.65	69.40	45.12	217	56.70	43.35	59.29	76.45	55.82
141	57.63	42.03	57.87	72.93	53.34	218	56.21	40.82	52.34	72.62	49.03
142	55.63	41.24	53.34	74.13	49.55	219	56.32	41.86	55.04	74.32	51.67
143	56.84	45.07	65.72	79.29	60.48	220	58.08	42.90	60.14	73.86	55.97
144	59.73	40.03	52.63	67.02	50.15	221	55.55	40.03	50.43	72.06	46.64
145	61.04	40.79	56.81	66.82	53.15	222	52.70	40.79	48.22	77.40	45.89
146	55.65	41.57	53.97	74.70	50.34	223	55.65	41.94	55.25	75.36	51.26
147	57.07	42.51	57.78	74.49	54.03	224	54.86	43.53	58.01	79.35	54.45
148	51.30	40.92	47.37	79.77	45.98	225	52.38	42.12	51.66	80.42	48.66
149	54.66	43.29	58.28	79.20	53.66	226	58.12	42.78	61.63	73.61	55.69
150	57.72	41.95	57.25	72.68	53.21	227	54.77	41.93	52.61	76.56	50.43
151	56.47	40.02	50.95	70.87	47.36	228	53.80	40.62	49.63	75.50	46.48
152	55.86	41.47	54.24	74.24	50.32	229	55.88	42.40	55.14	75.88	52.60
153	59.26	45.23	69.51	76.32	63.50	230	56.33	44.54	61.87	79.07	58.52
154	60.29	40.39	56.36	66.72	51.46	231	52.93	42.04	51.96	79.43	48.99
155	55.33	41.52	53.86	75.04	49.95	232	56.25	42.46	57.09	75.48	53.10
156	56.66	40.87	53.40	72.13	49.68	233	56.88	41.09	53.29	72.24	50.26
157	56.49	44.26	61.56	78.35	57.93	234	54.54	41.48	51.99	76.05	49.14
158	57.57	43.25	61.73	75.13	56.42	235	58.86	45.52	66.99	77.34	63.85
159	54.56	40.98	50.08	75.11	47.98	236	58.66	43.19	60.63	73.63	57.27
160	59.29	42.60	61.15	71.85	56.33	237	54.08	41.85	53.31	77.39	49.62
161	53.36	40.98	49.94	76.80	46.92	238	52.50	42.26	51.14	80.49	49.09
162	57.16	42.37	58.01	74.13	53.25	239	55.16	43.70	58.48	79.22	55.16
163	56.20	43.13	59.84	76.74	54.76	240	55.00	39.93	48.20	72.60	45.94
164	64.91	39.03	55.94	60.13	51.81	241	52.55	41.37	49.06	75.28	47.12
165	55.55	41.81	54.97	75.27	50.83	242	53.39	40.19	48.37	78.73	45.12
166	60.18	41.80	58.69	69.49	54.00	243	57.47	41.39	54.84	72.02	51.53
167	58.00	39.12	50.19	67.45	46.47	244	54.06	43.04	56.07	79.61	52.43
168	60.13	41.61	59.83	69.20	54.52	245	57.04	40.63	53.24	71.23	49.33
169	60.98	44.99	68.52	73.78	64.59	246	56.40	38.38	47.13	68.05	43.50
170	54.40	41.82	53.37	76.88	49.81	247	56.58	40.85	52.44	72.20	49.46
171	58.40	43.17	61.29	73.92	57.01	248	59.75	42.93	62.20	71.85	57.69
172	60.29	39.64	52.10	65.75	49.61	249	52.20	40.96	49.32	78.47	45.85
173	57.52	41.71	55.71	72.51	52.42	250	62.07	40.75	59.47	65.65	54.00
174	54.47	41.92	53.24	76.96	50.13	251	56.37	39.90	49.52	70.78	46.98
175	56.72	42.75	56.97	75.37	54.30	252	56.40	42.98	57.88	76.20	54.50
176	54.95	40.70	50.05	74.07	47.67	253	57.56	41.75	56.25	72.53	52.56
177	52.46	43.40	55.16	82.73	51.74	254	57.62	43.53	60.58	75.55	57.19
178	58.18	42.40	58.77	72.88	54.76	255	51.30	42.47	52.19	82.79	48.47
179	55.15	42.86	56.77	77.72	53.06	256	53.32	39.42	47.94	63.93	43.38
180	59.70	44.39	65.07	74.36	61.56	257	56.21	42.00	55.66	74.72	51.91
181	60.11	38.77	50.45	64.50	47.29	258	55.72	42.12	54.90	75.59	51.76
182	55.85	39.76	48.95	71.19	46.24	259	58.09	40.54	53.94	69.79	49.98

TABLE I.—*Giving the original measurements of the eggs comprising Series A of this bulletin (cf. p. 176)*—Continued.

Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.	Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.
	mm.	*mm.*	*gms.*	*Per ct.*	*c. c.*		*mm.*	*mm.*	*gms.*	*Per ct.*	*c. c.*
260	56.03	42.02	54.59	75.00	51.81	339	55.77	43.07	56.49	77.23	54.20
261	56.39	40.74	51.38	72.25	48.99	340	59.61	42.39	59.59	71.11	56.07
262	53.99	43.00	55.97	79.64	52.26	341	54.58	41.34	51.07	75.74	48.84
263	56.46	41.23	53.59	73.02	50.23	342	55.85	42.01	54.65	75.22	51.64
264	55.95	44.14	60.07	78.89	57.09	343	54.75	42.35	55.19	77.35	51.45
265	56.75	43.19	60.04	76.10	55.41	344	54.23	40.69	49.03	75.03	46.99
266	57.79	42.85	59.45	74.53	55.58	345	55.36	40.52	51.99	73.19	47.59
267	54.54	42.29	54.23	77.54	51.04	346	54.53	41.84	54.34	76.73	49.99
268	53.02	42.76	52.94	80.65	50.76	347	54.58	40.73	49.58	74.62	47.43
269	56.68	40.46	50.59	71.38	48.58	348	60.64	44.76	66.95	73.81	63.61
270	55.04	42.24	54.58	76.74	51.42	349	57.51	39.30	49.73	68.34	46.52
271	55.86	42.97	57.96	76.93	54.03	350	54.03	42.15	52.98	78.01	50.29
272	54.48	41.70	52.87	76.54	49.60	351	54.74	43.04	56.56	78.63	53.09
273	56.68	41.32	53.14	72.90	50.67	352	54.93	40.69	51.44	74.08	47.60
274	55.01	41.87	54.08	76.11	50.51	353	55.98	39.46	46.70	70.49	45.64
275	54.84	40.77	48.39	74.34	47.75	354	59.73	42.40	60.55	70.99	56.23
276	53.74	43.17	55.25	80.33	52.41	355	52.55	43.14	53.45	82.09	51.22
277	57.01	42.51	56.95	74.56	53.96	356	52.72	43.45	54.57	82.42	52.14
278	58.14	39.39	49.98	67.75	47.21	357	55.32	43.05	56.28	77.82	53.71
279	55.91	41.88	54.06	74.91	51.35	358	56.41	41.21	53.47	73.05	50.17
280	53.69	44.43	57.39	82.75	55.46	359	57.39	40.69	52.94	70.90	49.72
281	55.09	43.66	58.41	79.25	54.97	360	53.93	39.35	46.06	72.96	43.75
282	56.41	42.42	56.23	75.20	53.14	361	55.63	42.54	55.29	76.47	52.72
283	57.38	42.86	59.52	74.70	54.23	362	56.21	42.41	55.99	75.45	52.95
284	57.60	40.18	52.14	69.76	48.69	363	55.73	42.02	54.61	75.40	51.53
285	53.98	40.40	48.86	74.84	46.13	364	54.87	42.46	53.72	77.38	51.80
286	56.38	42.55	57.03	75.47	53.47	365	57.13	40.48	51.73	70.68	49.02
287	56.78	44.60	62.33	78.55	59.14	366	56.54	41.77	52.64	73.88	51.67
288	58.12	45.81	68.09	78.82	63.83	367	59.48	43.18	62.04	72.60	58.06
289	49.77	40.48	43.66	71.33	42.71	368	55.93	41.24	53.64	74.06	49.81
290	57.68	43.77	59.69	75.88	57.83	369	52.64	40.13	46.42	76.23	44.36
291	58.26	41.10	53.13	70.54	51.53	370	63.47	42.84	64.43	67.50	61.00
292	53.32	41.95	51.49	78.67	49.15	371	55.40	42.97	57.28	77.56	53.16
293	53.95	41.35	50.29	76.64	48.33	372	53.96	41.25	51.03	76.44	48.09
294	52.86	41.21	50.41	77.96	46.98	373	59.40	42.14	57.48	70.94	55.23
295	56.96	40.27	51.67	70.70	48.39	374	55.27	42.27	54.93	76.48	51.74
296	53.43	43.55	61.07	81.51	53.09	375	56.71	43.34	58.90	76.42	55.68
297	53.31	40.83	50.03	76.59	46.56	376	59.67	42.22	59.20	70.75	55.70
298	53.83	42.89	52.18	79.68	51.88	377	58.42	43.61	61.38	74.65	58.14
299	54.66	39.82	47.72	72.85	45.38	378	54.98	41.90	53.51	76.21	50.29
300	54.89	40.66	48.81	74.07	47.52	379	57.65	41.85	55.61	72.59	52.90
301	57.19	41.49	55.93	72.55	49.06	380	53.27	40.91	47.31	76.80	46.71
302	59.16	42.26	58.26	71.43	55.32	381	56.67	44.05	60.72	77.73	57.61
303	55.98	42.50	56.85	75.92	52.94	382	52.99	40.11	47.13	75.69	44.65
304	54.24	40.83	49.61	75.28	47.37	383	59.65	40.13	53.20	67.27	50.33
305	52.96	41.76	51.70	78.85	48.36	384	52.60	40.70	47.74	77.38	45.62
307	63.89	42.68	64.52	66.80	60.93	385	54.00	40.26	48.43	74.55	45.83
308	63.00	39.68	54.59	62.98	51.94	386	57.58	42.42	56.47	73.67	54.25
309	58.74	45.24	66.56	77.01	62.95	387	55.60	42.42	54.79	76.29	52.39
310	58.23	43.44	60.65	74.60	57.54	388	57.08	41.72	53.04	73.09	52.02
311	58.02	39.94	51.71	68.84	48.46	389	56.95	40.05	51.27	70.32	47.86
312	55.54	40.22	48.99	72.42	47.14	390	57.06	41.67	55.14	73.03	51.90
313	58.48	43.38	60.88	74.18	57.62	391	52.63	43.81	56.86	83.24	52.51
314	57.69	43.86	62.23	76.03	58.09	392	57.02	39.88	50.78	69.94	47.48
315	56.02	40.85	52.25	72.92	48.92	393	57.73	41.24	53.59	71.43	51.42
316	57.69	41.84	55.71	72.53	52.87	394	57.11	42.98	59.29	75.26	55.25
317	57.38	40.43	52.29	70.46	49.13	395	55.97	45.38	62.62	81.08	60.36
318	56.69	43.45	59.39	76.64	56.05	396	56.61	43.94	60.49	77.62	57.23
319	55.19	42.33	54.19	76.70	51.79	397	56.87	43.18	58.72	75.93	55.53
320	55.68	44.74	61.94	80.35	58.36	398	54.50	40.09	48.47	73.56	45.84
321	55.09	41.03	50.51	74.48	48.59	399	56.60	42.78	56.65	75.58	54.24
322	54.88	42.28	54.22	77.04	51.37	400	59.16	43.32	64.54	73.22	58.13
323	55.85	42.01	55.40	75.22	51.64	401	57.25	42.79	59.72	74.74	54.87
324	57.85	43.60	61.31	75.37	57.59	402	54.44	43.27	56.76	79.48	53.40
325	53.69	41.23	49.04	76.80	47.80	403	52.05	40.54	47.67	77.89	44.80
326	57.68	42.12	56.84	73.02	53.58	404	54.93	42.00	52.41	76.46	50.74
327	59.09	41.94	57.12	70.98	54.41	405	57.85	44.19	62.89	76.39	59.13
328	58.62	41.50	55.06	70.80	52.86	406	59.03	40.13	53.52	67.98	49.81
329	57.53	43.42	60.26	75.47	56.78	407	56.24	40.45	50.17	71.92	48.20
330	53.30	42.60	53.65	79.92	50.65	408	54.77	43.66	57.89	79.71	54.67
331	54.56	42.00	52.74	76.98	50.39	409	56.92	41.79	54.69	73.42	52.02
332	54.94	43.19	55.45	78.61	53.64	410	55.36	41.48	52.54	74.93	49.87
333	57.24	41.76	56.65	72.96	52.27	412	54.41	42.46	54.25	78.04	51.35
334	57.35	42.37	56.08	73.88	53.89	413	57.89	42.13	57.94	72.77	53.82
335	55.80	41.98	55.17	75.23	51.49	414	56.76	42.12	55.04	74.21	52.73
336	53.97	39.51	46.29	73.21	44.13	415	52.22	41.43	48.56	79.34	46.95
337	54.51	42.04	53.73	77.12	50.45	416	60.19	42.32	58.92	70.31	56.43
338	59.73	42.11	58.67	70.50	55.49	417	55.20	42.95	57.63	77.81	53.34

TABLE I.—*Giving the original measurements of the eggs comprising Series A of this bulletin (cf. p. 176)*—Continued.

Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.	Egg No.	Length.	Breadth.	Weight.	Index.	Bulk.
	mm.	*mm.*	*gms.*	*Per ct.*	*c. c.*		*mm.*	*mm.*	*gms.*	*Per ct.*	*c. c.*
418	56.65	43.49	58.54	76.77	56.09	436	57.57	41.82	54.78	72.64	52.73
419	54.58	41.60	54.74	76.22	49.46	437	57.70	40.73	53.60	70.59	50.14
420	54.11	42.21	53.61	78.00	50.46	438	57.44	41.41	53.61	72.09	51.59
421	56.48	43.37	57.46	76.79	55.65	439	54.45	41.60	52.16	76.40	49.44
422	57.27	40.63	51.53	70.94	49.60	440	54.47	39.74	46.27	72.96	45.05
423	59.51	41.71	57.15	70.09	54.24	441	56.48	39.96	49.06	70.75	47.22
424	57.37	43.78	61.51	76.31	57.56	442	54.24	43.53	56.23	80.25	53.84
425	59.13	42.79	60.02	72.36	56.67	443	54.17	41.89	52.37	77.33	49.74
426	57.15	42.91	58.83	75.08	55.13	444	56.22	39.77	49.03	70.74	46.59
427	56.79	44.03	60.50	77.53	57.66	445	58.11	43.32	60.77	74.55	57.11
428	55.27	44.03	58.28	79.66	56.13	446	54.04	43.29	57.08	80.11	53.00
429	55.98	43.53	57.71	77.76	55.57	447	51.46	40.35	48.51	78.41	43.89
430	59.52	40.83	53.80	68.60	51.98	448	55.60	40.15	47.92	72.21	46.95
431	59.05	43.41	62.17	73.51	58.30	449	58.12	41.15	55.90	70.80	51.56
432	53.82	43.74	56.50	81.27	53.91	450	54.11	40.76	48.25	75.33	47.08
433	55.58	42.51	54.73	76.48	52.62	451	56.82	43.29	58.71	76.19	55.73
434	54.80	39.06	46.58	71.28	43.78	452	59.56	41.02	56.73	68.87	52.47
435	59.79	44.87	67.99	75.04	63.05	453	58.85	40.81	55.67	69.35	51.30

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